Exploring Factors of Economic Health and Social Well-Being Among Populous US Counties

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Abstract

With over 3,000 counties spread across 51 states, including the District of Columbia, the United States has a very diverse population with a combination of social and economic factors that affect it. We are interested in finding the best model to predict average income per capita and the relationships between these factors based on 14 different characteristics. The data for this study has been collected for 440 of the most populous counties in the United States from the years 1990 and 1992 and is provided by the Geospatial and Statistical Data Center located at the University of Virginia. We performed general EDA and variable transformations in order to use various variable selection techniques. We discovered that many of the variables share associations and there are a few collinearity issues. Across the United States per-capita crime and per-capita income are positively related and baseline per-capita income differs significantly by region, except for the in the West. We were able to fit a model predicting per capita income with high predictive power while accounting for interpretability using both main effects and interactions. Overall, the findings are descriptive in describing how a county's economic health and social well-being can affect average income per capita.

1 Introduction

A county is described as a governmental unit in the United States that is bigger than a city but smaller than a state. With over 3,000 counties spread across 51 states, including the District of Columbia, the United States has a very diverse population with a combination of social and economic factors that affect it. *(1)* Social scientists are interested in looking at historical data to learn how average income per capita is associated with a county's economic health and social well-being based on factors such as population, land area, crime, education, and many other determinants. In addition to answering the main question posed above, we hope to also address the following concerns:

(2) Which variables seem to be related to which other variables in the data? Are these relationships expected and can these findings be explained in terms of the meanings of the variables?

(3) If we ignore all other variables, what is the relationship between per-capita income and crime rate and how is it affected by different regions of the country?

(4) Should we be worried about either the missing states or the missing counties? Why or why not?

It is important for the social scientists to inform county leaders and politicians of the factors influencing the lives of their citizens. The answers to these questions may lead to new policies or changes within the legislation of these counties in hopes of providing a better life for their populations.

2 Data

The data for this study has been collected for 440 of the most populous counties in the United States from the years 1990 and 1992. It is provided by the Geospatial and Statistical Data Center located at the University of Virginia. Each line of the data set has an identification number attached to a county and state abbreviation followed by information on 14 different characteristics, all described in table 1. Counties with missing data were deleted from the data set prior to this study.

Variable Number	Variable Name	Description
1	Identification number	1–440
2	County	County name
3	State	Two-letter state abbreviation
4	Land area	Land area (square miles)
5	Total population	Estimated 1990 population
6	Percent of population aged 18–34	Percent of 1990 CDI population aged 18–34
7	Percent of population 65 or older	Percent of 1990 CDI population aged 65 or old
8	Number of active physicians	Number of professionally active nonfederal physicians during 1990
9	Number of hospital beds	Total number of beds, cribs, and bassinets during 1990
10	Total serious crimes	Total number of serious crimes in 1990, including murder, rape, robbery, aggravated assault, burglary, larceny-theft, and motor vehicle theft, as reported by law enforcement agencies
11	Percent high school graduates	Percent of adult population (persons 25 years old or older) who completed 12 or more years of school
12	Percent bachelor's degrees	Percent of adult population (persons 25 years old or older) with bachelor's degree
13	Percent below poverty level	Percent of 1990 CDI population with income below poverty level
14	Percent unemployment	Percent of 1990 CDI population that is unemployed
15	Per capita income	Per-capita income (i.e. average income per person) of 1990 CDI population (in dollars)
16	Total personal income	Total personal income of 1990 CDI population (in millions of dollars)
17	Geographic region	Geographic region classification used by the US Bureau of the Census, NE (northeast region of the US), NC (north-central region of the US), S (southern region of the US), and W (Western region of the US)

Table 1: Variable definitions for CDI data from Kutner et al. (2005).

To create an ideal dataset for future modeling a few variables were immediately removed. ID was removed due to its replication of the row numbers. State and county were removed because they both contained too many unique values to be useful categorically, leaving region as the sole categorical variable (Appendix 1). Majority of the counties are located in the Southern region with 152 and the least in the Western region with 77 counties. The full distribution of region can be seen in table 2.

The social scientists are particularly interested in understanding how per capita income and crime are related, thus, in addition to the variables in table 1, we created a new variable called per-capita crime from total crime and total population. Both of these variables are right skewed. Both per-capita income and per-capita crime are measured in thousands. Per-capita income has a mean of approximately $18,560 and a standard deviation of $4,060. Per-capita crime has a mean of approximately 57,290 and a standard deviation of 27,330. A table of summary statistics for all continuous variables can be found under Appendix 1.

Figure 1 displays distributions for all continuous variables in the dataset. A substantial amount of the variables have a prominent right skew which will be discussed later in the study.

Figure 2 displays the associations between all of the continuous variables and the primary response variable, per-capita income. Due to the skewed distributions of some of the variables there are plots showing non linear associations which will also be assessed later in the study.

region	Freq.
NC	108
NE	103
S	152
W	77

Table 2: Frequency distribution of Region
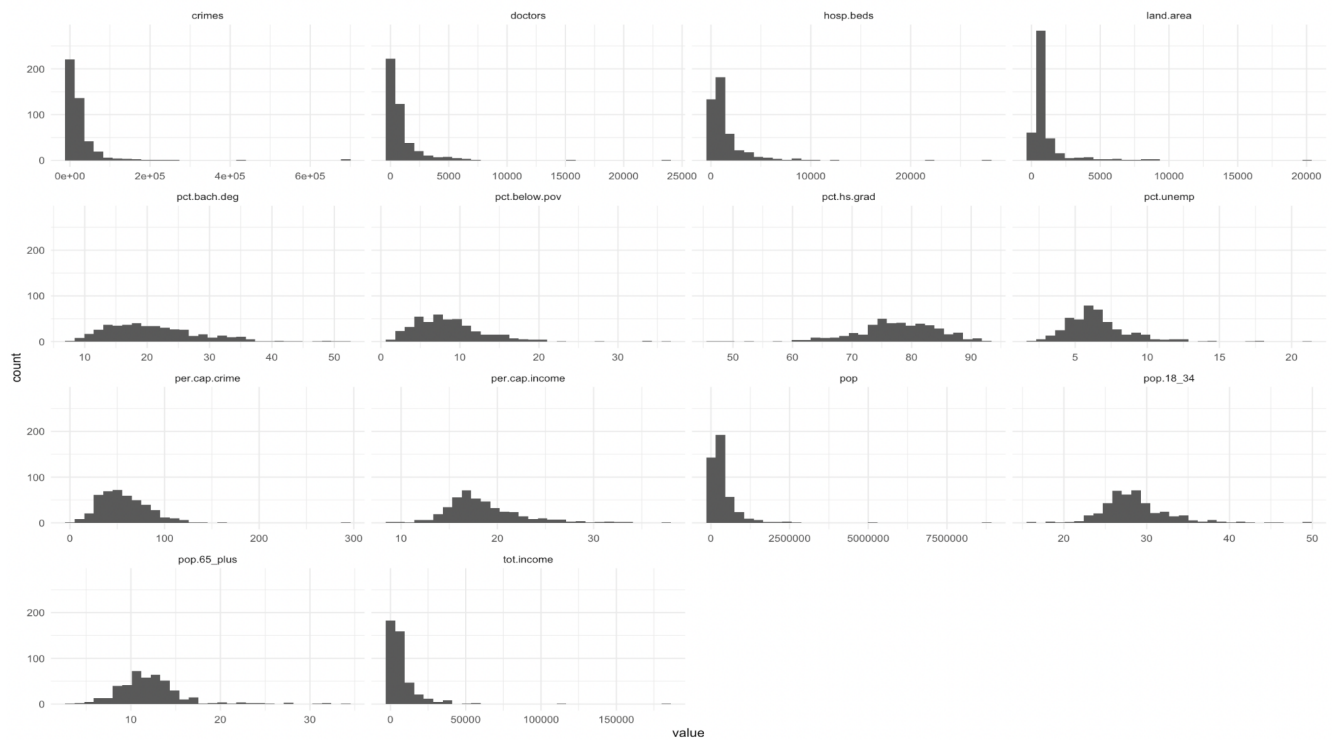
Figure 1: Distributions of continuous variables
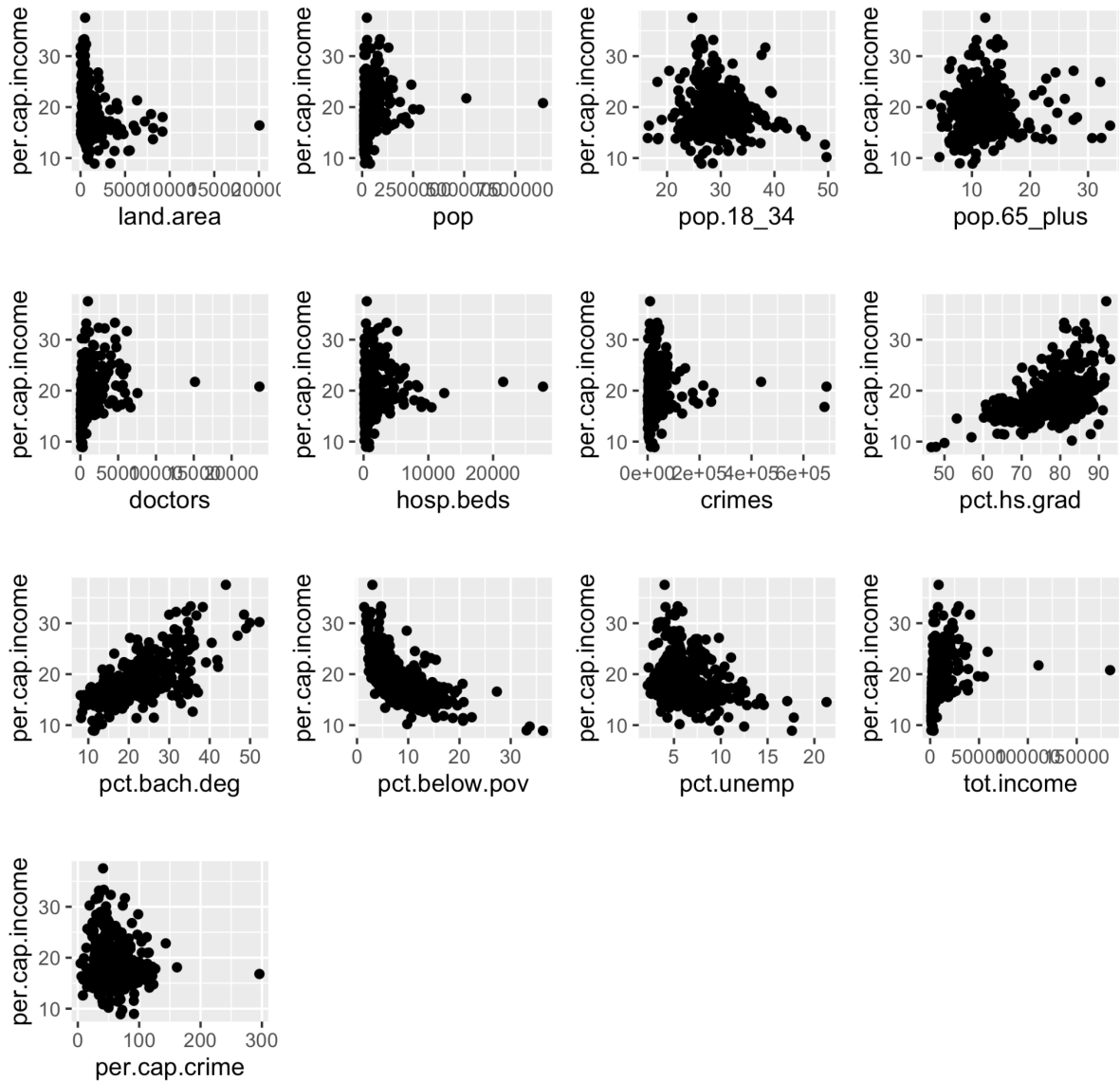
Figure 2: Relationship between per.cap.income and continuous variables

