Review report for Temporal Latent Space Network Model with VAR 1 Evolution of Latent Positions: Introduction

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1 Summary

I am asked to review section 2 and part of section up to page 7. The latent space network model (LSM) has an unobservable latent space in which each agent's distance from other agents determines the probability of having a tie with them. The LSM cannot account for evolution dynamics of network, so the author tried to extend it to a temporal model that allows each agent's latent position to have a VAR(1) process evolution. In the estimation section, the author mainly described the basic setup of Gibbs sampling procedure from the model and one problem: the model should be invariant to isometric transformations in the latent space. But the normal density function doesn't have the same property, making the estimation non-identifiable. This problem is an issue in drawing samples from the initial time point in Monte Carlo run. This result is shown as theorem 3.1. It is used to motivate the author's method to obtain an identifiable estimation (which is beyond the range of this review).

2 General comment

I think the sections I read are well organized and well written. I can see what was previously done on this and what the author is trying to accomplish (unless I made some obvious misstatement in the summary). The notation and formulas are in general clear. Most of my comments would be about some small details. See below.

3 Specific comments

Section 2, the formula between equation 3 and 4: I think the standard way of representing Kronecker product is \otimes. And I'm a little confused what you mean by a Kronecker delta product. But that could be just me knowing nothing about the field. Small notation problem, no reference.
Equation (4): I'm not sure how to address this problem. But the four lines here are progressive while in equation (2) and (3) the equations are 'parallel', in the sense they are about different variables/parameters. I'm a little concerned about them being presented in the same way. But this is probably not a big deal. Based on Lebrum's discussion about saving readers' energy.
Prior distribution for the sampling algorithm on page 6: on the last line it should be $\Phi_{ij} \sim \cdots\cdots, \forall i, j$. Small notation problem, no reference.
Page 7, proof of theorem 3.1: this might be just a problem of taste, but I think this proof has too much English when it can be simplified without sacrificing too much readability. I tried to

rewrite the proof from the equation on the third line. But it's totally possible this is inferior to the current form. This comment is based on the discussion during last lecture about writing proofs.

Since Σ^{-1} is assumed to be positive definite. Then both sides of the equation must be either (i) both 0; or (ii) equal to the same positive constant, call it c.

(i) By positive definiteness of Σ^{-1}, we have $X_t - \Phi Z_{t-1} = Z_t - \Phi Z_{t-1}$, or $X_t = Z_t$, contradicting our assumption that $Z_t \neq X_t$.

(ii) By $(X_t - \Phi Z_{t-1})^T \Sigma^{-1} (X_t - \Phi Z_{t-1}) = c > 0$, we know all eligible X_t's lie on an ellipsoid centered at ΦZ_{t-1}. Due to the positive definiteness of Σ^{-1}, the surface extends through all dimensions of the latent space.

On the other hand, since X_t is an arbitrary isometry of Z_t, i.e. $X_t = RZ_t$ for some rotation-reflection isometry R. All permissible X_t's are on the sphere centered at the origin.

Therefore a sphere centered at the origin must coincide with the ellipsoid centered at ΦZ_{t-1}, which would only have Lesbesgue measure 0.

By (i), (ii), we have shown to theorem to be true.