The Population Vector Could Implement Approximately Bayesian Inference

There is considerable interest in understanding how the brain might use populations of spiking neurons to encode, communicate, and combine sources of information optimally, as specified by Bayesian inference. For instance, Kording and Wolpert, in a 2004 paper, showed that performance on a sensorimotor task was consistent with optimal combination of sensory input and the statistics of the task that were learned during training, while Ma and colleagues in a 2008 paper proposed a neural modeling framework according to which Bayesian inferences could be computed. These works focused on the form in which inputs were combined to produce the posterior mean and variance.  $W_{i}$ show that population vectors based on point process inputs combine evidence in a form that closely resembles Bayesian inference, with each input spike carrying information about the tuning of the input neuron. We have investigated performance of population vector-based inference with various tuning functions. While it is exactly Bayesian for von Mises tuning functions, it remains approximately Bayesian for many other cases. We also suggest that encoding stability within short epochs of time could lead to nearly optimal sensorimotor integration.

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## Summary of Comments on Bayes\_PVA\_JO\_early\_draft\_Jan19.pdf

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Author: loaner Subject: Sticky Note Date: 1/29/2016 11:23:17 AM This seems to be a beginning of a new idea where you specifically describe what you are doing. You could start a new paragraph here. (2)

There is considerable interest in understanding how the brain might use populations of spiking neurons to encode and communicate probability, and to combine sources of information in an optimal, or nearly optimal way, as specified by Bayesian inference we thereview by (3). A population code represents information about a stimulus of behavioral feature using the simultaneous activity of a population of spiking neurons that are sensitive to that feature (4). Far from being deterministic, the neural response for the same action or stimulus varies from trial to trial. This suggests that the brain might encode features as probability distributions. For example, for a *center-out* reach action, the population code might represent a probability distribution with a central directional tendency  $\mu_{\theta}$ , and a measure of precision  $\kappa$ .

**Population code** Let us suppose that spikes from each neuron *i*, within a population of *N* neurons follow independent point processes  $r = \{r_i\}_{i=1,...,N}$  (see the inset panel of Figure 1).

$$P(r_i|\theta) = \frac{exp\{-f_i(\theta)\}f_i(\theta)^{r_i}}{r_i!}$$

(1)

(2)

The mean response  $f_i(\theta)$  depends on  $\theta - \theta_{PDi}$ , where  $\theta$  is the intended direction of reach, and  $\theta_{PDi}$  is the preferred direction for neuron *i*.

Here, we define  $f_i$  as a von Mises function, which defines an exponential family on the unit circle, analogous to the normal distribution on the real line.

$$f_i(\theta) = A_i exp\{B_i \cos(\theta - \theta_{PDi})\}$$

Where  $A_i$  and  $B_i$  are constants representing the  $i_{th}$  neuron's amplitude and precision, respectively. High precision indicates narrow tuning for a particular preferred stimulus  $\theta_{PDi}$ . The parameters  $\theta$ ,  $\theta_{PDi}$ , and  $\mu_{\theta}$  are directional values; for a two-dimensional workspace, they can be conveniently expressed in circular angles  $[0^{\circ}, 360^{\circ}]$  with  $0^{\circ}$  being equivalent to  $360^{\circ}$ . Figure 1 (left) shows the response vs. reach direction for a neuron with preferred direction of

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E	Author: loaner	Subject: Sticky Note	Date: 1/29/2016 11:25:27 AM	
Can you put this clause in a parenthesis or use it as a separate sentence? I find it distracting as a part of the first sentence.			it as a separate sentence? I find it distracting as a part of the first sentence.	
r	Author: loaner	Subject: Highlight Date:	L/29/2016 11:27:58 AM	
	"I don't see this phrase in the previous sentences, so I am a little confused where it came from. It might be clearer for someone in the area, but you could try to introduce population code in the first sentence.			
r	Author: loaner	Subject: Highlight Date:	L/29/2016 11:28:50 AM	
	<sup>22</sup> Is there a way to use neural response in the stress position of the earlier sentence?			
г	Author: loaner	Subject: Highlight Date:	L/29/2016 11:30:15 AM	
is this the same as stimulur or behavior feature?				
	Author: loaner	Subject: Sticky Note	Date: 1/29/2016 11:33:53 AM	

You could try to include equation as a part of the sentence and continue with. "where" instead of starting a new sentence there.

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Author: loaner Subject: Sticky Note Date: 1/29/2016 11:36:37 AM You could start a new paragraph here, since you are now finished talking about the figure and a new idea seems to be introduced in this sentence. 0.1  $\sum_{i=1}^{N} f_i(\theta)$ 87.33 0.05 0 10 20 30 response (spikes) 10 270 180 270 360 90 180 90 Reach direction 0 Reac irection 0

Figure 1: Encoding reach direction. The response distribution for one neuron with preferred direction of  $180^{\circ}$  is shown on the left panel (width at half amplitude=  $133^{\circ}$ ). The black solid line indicates the mean, and the blue dashed lines are  $\pm$  one standard deviation. The gray inset shows the poisson distribution for the neuron's response given the preferred direction stimulus. The right panel shows the mean response for a population of 12 neurons with equal precision and preferred directions spaced at  $30^{\circ}$ 

180°. For a poisson distribution, the variance of the response is dependent on the stimulus with a Fano factor (variance to mean ratio) of 1. Tuning curves for a population of N=12 neurons with equal precision  $(B_i - B)$  are shown on the right panel of Figure 1, the preferred directions are spaced by 30° predet the assumption that every neuron responds independently, the population response distribution becomes the product of the individual neuron response distributions as shown in equation 3. Experimental evidence shows that neural populations do exhibit correlations in firing rate, but for now we maintain the assumption of independence for mathematical simplicity.

$$P(r|\theta) = \prod_{i=1}^{N} P(r_i|\theta)$$
(3)

Now we discuss two ways of computing estimates of the stimulus from the population response: Bayesian inference and Population Vector.