3 Methods

All statistical modeling and visualizations were made using the R language and environment for statistical computing (R Core Team, 2017).

Before beginning to answer any of the concerns of the social scientists, we first decided to transform variables that were skewed, keeping interpretability in mind, in hopes of creating better distributions and providing accurate model results. After transformations, we wanted to characterize the associations between variables. To do this we created a heat map correlation matrix to check for high correlations among the predictors and raise awareness to any problems of multicollinearity.

Next we considered two regression models. For the first model, predicting per-capita income based on crime rate and region, we assessed for a model that was best in terms of interpretability and predictive power. We looked at differences between additive models and interaction models using Partial F-Tests, AIC/BIC, and checking model assumptions with residual plots. Details on the model selection can be found under Appendix 3. For the second model, we wanted to find the best combination of variables to predict per-capita income. To do this we used a collection of variable selection techniques such as checking VIF's for collinearity problems, all subsets regression, stepwise regression, and lasso regression with cross validation. Details on the model selection can be found under Appendix 3.

To complete the analysis we discussed the possible implications of the missing states and counties in the dataset by considering the representation of each of the categorical variables.

4 Results

Correlations Between Predictors

Based on the histograms (Figure 1) and scatter plots (Figure 2) we decided to transform the variables crimes, tot.income, land.area, doctors, hosp.beds, per.cap.crime, and per.cap.income with log transformations. Other variables like pct.unemp, pce.below.pov, and pop could have also used transformations, however to adhere to the integrity of interpretability we only considered those that improved with a log transformation. The distribution of the newly transformed variables can be seen under Appendix 2. The predictors now show clear linear associations and can safely be used in modeling with the main response variable, log.per.cap.income. The log transformed variables will be used throughout the duration of the modeling.

The heat map correlation matrix, seen in figure 3, shows some interesting associations between variables. Many of the associations are to be expected and can be explained in terms of the variable descriptions. The log.doctors variable has some strong positive associations with log.crime, log.tot.income, and log.hosp.beds. Doctors probably have a high income thus influencing the total income, the amount of hospital beds is related to the number of doctors there are to attend to them, and doctors are treating criminals and their victims when needed. The log.hosp.beds variable shares the same strong positive associations with log.crimes, log.tot.income, and log.doctors. The variable log.tot.income has other strong positive correlations with pop and log.crimes.

In terms of collinearity issues, there is strong association between log.crimes and log.per.cap.crime. This is expected due to the fact that crimes was used to calculate per.cap.crime. There is also a strong positive correlation between pct.bach.deg and pct.hs.grad since those who have graduated highschool are included in those who have bachelor degrees. We assume these collinearities will be addressed in the variable selection methods. The main

response variable log.per.cap.income has some moderate associations which means there is hope that good models can be made with it. The strongest of these being pct.hs.grad, pct.bach.deg, and pct.below.pov.
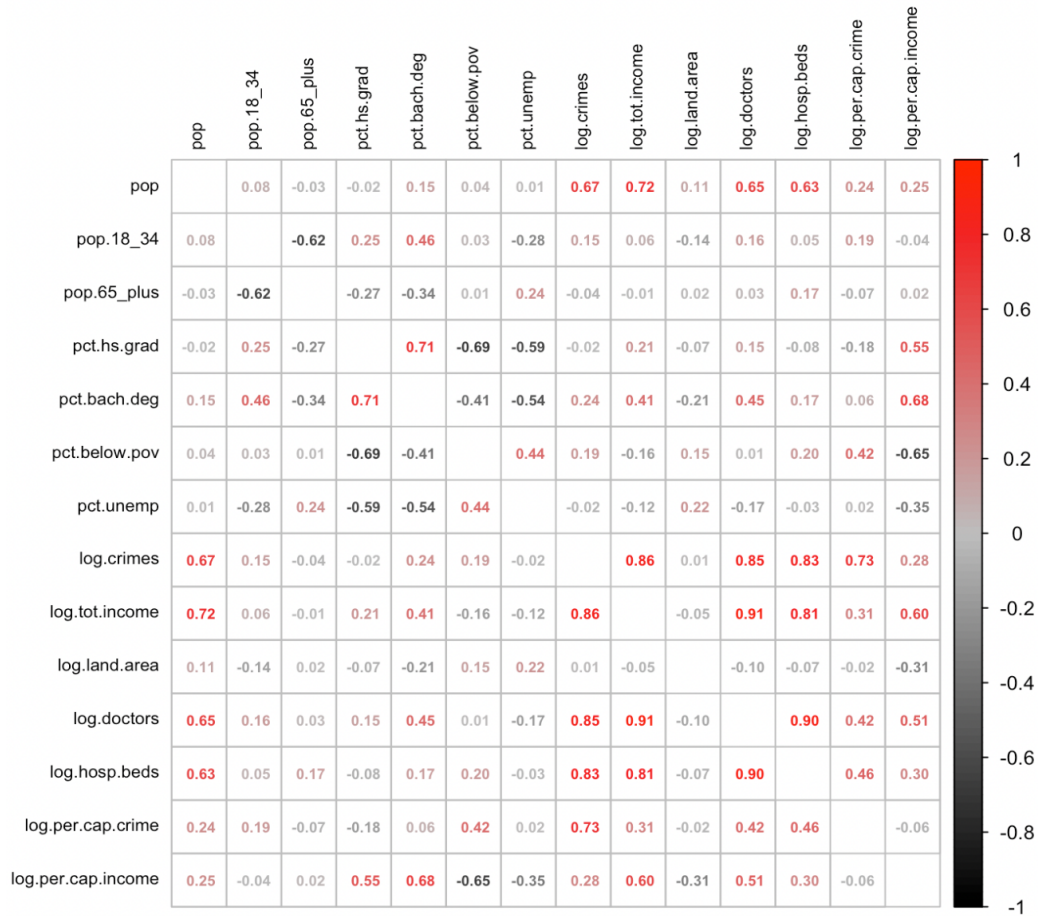
	pop	pop.18_34	pop.65_plus	pct.hs.grad	pct.bach.deg	pct.below.pov	pct.unemp	log.crimes	log.tot.income	log.land.area	log.doctors	log.hosp.beds	log.per.cap.crime	log.per.cap.income
pop		0.08	-0.03	-0.02	0.15	0.04	0.01	**0.67**	**0.72**	0.11	**0.65**	**0.63**	0.24	0.25
pop.18_34	0.08		-0.62	0.25	0.46	0.03	-0.28	0.15	0.06	-0.14	0.16	0.05	0.19	-0.04
pop.65_plus	-0.03	-0.62		-0.27	-0.34	0.01	0.24	-0.04	-0.01	0.02	0.03	0.17	-0.07	0.02
pct.hs.grad	-0.02	0.25	-0.27		**0.71**	-0.69	-0.59	-0.02	0.21	-0.07	0.15	-0.08	-0.18	**0.55**
pct.bach.deg	0.15	0.46	-0.34	**0.71**		-0.41	-0.54	0.24	0.41	-0.21	0.45	0.17	0.06	**0.68**
pct.below.pov	0.04	0.03	0.01	-0.69	-0.41		0.44	0.19	-0.16	0.15	0.01	0.20	0.42	-0.65
pct.unemp	0.01	-0.28	0.24	-0.59	-0.54	0.44		-0.02	-0.12	0.22	-0.17	-0.03	0.02	-0.35
log.crimes	**0.67**	0.15	-0.04	-0.02	0.24	0.19	-0.02		**0.86**	0.01	**0.85**	**0.83**	**0.73**	0.28
log.tot.income	**0.72**	0.06	-0.01	0.21	0.41	-0.16	-0.12	**0.86**		-0.05	**0.91**	**0.81**	0.31	**0.60**
log.land.area	0.11	-0.14	0.02	-0.07	-0.21	0.15	0.22	0.01	-0.05		-0.10	-0.07	-0.02	-0.31
log.doctors	**0.65**	0.16	0.03	0.15	0.45	0.01	-0.17	**0.85**	**0.91**	-0.10		**0.90**	0.42	**0.51**
log.hosp.beds	**0.63**	0.05	0.17	-0.08	0.17	0.20	-0.03	**0.83**	**0.81**	-0.07	**0.90**		0.46	0.30
log.per.cap.crime	0.24	0.19	-0.07	-0.18	0.06	0.42	0.02	**0.73**	0.31	-0.02	0.42	0.46		-0.06
log.per.cap.income	0.25	-0.04	0.02	**0.55**	**0.68**	-0.65	-0.35	0.28	**0.60**	-0.31	**0.51**	0.30	-0.06	

Figure 3: Heat map correlation matrix of variables.

