A matrix Method for Computing the Derivatives of Interval Uniform B-Spline Curves

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***Abstract*—The matrix forms for curves and surfaces were largely promoted in CAD. These formulations are very compact to write, simple to program, and clear to understand. They manifest the desired basis as a matrix transformation of the common power basis. Furthermore, this implementation can be made extremely fast if appropriate matrix facilities are available in either hardware or software. Derivatives are very important in computation in engineering practice on graphics structures. B-spline functions are defined recursive, so direct computation is very difficult. A method for obtaining the matrix representations of uniform B-splines and Bezier curves of arbitrary degrees have been presented in this paper. By means of the basis matrix, the matrix representations of uniform B-splines and Bezier curves are unified by a recursive formula. The four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves) associated with the original interval uniform B-spline curve are obtained in matrix form. The fixed control points of the r^{th} derivatives of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves) are found. Finally the interval control points of the r^{th} derivative of the interval B-spline curve is computed from the fixed control points of the r^{th} derivatives of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves). A numerical example is included in order to demonstrate the effectiveness of the proposed method.**

***Index Term*— Recursive matrix representations, interval B-spline curve, CAD, derivatives of B-spline curve, CAGD.**

I. INTRODUCTION

The curve is the most basic design element to determine shapes and silhouettes of industrial products and works for shape designers and it is inevitable for them to make it aesthetic and attractive to improve the total quality of the shape design. The B-spline curves and surfaces have been widely applied in many CAD/CAM systems. They are gaining popularity and have become a standard tool in industry of geometric modeling because they provide a common mathematical form for analytical geometry and free-form curves and surfaces.

Computer aided design (CAD) is concerned with the representation and approximation of curves and surfaces, when these objects have to be processed by a computer. Parametric representations are widely used since they allow considerable flexibility for shaping and design. In computer aided design and geometric modeling, there are considerable interests in approximating curves and surfaces with simpler forms of curves and surfaces. This problem arises whenever CAD data need to be shared across heterogeneous systems which use different proprietary data structures for model representations. For example, some systems restrict themselves to polynomial forms or limit the polynomial degree that they accommodate. Geometric modeling and computer graphics have been interesting and important subjects for many years from the point of view of scientists and engineers. One of the main and useful applications of these concepts is the treatment of curves and surfaces in terms of control points, a tool extensively used in CAGD.

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There are several kinds of polynomial curves in CAGD, e.g., Bezier [1], [2], [3], [4] Said-Ball [5], Wang-Ball [6], [7], [8], B-spline curves [9] and DP curves [10], [11]. These curves have some common and different properties. All of them are defined in terms of the sum of product of their blending functions and control points. They are just different in their own basis polynomials. In order to compare these curves, we need to consider these properties. The common properties of these curves are control points, weights, and their number of degrees. Control points are the points that affect to the shape of the curve. Weights can be treated as the indicators to control how much each control point influences to the curve. Number of degree determines the maximum degree of polynomials. In different curves, these properties are not computed by the same method. To compare different kinds of curves we need to convert these curves into an intermediate form.

Parametric representation for curves is important in computer-aided geometric design, medical imaging, computer vision, computer graphics, shape matching, and face/object recognition. They are far better alternatives to free form representation, which are plagued with unboundedness and stability problems. Parametric representations are widely used since they allow considerable flexibility for shaping and design. A curve that actually passes through each control point is called an interpolating curve; a curve that passes near to the control points but not necessarily through them is called an approximating curve.

B-splines are defined by piecewise functions, represented by a set of knots. To add an extra piece to the curve, only one knot is required. Comparing with piecewise linear functions, B-splines are smoother. These advantages enable B-splines to be favored in CAD tools for providing smooth curves. In fact, knots only indirectly affect the shape of curves. Manual and interactive refinement to the knots using CAD tools is sometime necessary to obtain an ideal spline curve.

There are several types of B-splines. In the uniform (also called periodic) B-spline, the knot values are uniformly spaced and all the weight functions have the same shape and are shifted with respect to each other. In the nonuniform B-spline, the knots are specified by the user and the weight functions are generally different. There is also an open uniform B-spline, where the knots are not uniform but are specified in a simple way. The B-spline is an approximating curve based on control points, but there is also an interpolating version that passes through the points.