Relationship Between Per-Capita Income, Crime Rate, and Region

There are two main factors we wanted to consider when trying to find a model to predict per.cap.income based on crime and region. One, is the additive or the interaction model preferred and two, is per.cap.crime a better predictor than crimes. As a reminder, log transformations were done previously to crimes, per.cap.crime, and per.cap.income and will be used in the models. We fit a total of 6 linear models seen in table 3; 3 with log.crimes and 3 with log.per.cap.crime including each null model. The residual plots for all models are acceptable meaning the models meet assumption requirements and we can continue on to using partial f tests to compare. All regression outputs and residual plots can be found under Appendix 3.

Name	Model
null	log.per.cap.income ~ log.crimes

null.1	log.per.cap.income ~ log.per.cap.crime
lm.1	log.per.cap.income ~ log.crimes + region
lm.2	log.per.cap.income ~ log.per.cap.crime + region
lm.3	log.per.cap.income ~ log.crimes*region
lm.4	log.per.cap.income ~ log.per.cap.crime*region

Table 3: Models predicting log.per.cap.income.

We performed two f tests; one for the models with log.crimes and one for the models with log.per.cap.crime. Both tests preferred the additive model over the interaction model. These outputs can be found under Appendix 3. To compare the two additive models we used AIC and BIC estimations. Both AIC and BIC estimations prefer the model using log.crimes which can also be found under Appendix 3. Since interpretability is one of the most important things to keep in mind, we decided to pick the model using log.per.cap.crime over the model using log.crimes despite the results of the AIC and BIC estimates. This way both the response and the predictor are in the same units and have the same transformation.

As stated before the residual plots satisfy modeling assumptions coupled with the enhanced interpretability we are confident on choosing lm.2 (table 3) as the final and best model. The regression output is shown in table 4. Across the United States for every 1% increase in per-capita crime, there is an 0.04% increase in per-capita income. Baseline per-capita incomes differ by region with North Central (NC) being $15,490, North East (NE) being $17,290, South (S) being $14,440, and West (W) being $15,180. All regions, except for the W, have baseline incomes that are significantly different from the NC baseline. According to the model, income level varies by region but not in the way it varies with crime.

Variable	Coefficient Estimate	P-Value
Intercept	2.74	< 2e-16 ***
log.per.cap.crime	0.04	0.05 *
regionNE	0.11	3.99e-5 ***
regionS	-0.07	0.005 **
regionW	-0.02	0.42

Table 4: Regression output for model lm.2 (final model).

Predicting Per Capita Income Based on All Predictors

To begin finding the best model to predict log.per.cap.income based on all predictors, we first decided to remove pop, log.tot.income, and log.crimes from the dataset. Pop and

log.tot.income were used in the calculation of log.per.cap.income thus removing them was necessary. We used the VIF calculations (Appendix 3) to check our logic and see if any other variables should be removed due to collinearity. Both log.per.cap.crime and log.crimes had high VIF's, as log.crimes was used in the calculation of log.per.cap.crime, but log.per.cap.crime was significantly lower and it is in the same units as the response, thus we kept it over log.crimes. None of the variable selection methods are particularly good at working with categorical variables, so region was left out of the original variable selection but will be added back later.

We chose all subsets regression as the primary variable selection method because it considers all possible combinations of models. Using BIC generated by the all subsets regression, we determined the best model was model 7 with the lowest BIC of -772.0715. Table 5 shows the variables in model 7. The complete all subsets output can be found in the technical appendix under Appendix 3. To check the validity of the model we refit it in a standard linear regression to show the residual plots. The residual plots show that the model is valid as it meets modeling assumptions. The regression output and residual plots can be found under Appendix 3. Lastly, to confirm this base model we looked at the marginal model plots to check for any missing interactions and transformations. The plots, shown in the technical appendix (Appendix 3), also upholds the validity of our model.

Model 7
pop.18_34
pct.hs.grad
pct.bach.deg
pct.below.pov
pct.unemp
log.land.area
log.doctors

Table 5: Variables in the best model selected by all subsets regression.

Since we have a good starting model, we wanted to introduce region. To do so we fit a model with region interacting with all of the variables in model 7. The regression output and residual plots can be found under Appendix 3. From the output we dropped log.land.area:region, pop.18_34:region, and log.doctors:region because none of the interactions were significant. The AIC and BIC estimates (Appendix 3) that were calculated for each model showed that both prefer the model with interactions, thus that is the final model.

Variable	Coefficient Estimate	P-Value

Intercept	3.22	< 2e-16 ***
pop.18_34	-0.02	< 2e-16 ***
pct.hs.grad	-0.003	0.43
pct.bach.deg	0.01	5.24e-11 ***
pct.below.pov	-0.02	7.30e-12 ***
pct.unemp	0.02	0.0002 ***
log.land.area	-0.03	3.50e-10 ***
log.doctors	0.06	< 2e-16 ***
regionNE	0.22	0.47
regionS	-0.06	0.82
regionW	1.63	6.86e-06 ***
pct.hs.grad:regionNE	-0.004	0.35
pct.hs.grad:regionS	0.002	0.57
pct.hs.grad:regionW	-0.02	8.85e-06 ***
pct.bach.deg:regionNE	0.006	0.02 *
pct.bach.deg:regionS	-0.001	0.58
pct.bach.deg:regionW	0.006	0.02 *
pct.below.pov:regionNE	-0.002	0.60
pct.below.pov:regionS	0.007	0.05 .
pct.below.pov:regionW	-0.015	0.003 **
pct.unemp:regionNE	-0.008	0.27
pct.unemp:regionS	-0.02	0.0001 ***
pct.unemp:regionW	-0.02	0.003 **

Table 6: Regression output for final model from all subsets with interaction with region.

Next we repeated the same process with stepwise regression, starting without region and adding it in later. The BIC model is the exact same as the all subsets model which helps us confirm that the all subsets model is valid. The AIC model added pct.65_plus however, we know

when we add the region interaction the number of terms increases thus BIC may be a better indicator since it is primarily used for larger models. The stepwise BIC model with the region interaction produces the exact same model as well, again helping us confirm our final model. As a last check, we performed lasso regression with cross validation and the 1se lambda. This variable selection technique also found the same base model as the all subsets and would likely find the same interaction model with region. The full outputs for both the stepwise regression and the lasso regression can be seen in the technical appendix under Appendix 3.

Now that we have confirmed a final model we can begin to interpret the outputs in table 6. For the sake of focusing on the most important factors, we will only interpret the significant variables. For every 1 percentage point increase in the percent of the population between 18 and 34 , per capita income decreases by 1.98%. This is most likely due to the fact that younger people make less money. For every 1 percentage point increase in the percent of the population that has a bachelor's degree, per capita income increases by 1%. This can most likely be attributed to the fact that more education can lead to higher income. For every 1 percentage point increase in the percent of the population that is below the poverty rate, per capita income decreases by 1.98%. For every 1 percentage point increase in the percent of the population that is unemployed, per capita income increases by 2.02%. This is an interesting finding as we may have expected the opposite. A possible explanation is that government assistance for people in poverty is resulting in the increase. For every 1% increase in land area, there is a 0.03% decrease in per-capita income. For every 1% increase in the amount of doctors, there is a 0.06% increase in per-capita income. This is attributed to the high income of doctors compared to the general population. The baseline per capita income for the West is $5,100. The West is also the only region with a significant difference from the North Central region in each interaction. This could be due to the low sample size for the Western region, thus each observation is weighted more heavily. The North East region has a significant difference in income from the North Central region in terms of the percentage of those with bachelor's degrees. The Southern region has a significant difference in income from the North Central region in terms of the percentage of people that are unemployed. Using a larger confidence interval the same applies to those below the poverty line. There are probably a lot of lurking variables that are influencing these results thus we can not directly characterize these differences.

Missing Counties and States
Addressing the concern of missingness in the dataset, we found that there are 373 counties present in the dataset out of a total of 3,000 and 48 out of a total 51 states. Problems could arise if we considered using county as a predictor within our model. There are only about 12% of counties represented therefore, it may not be a big enough sample to be generalized for all counties. We also are not sure whether the sample was randomly selected or if all 3,000 counties were sampled but these 373 were the only ones that remained. However, if we were to consider state in the model, there may not be a problem with the missingness. Only 3 states are missing from the data set so about 94% of the US is represented. Lastly, in all of our modeling

we used region as a location classifier instead of county or state. All the regions are represented in the dataset, thus missing data is not a big concern in the current analysis.

5 Discussion

In this study, we were examining four main concerns related to average income per capita and its association with a county's economic health and social well-being. First, we wanted to characterize the relationships of all the variables in the dataset independently and explain them in terms of the variable descriptions. We determined that many of the associations are to be expected and can be explained in terms of the variable descriptions. Due to the similarity of the variables and possible collinearities doctors and hospital beds share some of the same strong correlations. Total income and per-capita crime both had high correlations to some variables due to them being used in the calculations of other variables. The education variables, pct.bach.deg and pct.hs.grad, had high correlations since those who have graduated highschool are included in those who have bachelor degrees. Per capita income made a great response variable because of its moderate associations with the majority of the variables.

Second, we wanted to see the relationship between per-capita income and crime rate and how it is affected by different regions of the country. Instead of using the raw crime rate variable we created the per capita crime variable so that both the response and the predictor were measured in the same units. We found that across the United States for every 1% increase in per-capita crime, there is a 0.04% increase in per-capita income and baseline per-capita income differs significantly by region, except for the West. Overall, income level varies by region but not in the way it varies with crime.

Third, the most important concern we addressed was to find the best combination of variables to predict per-capita income. Using all subsets as our primary variable selection method and checking with other selection methods, we were able to fit a model with high predictive power while accounting for interpretability. The final model determined that possibly due to the younger population making less money, for every 1 percentage point increase in the percent of the population between 18 and 34, per capita income decreases by 1.98%. It also found that for every 1 percentage point increase in the percent of the population that has a bachelor's degree, per capita income increases by 1%. This can most likely be attributed to the fact that more education usually leads to higher income. A 1 unit percent increase of the population that is below the poverty rate decreases per capita income by 1.98%. For every 1 percentage point increase in the percent of the population that is unemployed, per capita income increases by 2.02%. This is an interesting finding as we may have expected the opposite. A possible explanation is that government assistance for people in poverty is actually resulting in an increase in average income. For every 1% increase in land area, there is a 0.03% decrease in per-capita income. For every 1% increase in the amount of doctors, there is a 0.06% increase in per-capita income. This is most likely attributed to the high income of doctors compared to the general population. Possibly due to the low sample size, the West is the only region with a significant difference from the North Central region in each interaction. The North East region

has a significant difference in income from the North Central region in terms of the percentage of those with bachelor's degrees. The Southern region has a significant difference in income from the North Central region in terms of the percentage of people that are unemployed. Generally, there are probably lurking variables that are influencing these results contributing to these differences.

We wanted to be more accurate in our interpretations of the region interactions however, we would like the social scientists to be able to understand the findings. We left them at broad generalizations that are still useful and hope in the future we can explore them more thoroughly. Had time permitted we would have also liked to explore interactions between the continuous variables. This may have led to an even more accurate model with higher predictive power. Lastly, we decided that the missingness of counties and states in the dataset was not the biggest concern right now since only region was used in the current analysis and was fully represented in the data set. In the future, we would like to consider a way to work with the state variable as our main categorical location variable because it also had good representation in the data. Using state would provide even more distinct results on how per capita income is affected given the different factors measured in the data.

Overall, all findings are descriptive and fairly accurate in describing how a county's economic health and social well-being can affect average income per capita.

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Appendices: CDI Analyses

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Contents

Appendix 1. Initial Data/Library Imports & Exploration

Loaded packages needed for the analyses along with the initial data set.

```
library(gtsummary)
library(readr)
library(tidyverse)
library(dplyr)
library(leaps)
library(MASS)
library(car)
library(glmnet)
library(kableExtra)
library(GGally)
library(grid)
library(gridExtra)
library(ggplotify)
library(reshape2)
library(ggplot2)
library(arm)
library(corrplot)

cdi <- read.table("cdi.dat")
```

Table 1: Summary of Continuous Variables

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	SD
land.area	15.0	451.25	656.50	1041.41	946.75	20062.00	1549.92
pop	100043.0	139027.25	217280.50	393010.92	436064.50	8863164.00	601987.02
pop.18_34	16.4	26.20	28.10	28.57	30.02	49.70	4.19
pop.65_plus	3.0	9.88	11.75	12.17	13.62	33.80	3.99
doctors	39.0	182.75	401.00	988.00	1036.00	23677.00	1789.75
hosp.beds	92.0	390.75	755.00	1458.63	1575.75	27700.00	2289.13
crimes	563.0	6219.50	11820.50	27111.62	26279.50	688936.00	58237.51
pct.hs.grad	46.6	73.88	77.70	77.56	82.40	92.90	7.02
pct.bach.deg	8.1	15.28	19.70	21.08	25.33	52.30	7.65
pct.below.pov	1.4	5.30	7.90	8.72	10.90	36.30	4.66
pct.unemp	2.2	5.10	6.20	6.60	7.50	21.30	2.34
per.cap.income	8.9	16.12	17.76	18.56	20.27	37.54	4.06
tot.income	1141.0	2311.00	3857.00	7869.27	8654.25	184230.00	12884.32
per.cap.crime	4.6	38.10	52.43	57.29	72.60	295.99	27.33

Started by creating a data set with only the continuous variables to be used to created the table of summary statistics.

Created table of unique values of each variable. This is mostly to see which categorical variable will be best to include in the modeling later in the analyses.

Created table of summary statistics for continuous variables and a frequency table for the best categorical variable, region.

```
cdinumeric <- cdi[,-c(1,2,3,17)] %>%
  mutate(per.cap.crime=(cdi$crimes/cdi$pop)*1000, per.cap.income=per.cap.income/1000)



apply(cdinumeric,2,function(x) c(summary(x),SD=sd(x))) %>%
  as.data.frame %>%
  t() %>%
  round(digits=2) %>%
  kbl(booktabs=T,caption="Summary of Continuous Variables") %>%
  kable_minimal()



apply(cdi,2,function(x) {length(unique(x))}) %>%
  kbl(booktabs=T,col.names="unique values",caption="Unique Values") %>%
  kable_minimal(full_width=F)



cdi_region <- cdi %>%
  dplyr::select(region) %>%
  group_by(region) %>%
  summarise(Freq.=n())

cdi_region %>%
  kbl(booktabs=T,caption="Frequency of Region", align = "ll") %>%
  kable_minimal()
```

Table 2: Unique Values

	unique values
id	440
county	373
state	48
land.area	384
pop	440
pop.18_34	149
pop.65_plus	137
doctors	360
hosp.beds	391
crimes	437
pct.hs.grad	223
pct.bach.deg	220
pct.below.pov	155
pct.unemp	97
per.cap.income	436
tot.income	428
region	4

Table 3: Frequency of Region

region	Freq.
NC	108
NE	103
S	152
W	77

Created per.cap.crime variable and mutated per.cap.income so that they are measured in the same units.
Visualized variables to see if any transformations were needed using histograms and scatter plots.

```r
cdinumeric <- cdi[,-c(1,2,3,17)] %>%
  mutate(per.cap.crime=(cdi$crimes/cdi$pop)*1000, per.cap.income=per.cap.income/1000)
```

```r
ggplot(gather(cdinumeric), aes(value))+
  geom_histogram(bins=30) +
  facet_wrap(~key, scales ='free_x')+
  theme_minimal()
```
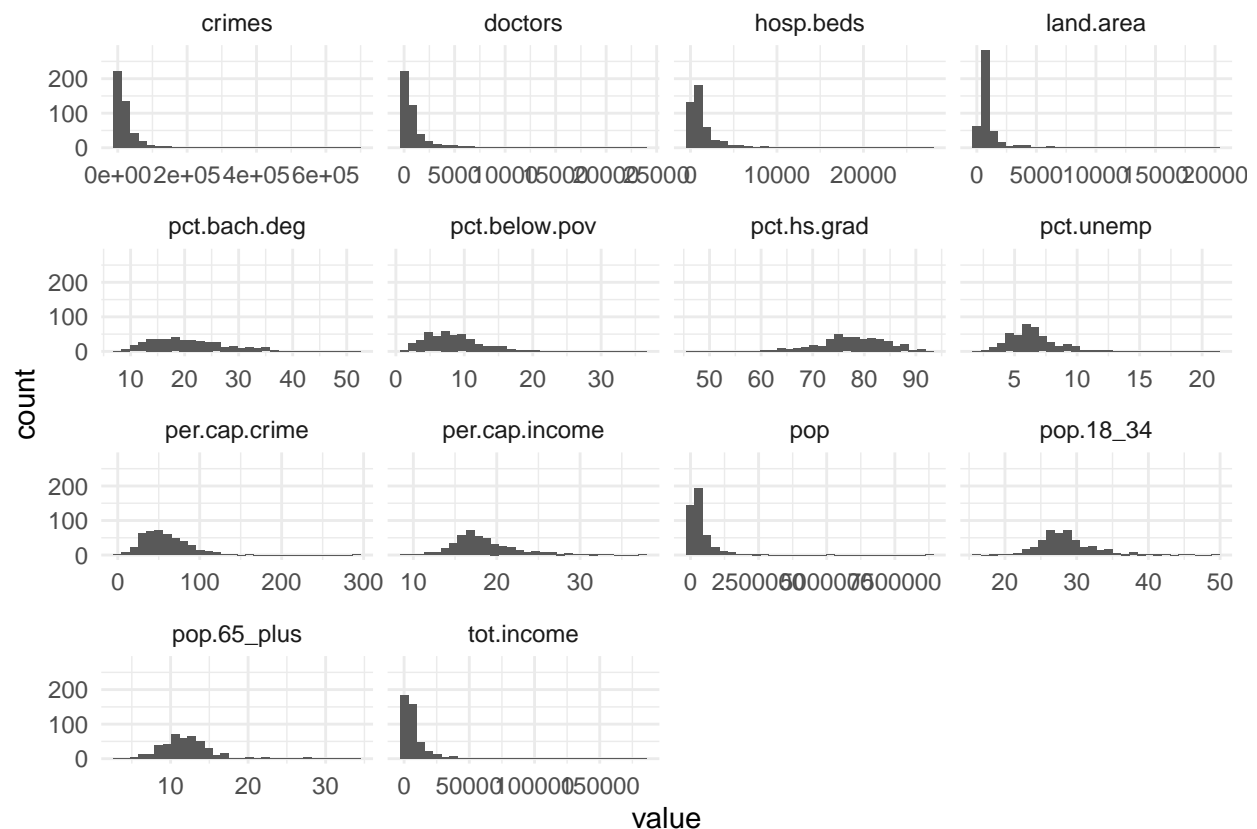


```r
cdinumeric.reg <-data.frame(cdinumeric,region=cdi$region)

scatter.builder <-function(df,yvar="per.cap.income") {
  result <- NULL
  y.index <- grep(yvar,names(df))
  for (xvar in names(df)[-y.index]) {
    d <- data.frame(xx=df[,xvar],yy=df[,y.index])
    if(mode(df[,xvar])=="numeric") {
      p <- ggplot(d,aes(x=xx,y=yy)) +
        geom_point() +
        ggtitle("") +
        xlab(xvar) +
        ylab(yvar)
```

```
    }
    else {
      p <- ggplot(d,aes(x=xx,y=yy)) +
      geom_boxplot(notch=F) +
      ggtitle("") +
      xlab(xvar) +
      ylab(yvar)
    }
    result <- c(result,list(p))
    }
  return(result)
  }
save <- grid.arrange(grobs=scatter.builder(cdinumeric))
```
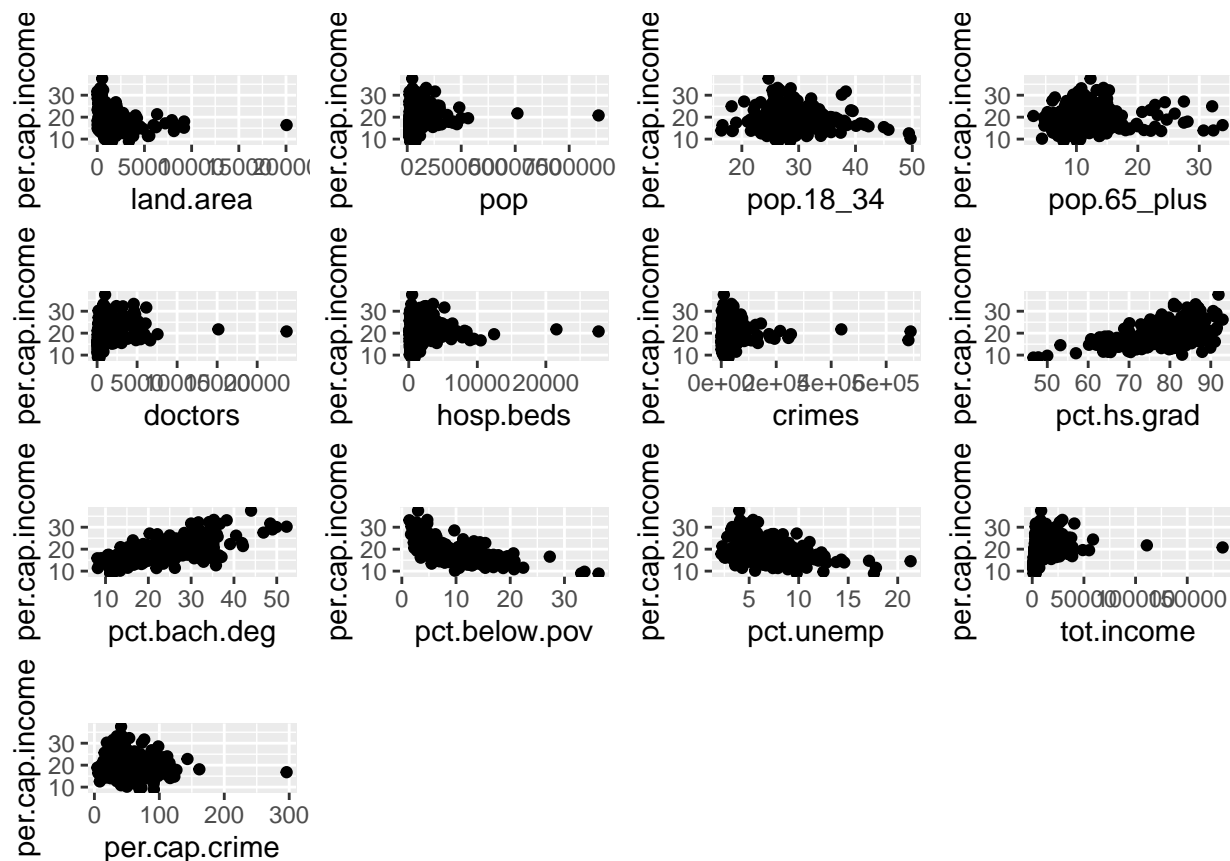