A B-spline is a convenient form for representing complicated, smooth curves. A uniform B-spline of order k is a piecewise order k Bezier curve, and is $C^{k-2}-continuous$ (i.e., the 0^{th} through $(k-2)^{th}$ derivatives are continuous). The form of a B-spline is in general chosen because it is easy to manipulate, not because it is the solution to an optimization problem, like smoothing splines. B-splines are particularly easy to manipulate because they have local control: parameters of the B-spline only affect a small part of the entire spline.

This paper is organized as follows. Section II contains the basic results, while section III gives computing derivatives of interval uniform B-spline curves whereas section IV shows a numerical example and the final section offers conclusions.

II. THE BASIC RESULTS

An interval polynomial is a polynomial whose coefficients are interval. We shall denote such polynomials in the form $Q^I(u)$ to distinguish them from ordinary (single-valued) polynomials. Let $Q^I(u)$ be the position vector along the interval curve as a function of the parameter u. In general we express an interval B-spline curve of degree n in the form:

$$Q_n^I(u) = \sum_{i=0}^{n}[q_i^-, q_i^+]\, B_{i,k}(u) = \sum_{i=0}^{n} Q_i^I\, B_{i,k}(u)$$
$$u_{min} \le u < u_{max} \quad and \quad 2 \le k \le n \tag{1}$$

with knot vector $T = \{t_0, t_1, \cdots\cdots, t_{n+k}\}$, where the $Q_i^I = [q_i^-, q_i^+]$ for $(i = 0, 1, \cdots, n)$ are the interval position vectors of the $(n+1)$ interval control polygon vertices, and the $B_{i,k}$ are the normalized B-spline basis functions. For the ith normalized B-spline basis function of order k $(degree\ k-1)$, the basis functions $B_{i,k}(u)$ are defined by the Cox-de Boor recursion formulas [12], [13]. Specifically:

$$B_{i,1}(u) = \begin{Bmatrix} 1, & if\ \ t_i \le u \le t_{i+1} \\ 0, & elsewhere \end{Bmatrix}$$
$$B_{i,k}(u) = \frac{(u-t_i)B_{i,k-1}(u)}{t_{i+k-1}-t_i} + \frac{(t_{i+k}-u)B_{i+1,k-1}(u)}{t_{i+k}-t_{i+1}} \tag{2}$$

The values of t_i are elements of a knot vector satisfying the relation $t_i < t_{i+1}$. The parameter u varies from u_{min} to u_{max} along the interval curve $Q_n^I(u)$. The convention $(0/0 = 0)$ is adopted. Formally, a B-spline curve is defined as a polynomial spline function of order k $(degree\ k-1)$, because it satisfies the following two conditions: (1) $Q_n^I(u)$ is an interval polynomial of degree $(k-1)$ on each interval $t_i \le u < t_{i+1}$. (2) $Q_n^I(u)$ and its derivatives of order $(1, 2, \cdots, k-2)$ are all continuous over the entire interval curve.

Vector-valued interval $Q^I(u)$ in the most general terms is defined as any compact set of points (x, y) dimensions as tensor products of scalar intervals:

$$\boldsymbol{Q^I} = [\boldsymbol{c_1}, \boldsymbol{d_1}] \times [\boldsymbol{c_2}, \boldsymbol{d_2}] = \{(\boldsymbol{x}, \boldsymbol{y}) \mid \boldsymbol{x} \in [\boldsymbol{c_1}, \boldsymbol{d_1}] \ \ \boldsymbol{and} \ \ \boldsymbol{y} \in [\boldsymbol{c_2}, \boldsymbol{d_2}]\} \tag{3}$$

such vector-valued intervals are clearly just rectangular regions in plane [14].

For each $t_{min} \le u < t_{max}$, the value $Q_n^I(u)$ of the interval curve (1) is an interval vector that has the following significance: For any fixed curve $Q_n(u)$ whose control points satisfy $q_i \in [q_i^-, q_i^+]$ for $(i = 0, 1, \cdots, n)$ we have $Q_n(u) \in Q_n^I(u)$. Likewise, the entire interval curve (1) defines a region in the plane that contains all curves whose control points satisfy $q_i \in [q_i^-, q_i^+]$ for $(i = 0, 1, \cdots, n)$.

By means of basis translation from B-spline to power basis [15], [16], $B_{i,k-1}(u)$ can be represented as follows:

$$B_{i,k-1}(u) = [1 \quad u \quad u^2 \quad \cdots \quad u^{k-2}] \cdot \begin{bmatrix} B_{0,i}^{k-1} \\ B_{1,i}^{k-1} \\ \vdots \\ B_{k-2,i}^{k-1} \end{bmatrix} \tag{4}$$

and

$$u = \frac{(t-t_i)}{(t_{i+1}-t_i)}, \quad for \ \ u \in [0,1)$$

B-spline basis function $B_{i,k}(t)$ are piecewise polynomials of degree $(k-1)$. If $t \in [t_i, t_{i+1})$ and $t_i < t_{i+1}$, there are k B-spline basis functions of degree $(k-1)$ that are nonzero $B_{i,k}(t)$, for $i = [(l-k+1), (l-k+2), \cdots\cdots, l]$. They can be represented in a matrix equation as follows:

$$[B_{l-k+1,k}(u) \quad B_{l-k+2,k}(u) \quad \cdots \quad B_{l,k}(u)] = [1 \quad u \quad u^2 \quad \cdots \quad u^{k-1}] B_k^j(l) \tag{5}$$

where,

$$u = \frac{(t-t_i)}{(t_{i+1}-t_i)}, \quad for \ \ u \in [0,1)$$

and

$$B_k^j(l) = \begin{bmatrix} B_{0,l-k+1}^{k,j} & B_{0,l-k+2}^{k,j} & \cdots & B_{0,l}^{k,j} \\ B_{1,l-k+1}^{k,j} & B_{1,l-k+2}^{k,j} & \cdots & B_{1,l}^{k,j} \\ \vdots & \vdots & \cdots & \vdots \\ B_{k-1,l-k+1}^{k,j} & B_{k-1,l-k+2}^{k,j} & \cdots & B_{k-1,l}^{k,j} \end{bmatrix}$$

and t_j are the knots.

Let $\alpha_{i,n}^j$ be the control vertices of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves). The B-spline curve segment is:

$$S_{l-k+1}^{j}(u) = [B_{l-k+1,k}(u) \quad B_{l-k+2,k}(u) \quad \cdots \quad B_{l,k}(u)] \cdot \begin{bmatrix} \alpha_{l-k+1,n}^{j} \\ \alpha_{l-k+2,n}^{j} \\ \vdots \\ \alpha_{l,n}^{j} \end{bmatrix}$$
$$= [1 \quad u \quad u^2 \quad \cdots \quad u^{k-1}] \cdot B_k^j(l) \cdot \begin{bmatrix} \alpha_{l-k+1,n}^{j} \\ \alpha_{l-k+2,n}^{j} \\ \vdots \\ \alpha_{l,n}^{j} \end{bmatrix} \tag{6}$$

where,

$$u = \frac{(t - t_i)}{(t_{i+1} - t_i)}, \quad for \quad u \in [0,1)$$

and $B_k^j(l)$ are referred to as the l^{th} basis matrices of the B-spline basis functions of degree $(k-1)$.

The four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves) [17] associated with the original interval uniform B-spline curve are:

$$\begin{aligned} Q_n^1(u) &= q_0^- + q_1^- u + q_2^+ u^2 + q_3^+ u^3 + q_4^- u^4 + q_5^- u^5 + \cdots \\ &\equiv \alpha_{0,n}^1 + \alpha_{1,n}^1 u + \alpha_{2,n}^1 u^2 + \cdots + \alpha_{n,n}^1 u^n \\ Q_n^2(u) &= q_0^- + q_1^+ u + q_2^+ u^2 + q_3^- u^3 + q_4^- u^4 + q_5^+ u^5 + \cdots \\ &\equiv \alpha_{0,n}^2 + \alpha_{1,n}^2 u + \alpha_{2,n}^2 u^2 + \cdots + \alpha_{n,n}^2 u^n \\ Q_n^3(u) &= q_0^+ + q_1^+ u + q_2^- u^2 + q_3^- u^3 + q_4^+ u^4 + q_5^+ u^5 + \cdots \\ &\equiv \alpha_{0,n}^3 + \alpha_{1,n}^3 u + \alpha_{2,n}^3 u^2 + \cdots + \alpha_{n,n}^3 u^n \\ Q_n^4(u) &= q_0^+ + q_1^- u + q_2^- u^2 + q_3^+ u^3 + q_4^+ u^4 + q_5^- u^5 + \cdots \\ &\equiv \alpha_{0,n}^4 + \alpha_{1,n}^4 u + \alpha_{2,n}^4 u^2 + \cdots + \alpha_{n,n}^4 u^n \end{aligned} \tag{7}$$

Lemma 1: The basis matrices of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-splines curves) satisfy the following recursive formula:

$$B_k^j = \frac{1}{k-1}\left\{\begin{bmatrix} B_{k-1}^j \\ 0 \end{bmatrix}\begin{bmatrix} 1 & k-2 & & & 0 \\ & 2 & k-3 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 0 \\ 0 & & & & k-1 \end{bmatrix} + \begin{bmatrix} 0 \\ B_{k-1}^j \end{bmatrix}\begin{bmatrix} -1 & 1 & & & 0 \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 1 \\ 0 & & & & -1 \end{bmatrix}\right\}$$
$$(j = 1,2,3,4) \tag{8}$$

where, $B_1^j = [1]$ and $u = \frac{(t-t_i)}{(t_{i+1}-t_i)}$ for $u \in [0,1]$.

Proof: The proof follows immediately from [18].

The basis matrices of four fixed uniform Kharitonov's polynomials (four fixed uniform B-splines curves) of degree $k-1$ are independent of t_i.

Using equation (8) recursively step by step, one can also obtain the following matrices:

$$B_k^j = \begin{bmatrix} b_{0,0}^j & b_{0,1}^j & \cdots & b_{0,k-1}^j \\ b_{1,0}^j & b_{1,1}^j & \cdots & b_{1,k-1}^j \\ \vdots & \vdots & \cdots & \vdots \\ b_{k-1,0}^j & b_{k-1,1}^j & \cdots & b_{k-1,k-1}^j \end{bmatrix} \tag{9}$$

where,

$$\left\{\begin{aligned} b_{i,l}^j &= \frac{1}{(k-1)!} C_{k-1}^{k-1-i} \sum_{s=l}^{k-1} (-1)^{s-l} C_k^{s-l} (k-s-1)^{k-1-i} \\ C_n^i &= \frac{n!}{i!\,(n-i)!} \end{aligned}\right\} \tag{10}$$

Both equations (8) and (9) can be used to calculate the basis matrices of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-splines curves).

Lemma 2: The basis matrices of the four fixed uniform Kharitonov's polynomials (four fixed uniform Bezier curves) satisfy the following recursive formula:

$$B_k^j = \begin{bmatrix} B_{k-1}^j & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ B_{k-1}^j \end{bmatrix}\begin{bmatrix} -1 & 1 & & & 0 \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 1 \\ 0 & & & & -1 \end{bmatrix}$$
$$(j = 1,2,3,4) \tag{11}$$

where, $B_1^j = [1]$ and $u = \frac{(t-t_i)}{(t_{i+1}-t_i)}$ for $u \in [0,1]$.

Proof: The proof follows immediately from [18].

The basis matrices of four fixed uniform Kharitonov's polynomials (four fixed uniform Bezier curves) of degree $k-1$ are independent of t_i.