Appendix 2. Transformations and Correlations

Created a new data set with transformed variables and displayed the new histograms.

```
cdilogs <- cdinumeric %>%
  mutate(log.crimes=log(crimes),
         log.tot.income=log(tot.income),
         log.land.area=log(land.area),
         log.doctors=log(doctors),
         log.hosp.beds=log(hosp.beds),
```

```
          log.per.cap.crime=log(per.cap.crime),
          log.per.cap.income=log(per.cap.income)) %>%
  dplyr::select(-c(crimes, tot.income,
                   land.area, doctors,
                   hosp.beds,per.cap.crime,
                   per.cap.income))

cdilogs.reg <- data.frame(cdilogs,region=cdi$region)

ggplot(gather(cdilogs), aes(value))+
  geom_histogram(bins=30) +
  facet_wrap(~key, scales ='free_x')+
  theme_minimal()
```
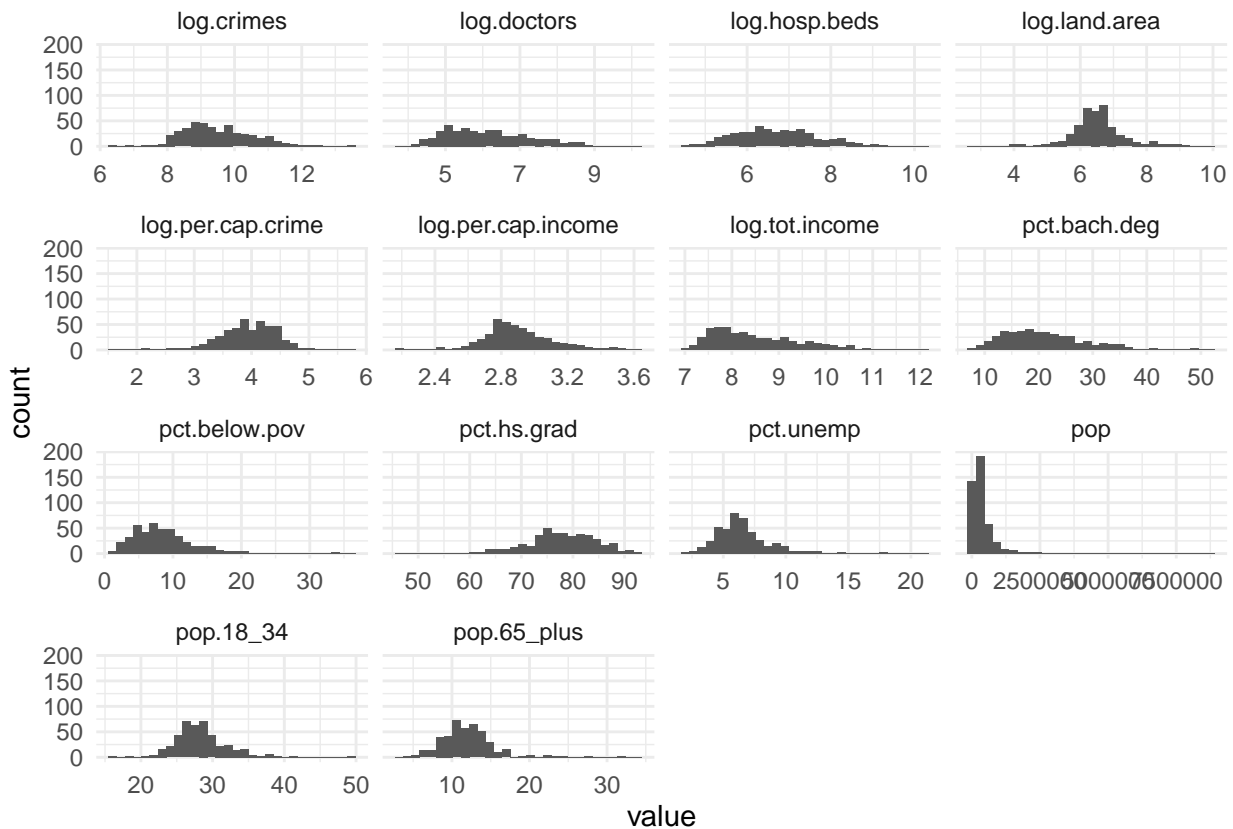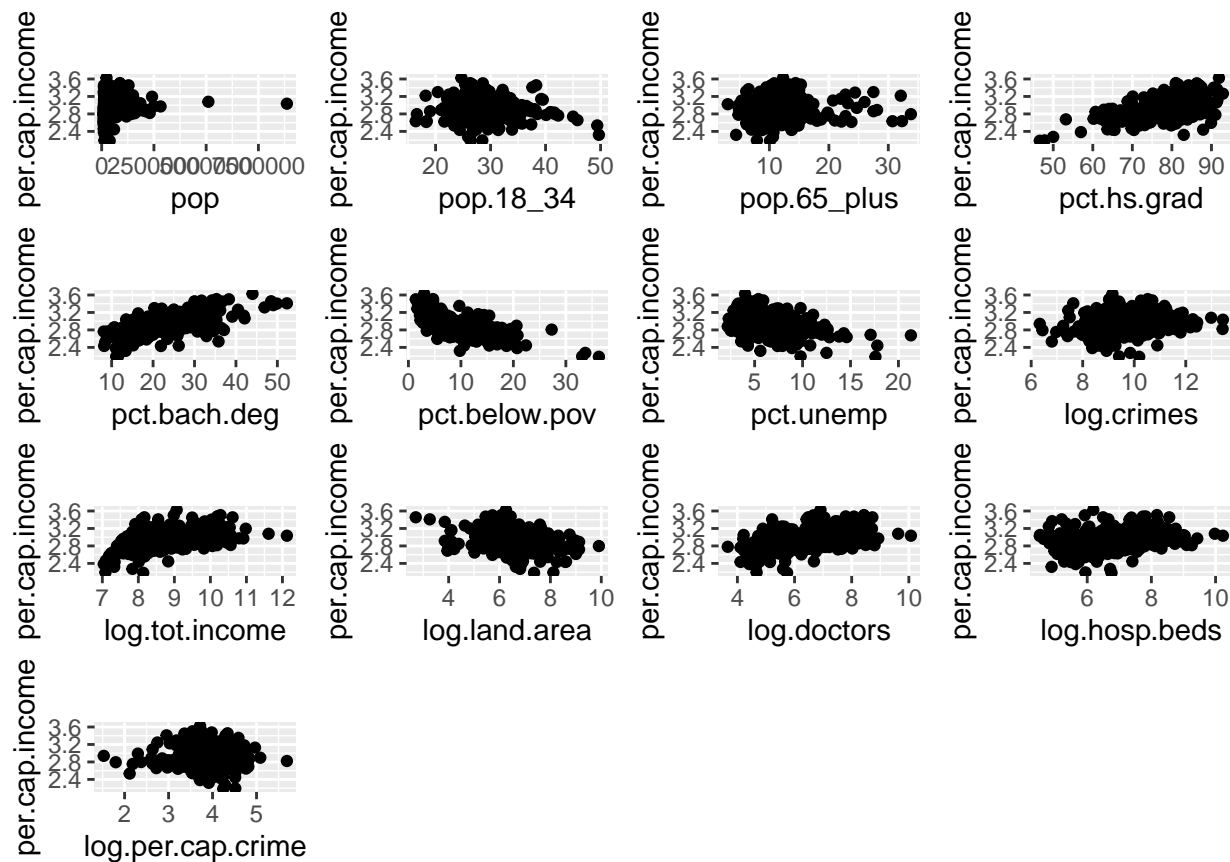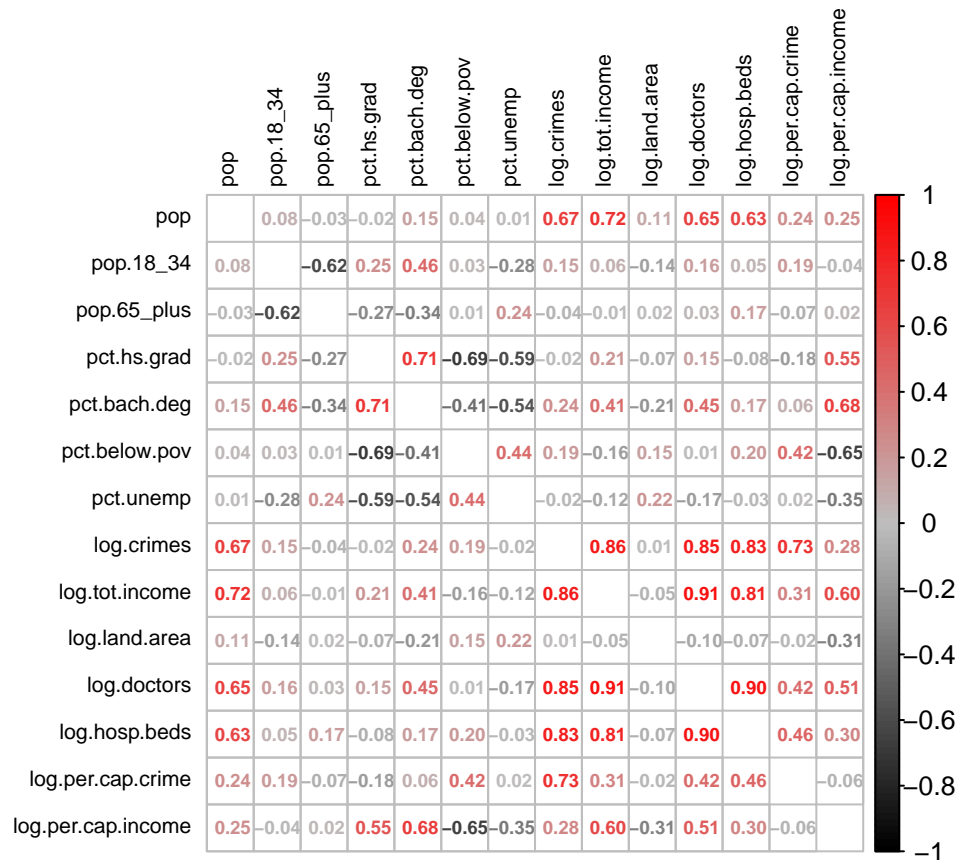


```
grid.arrange(grobs=scatter.builder(cdilogs))
```

Created a heat map correlation matrix to check for high correlations among the predictors and raise awareness to any problems of multicollinearity.

```r
m <- cor(cdilogs)
msave <- corrplot(m,
                  method=c("number"),
                  diag=FALSE,
                  number.cex=0.6,
                  tl.cex = .7,
                  tl.col = "black",
                  col = colorRampPalette(c("black","gray","red"))(100))
```

	pop	pop.18_34	pop.65_plus	pct.hs.grad	pct.bach.deg	pct.below.pov	pct.unemp	log.crimes	log.tot.income	log.land.area	log.doctors	log.hosp.beds	log.per.cap.crime	log.per.cap.income
pop		0.08	-0.03	-0.02	0.15	0.04	0.01	0.67	0.72	0.11	0.65	0.63	0.24	0.25
pop.18_34	0.08		-0.62	0.25	0.46	0.03	-0.28	0.15	0.06	-0.14	0.16	0.05	0.19	-0.04
pop.65_plus	-0.03	-0.62		-0.27	-0.34	0.01	0.24	-0.04	-0.01	0.02	0.03	0.17	-0.07	0.02
pct.hs.grad	-0.02	0.25	-0.27		0.71	-0.69	-0.59	-0.02	0.21	-0.07	0.15	-0.08	-0.18	0.55
pct.bach.deg	0.15	0.46	-0.34	0.71		-0.41	-0.54	0.24	0.41	-0.21	0.45	0.17	0.06	0.68
pct.below.pov	0.04	0.03	0.01	-0.69	-0.41		0.44	0.19	-0.16	0.15	0.01	0.20	0.42	-0.65
pct.unemp	0.01	-0.28	0.24	-0.59	-0.54	0.44		-0.02	-0.12	0.22	-0.17	-0.03	0.02	-0.35
log.crimes	0.67	0.15	-0.04	-0.02	0.24	0.19	-0.02		0.86	0.01	0.85	0.83	0.73	0.28
log.tot.income	0.72	0.06	-0.01	0.21	0.41	-0.16	-0.12	0.86		-0.05	0.91	0.81	0.31	0.60
log.land.area	0.11	-0.14	0.02	-0.07	-0.21	0.15	0.22	0.01	-0.05		-0.10	-0.07	-0.02	-0.31
log.doctors	0.65	0.16	0.03	0.15	0.45	0.01	-0.17	0.85	0.91	-0.10		0.90	0.42	0.51
log.hosp.beds	0.63	0.05	0.17	-0.08	0.17	0.20	-0.03	0.83	0.81	-0.07	0.90		0.46	0.30
log.per.cap.crime	0.24	0.19	-0.07	-0.18	0.06	0.42	0.02	0.73	0.31	-0.02	0.42	0.46		-0.06
log.per.cap.income	0.25	-0.04	0.02	0.55	0.68	-0.65	-0.35	0.28	0.60	-0.31	0.51	0.30	-0.06	

Appendix 3. Variable Selection and Regression Analysis

Per Capita Income ~ Per Capita Crime and Region

The first modeling was done to predict per-capita income based on crime rate and region.