Using equation (11) recursively step by step, one can also obtain the following matrices for the four fixed uniform Kharitonov's polynomials (four fixed uniform Bezier curves):

$$B_k^j = \begin{bmatrix} b_{0,0}^j & 0 & 0 & \cdots & 0 & 0 \\ b_{1,0}^j & b_{1,1}^j & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ b_{k-2,0}^j & b_{k-2,1}^j & \cdots & \cdots & b_{k-2,k-2}^j & 0 \\ b_{k-1,0}^j & b_{k-1,1}^j & b_{k-1,2}^j & \cdots & b_{k-1,k-2}^j & b_{k-1,k-1}^j \end{bmatrix} \tag{12}$$

where,

$$b_{i,l}^j = \left\{\begin{matrix} (-1)^{i-l} C_{k-1}^l C_{k-1-l}^{i-l}, & for \quad i \geq l \\ 0, & for \quad i < j \end{matrix}\right\} \tag{13}$$

The matrices B_k^j for $(j = 1,2,3,4)$ are lower triangular $k \times k$ matrices.

Both equations (11) and (12) can be used to calculate the basis matrices of the four fixed uniform Kharitonov's polynomials (four fixed uniform Bezier curves).

III. COMPUTING DERIVATIVES OF INTERVAL UNIFORM B-SPLINE CURVES

Assume there is an interval uniform B-spline curve of degree $(k-1)$.

$$Q_n^I(u) = \sum_{i=0}^{n} Q_i^I\, B_{i,k}(u) \tag{14}$$

defined on the knot vector,

$$T = \Big\{\underbrace{0,\cdots,0}_{k+1}, t_{k+1}, \cdots, t_{m-k-1}, \underbrace{1,\cdots,1}_{k+1}\Big\}$$

Equation (14) can be written in matrix form as follows:

$$\begin{gathered} S_{l-k+1}^I(u) = U^k B_k^j(l) Q^I(l) \\ for\ \ l = k-1, k, \quad \cdots\cdots, n \\ and\ \ u = \frac{(t-t_i)}{(t_{i+1}-t_i)} \ \text{ for } u \in [0,1] \end{gathered} \tag{15}$$

$$U^k = [1 \quad u \quad u^2 \quad \cdots \quad u^{k-1}]$$

$$Q^I(l) = [[q_{l-k+1}^-, q_{l-k+1}^+] \quad [q_{l-k+2}^-, q_{l-k+2}^+] \quad \cdots\cdots \quad [q_l^-, q_l^+]]^T$$

The problem is to find the r^{th} derivative of the $Q_n^I(u)$, (i.e., $Q_n^{(r)I}(u)$ where,

$$Q_n^{(r)I}(u) = \sum_{i=0}^{n-r} Q_i^{(r)I}\, B_{i,k-r}(u) \tag{16}$$

with knot vector,

$$T^{(r)} = \Big\{\underbrace{0,\cdots,0}_{k-r+1}, t_{k+1}, \cdots, t_{m-k-1}, \underbrace{1,\cdots,1}_{k-r+1}\Big\}$$

The four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves) [17] associated with the original interval uniform B-spline curve are obtained in matrix form as shown below:

$$\begin{gathered} S_{l-k+1}^j(u) = U^k B_k^j(l) \alpha^j(l) \\ for\ \ l = k-1, k, \quad \cdots\cdots, n \\ and\ \ u = \frac{(t-t_i)}{(t_{i+1}-t_i)} \ \text{ for } u \in [0,1] \end{gathered} \tag{17}$$

$$\alpha^j(l) = \left[\alpha_{l-k+1,n}^j \quad \alpha_{l-k+2,n}^j \quad \cdots\cdots \quad \alpha_{l,n}^j\right]^T$$

Then,

$$\begin{gathered} \frac{d^r}{du^r} S_{l-k+1}^j(u) = \left(\frac{d^r}{du^r} U^k\right) B_k^j(l) \alpha^j(l) \\ (j = 1,2,3,4) \end{gathered} \tag{18}$$

where, the fixed control points of the r^{th} derivatives of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves) can easily be computed $\left(\left(\frac{d^r}{du^r} U^k\right)\right.$, can easily be obtained, for instance, $(dU^k/du) = [0 \quad 1 \quad 2u \quad \cdots\cdots \quad (k-1)u^{k-2}]$). Finally the interval control points of the r^{th} derivative of the interval uniform B-spline curve is found from the fixed control points of the r^{th} derivatives of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves) as follows:

$$\begin{gathered} \frac{d^r}{du^r} S_{l-k+1}^I(u) = \left[\min\left(\frac{d^r}{du^r} S_{l-k+1}^j(u)\right), \max\left(\frac{d^r}{du^r} S_{l-k+1}^j(u)\right)\right] \\ (j = 1,2,3,4) \quad and \quad (l = k-1, k, \quad \cdots\cdots, n) \end{gathered} \tag{19}$$