We fit a total of 6 linear models seen in table 3; 3 with log.crimes and 3 with log.per.cap.crime including each null model. The best model was assesed using Partial F-Tests, AIC/BIC, and checking model assumptions with residual plots.

```
null <- lm(log.per.cap.income ~ log.crimes, data=cdilogs.reg)
par(mfrow=c(2,2))
plot(null)
```
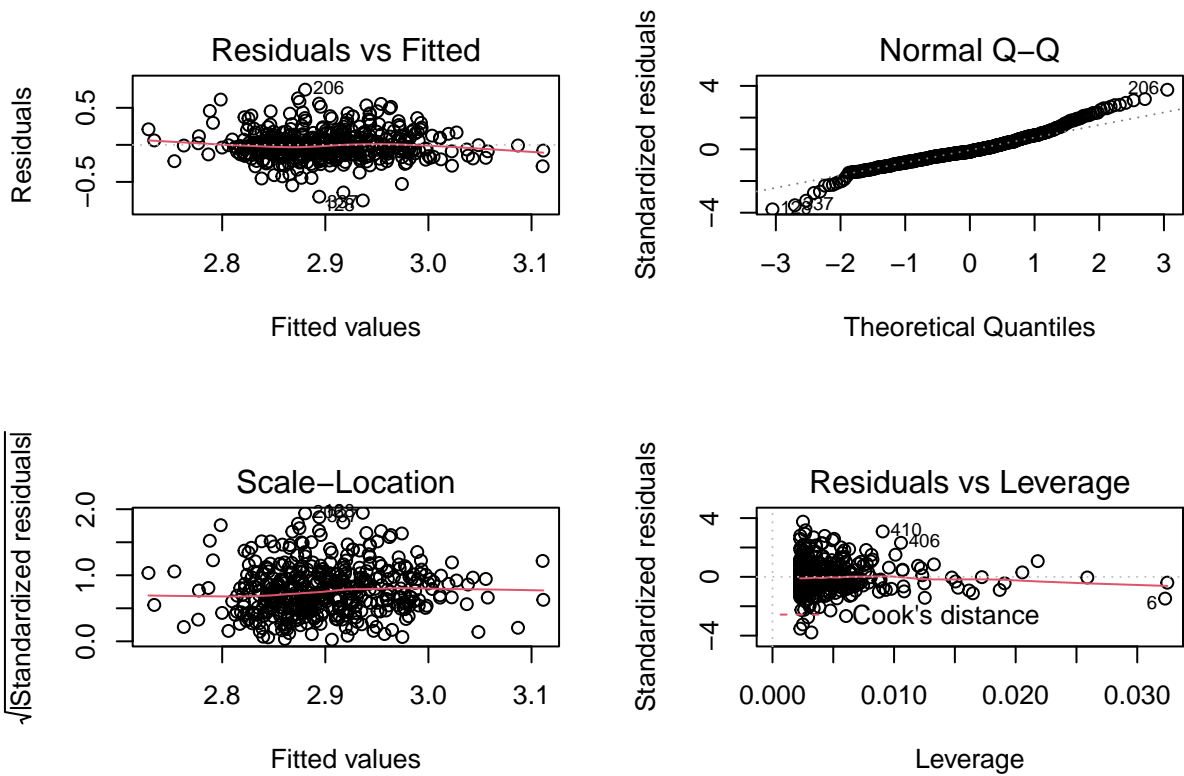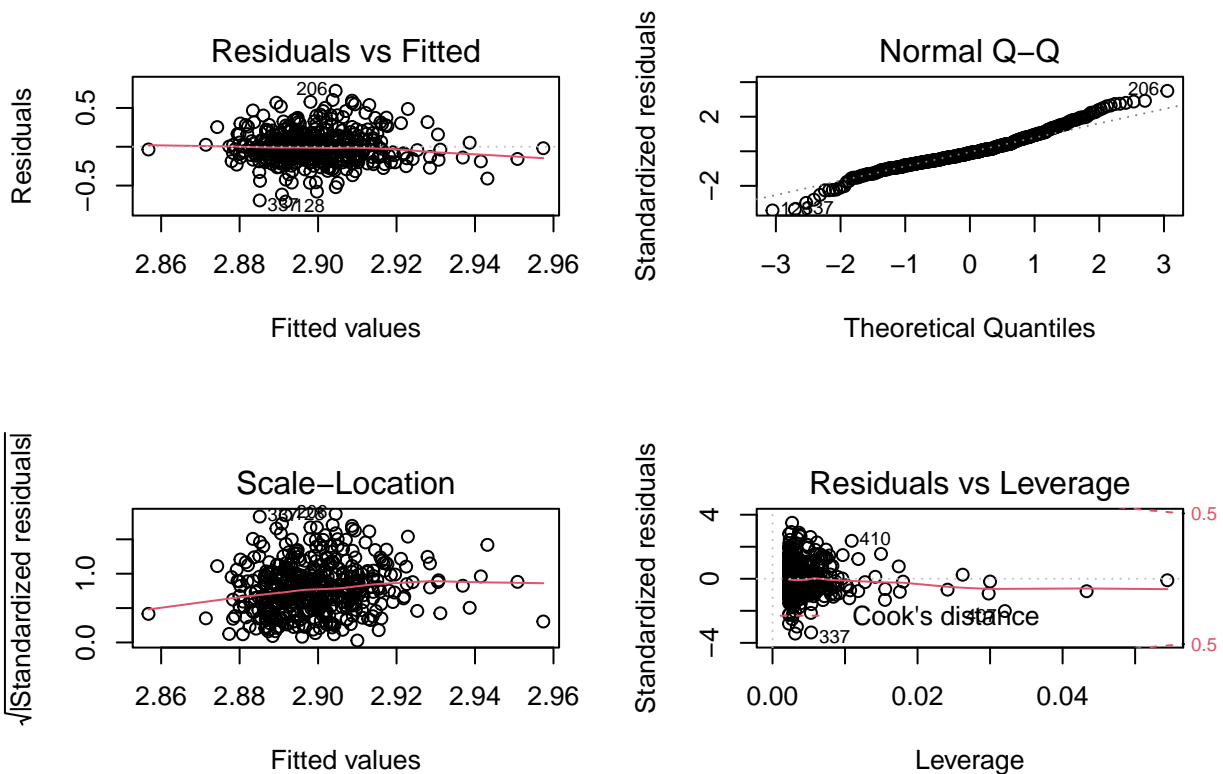
```
null.2 <- lm(log.per.cap.income ~ log.per.cap.crime, data=cdilogs.reg)
par(mfrow=c(2,2))
plot(null.2)
```
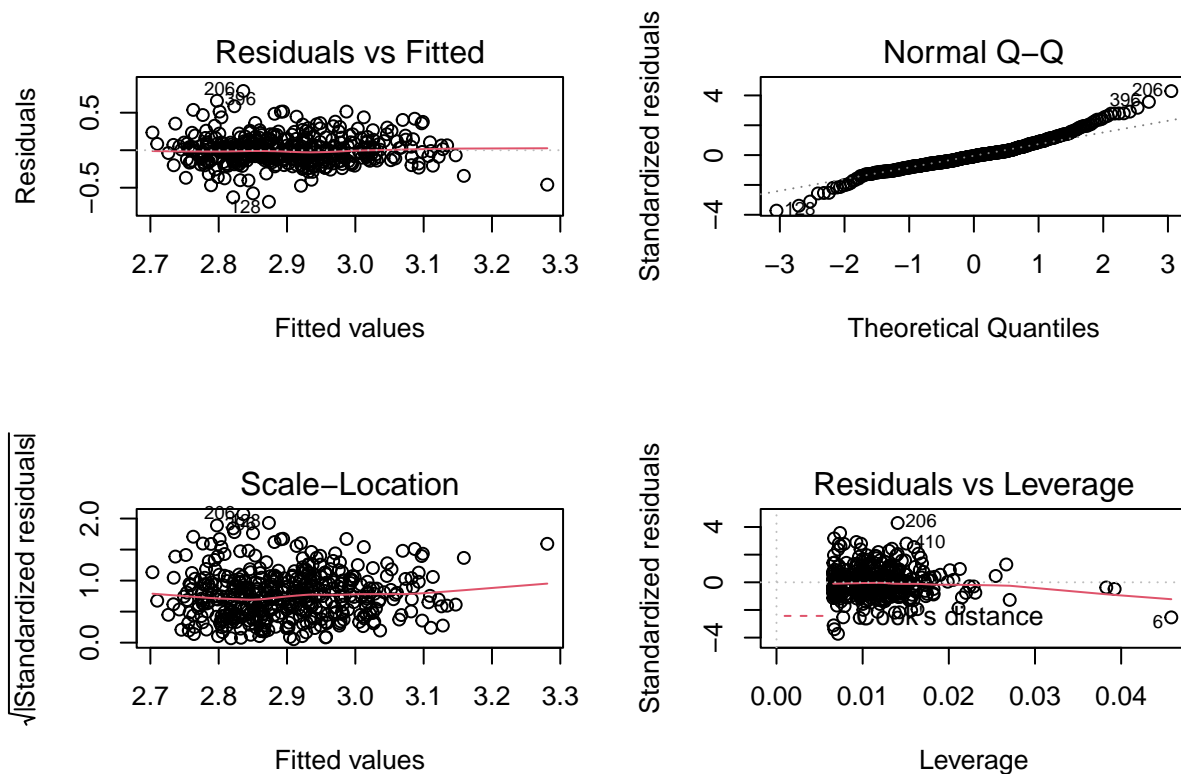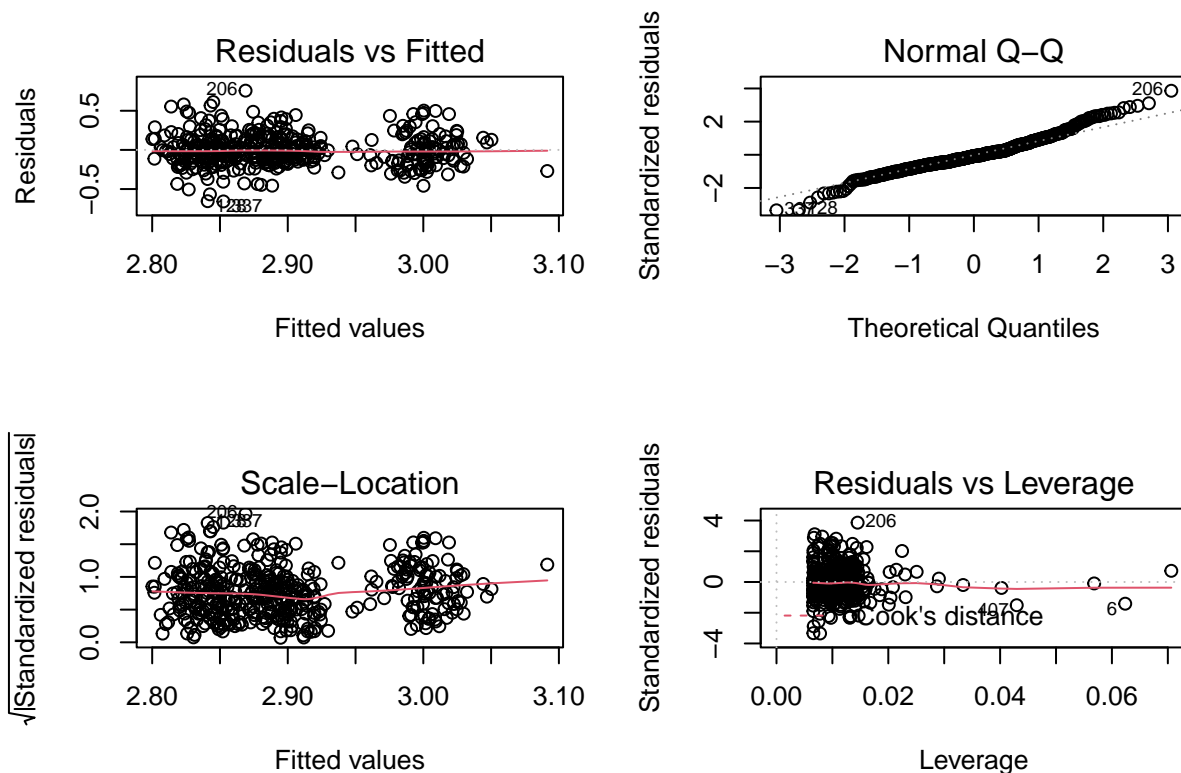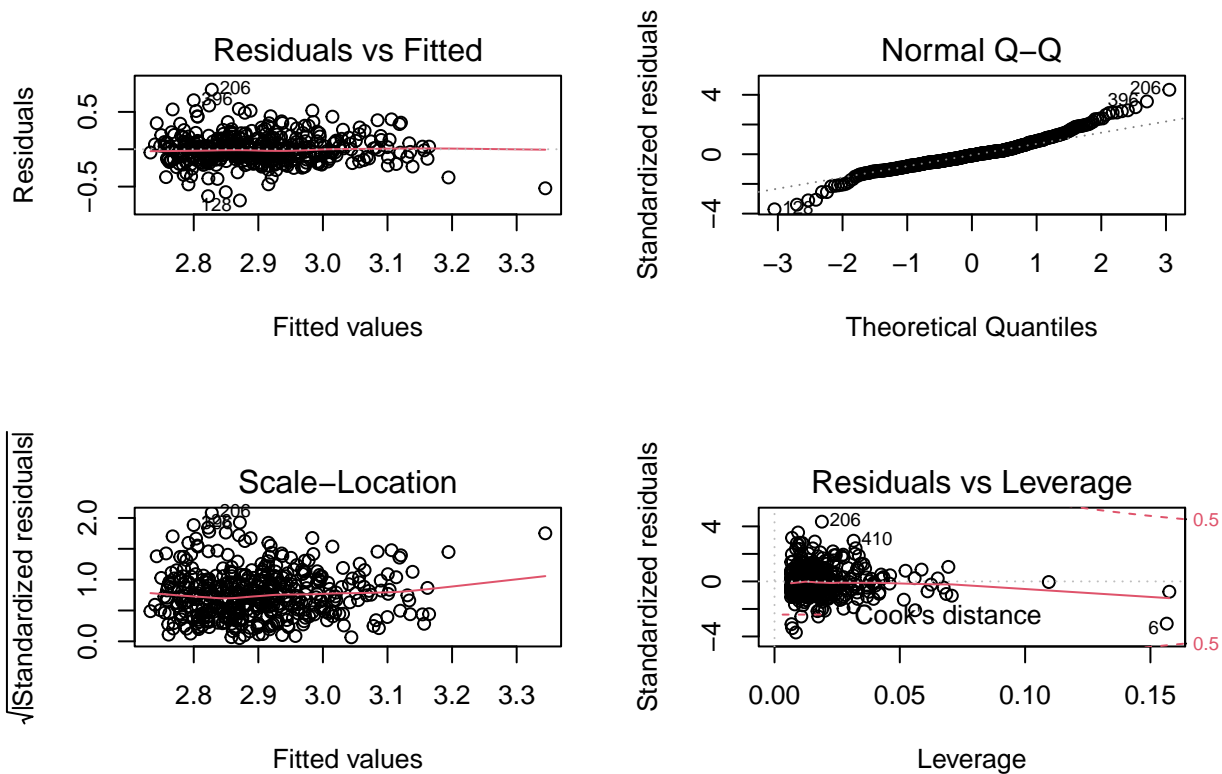
```
lm.1 <- lm(log.per.cap.income ~ log.crimes + region, data=cdilogs.reg)
summary(lm.1)
```

```
##
## Call:
## lm(formula = log.per.cap.income ~ log.crimes + region, data = cdilogs.reg)
##
## Residuals:
##      Min      1Q   Median      3Q      Max
## -0.68757 -0.10557 -0.01422  0.08905  0.78946
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.280676   0.079812  28.575  < 2e-16 ***
## log.crimes   0.066695   0.008421   7.920 2.00e-14 ***
## regionNE     0.104458   0.025531   4.091 5.11e-05 ***
## regionS     -0.086983   0.023618  -3.683  0.00026 ***
## regionW     -0.055280   0.028167  -1.963  0.05033 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1854 on 435 degrees of freedom
## Multiple R-squared:  0.2032, Adjusted R-squared:  0.1959
## F-statistic: 27.74 on 4 and 435 DF,  p-value: < 2.2e-16
```

```
par(mfrow=c(2,2))
plot(lm.1)
```

```
lm.2 <- lm(log.per.cap.income ~ log.per.cap.crime + region, data=cdilogs.reg)
summary(lm.2)
```

```
##
## Call:
## lm(formula = log.per.cap.income ~ log.per.cap.crime + region,
##      data = cdilogs.reg)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.65832 -0.11431 -0.01548  0.10838  0.75657
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        2.73543    0.08386  32.617  < 2e-16 ***
## log.per.cap.crime  0.04243    0.02148   1.975  0.04885 *
## regionNE           0.11457    0.02760   4.151 3.99e-05 ***
## regionS           -0.07456    0.02624  -2.841  0.00471 **
## regionW           -0.02426    0.03002  -0.808  0.41952
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1974 on 435 degrees of freedom
## Multiple R-squared:  0.09645,    Adjusted R-squared:  0.08814
## F-statistic: 11.61 on 4 and 435 DF,  p-value: 5.776e-09
```

```
par(mfrow=c(2,2))
plot(lm.2)
```

```
lm.3 <- lm(log.per.cap.income ~ log.crimes*region, data=cdilogs.reg)
summary(lm.3)
```

```
##
## Call:
## lm(formula = log.per.cap.income ~ log.crimes * region, data = cdilogs.reg)
##
## Residuals:
##      Min      1Q   Median      3Q      Max
## -0.68552 -0.10418 -0.01444  0.08302  0.79755
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)         2.42902    0.14579  16.661  < 2e-16 ***
## log.crimes          0.05064    0.01566   3.233  0.00132 **
## regionNE           -0.18407    0.21515  -0.856  0.39272
## regionS            -0.19717    0.21211  -0.930  0.35312
## regionW            -0.31439    0.24465  -1.285  0.19947
## log.crimes:regionNE  0.03122    0.02311   1.351  0.17749
## log.crimes:regionS   0.01211    0.02228   0.544  0.58696
## log.crimes:regionW   0.02727    0.02523   1.081  0.28028
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1855 on 432 degrees of freedom
## Multiple R-squared:  0.2073, Adjusted R-squared:  0.1945
## F-statistic: 16.14 on 7 and 432 DF,  p-value: < 2.2e-16
```

```
par(mfrow=c(2,2))
plot(lm.3)
```

```
lm.4 <- lm(log.per.cap.income ~ log.per.cap.crime*region, data=cdilogs.reg)
summary(lm.4)
```

```
##
## Call:
## lm(formula = log.per.cap.income ~ log.per.cap.crime * region,
##     data = cdilogs.reg)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.65410 -0.11829 -0.01708  0.10399  0.76628
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                 2.765458   0.127953  21.613   <2e-16 ***
## log.per.cap.crime           0.034535   0.033270   1.038    0.300
## regionNE                    0.008122   0.194080   0.042    0.967
## regionS                    -0.025139   0.226872  -0.111    0.912
## regionW                    -0.164523   0.375669  -0.438    0.662
## log.per.cap.crime:regionNE  0.029236   0.052324   0.559    0.577
## log.per.cap.crime:regionS  -0.011035   0.055544  -0.199    0.843
## log.per.cap.crime:regionW   0.034948   0.092675   0.377    0.706
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.198 on 432 degrees of freedom
## Multiple R-squared:  0.09773,    Adjusted R-squared:  0.08311
## F-statistic: 6.685 on 7 and 432 DF,  p-value: 1.575e-07
```