IV. NUMERICAL EXAMPLE

Consider the interval uniform B-spline curve $Q_3^I(u)$ of order $(k = 4)$, with interval control points:

$$\begin{aligned} [p_0^-, p_0^+] &= ([16.0000{,}18.0000] \times [20.0000{,}24.0000]) \\ [p_1^-, p_1^+] &= ([22.0000{,}26.0000] \times [30.0000{,}34.0000]) \\ [p_2^-, p_2^+] &= ([42.0000{,}44.0000] \times [50.0000{,}56.0000]) \\ [p_3^-, p_3^+] &= ([50.0000{,}56.0000] \times [62.0000{,}68.0000]) \end{aligned}$$

The problem is to determine the interval control points of the first derivative of $Q_3^I(u)$.

As explained in sections II and III, the four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves) are found, and the basis matrices of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-splines curves) are obtained using recursive formula as follows:

$$\begin{gathered} B_4^j(l) = \frac{1}{3!}\begin{bmatrix} 1 & 4 & 1 & 0 \\ -3 & 0 & 3 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \\ l = 4 \\ j = 1,2,3,4 \end{gathered}$$

The fixed control points of the first derivatives of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves) are obtained, and the interval control points of the first derivative of the interval uniform B-spline curve $Q_3^I(u)$ is found from the fixed control points of the first derivatives of the four fixed uniform Kharitonov's Polynomials (four fixed uniform B-spline curves) as:

$$\begin{aligned} \left[q_0^{(1)-}, q_0^{(1)+}\right] &= ([12.0000{,}14.0000] \times [8.0000{,}13.0000]) \\ \left[q_1^{(1)-}, q_1^{(1)+}\right] &= ([8.0000{,}16.0000] \times [6.0000{,}16.0000]) \\ \left[q_2^{(1)-}, q_2^{(1)+}\right] &= ([-13.0000, -8.0000] \times [-15.0000, -5.0000]) \end{aligned}$$

The first derivative of the given interval uniform B-spline curve has order $(k = 3)$.

V. CONCLUSIONS

Industrial Geometry aims at unifying existing and developing new methods and algorithms for a variety of application areas with a strong geometric component. These include CAD, CAM, geometric modelling, robotics, computer vision and image processing, computer graphics and scientific visualization. B-spline functions have become, in interactive CAD systems, the most commonly used mathematical tool for definition and representation of free-shape curves and surfaces. Matrix representation is very useful in computer aided geometric design, since the matrix is an important and basic tool in mathematics. B-splines play an important role in computer-aided design (CAD) and signal processing. Matrix formulas of B-spline curves and surfaces have the advantages of both simple computation of points on curves or surfaces and their derivatives and of easy analysis of the geometric properties of B-spline curves and surfaces. Using the basis matrix proposed in this paper, the matrix representations of uniform B-splines and Bezier curves can be unified by a recursive formula. Derivatives are a very important tool of computations in an engineering practice on the graphics structures. B-Spline functions are defined recursively, so the direct computation is very difficult. The four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves) associated with the original interval uniform B-spline curve are obtained in matrix form. The fixed control points of the r^{th} derivatives of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves) are found. Finally the interval control points of the r^{th} derivative of the interval uniform B-spline curve is computed from the fixed control points of the r^{th} derivatives of the four fixed uniform Kharitonov's polynomials (four fixed uniform B-spline curves). Whenever free-form curves and surfaces are represented mathematically, as they are in computer-aided design, analysis, and manufacturing, B-splines are the foundation of an efficient implementation. B-splines are especially important in the aircraft and automotive industries, where shape is all important.

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