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```
par(mfrow=c(2,2))
plot(lm.4)
```
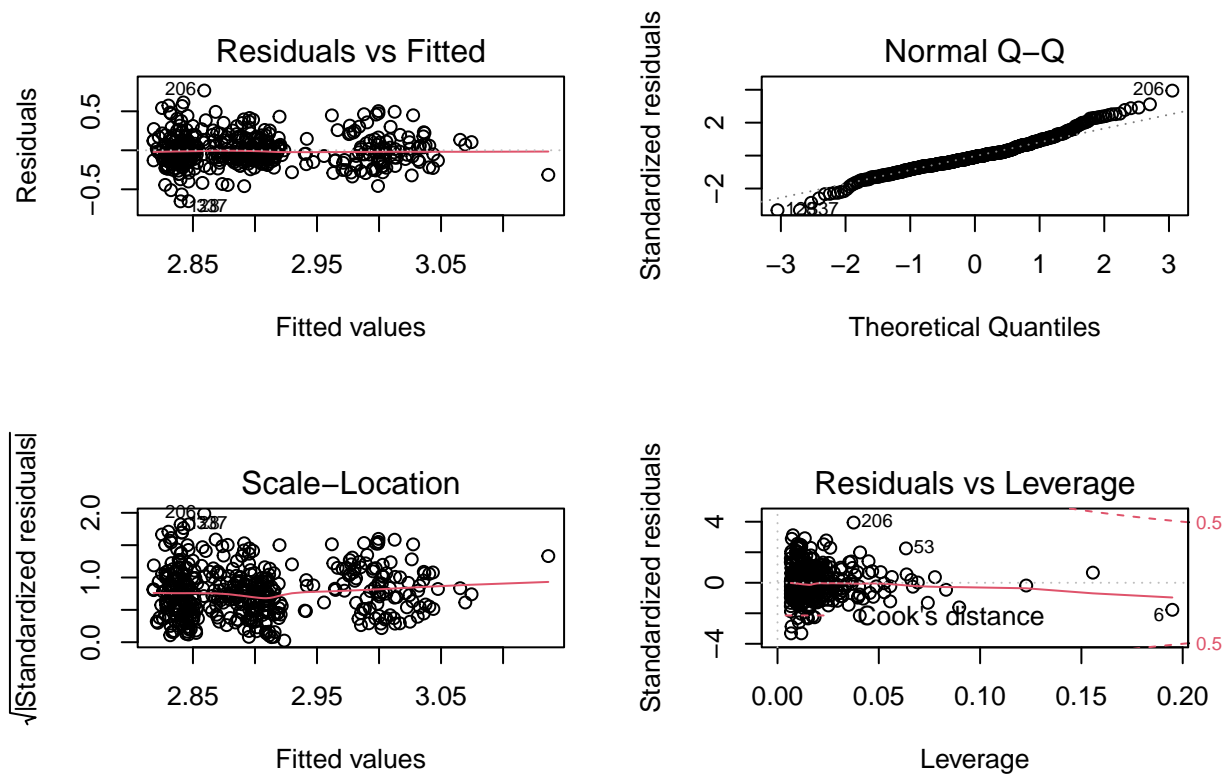


```
anova(null.2, lm.2, lm.4)
```

```
## Analysis of Variance Table
##
## Model 1: log.per.cap.income ~ log.per.cap.crime
## Model 2: log.per.cap.income ~ log.per.cap.crime + region
## Model 3: log.per.cap.income ~ log.per.cap.crime * region
##   Res.Df    RSS Df Sum of Sq       F    Pr(>F)
## 1    438 18.697
## 2    435 16.952  3   1.74465 14.8407 3.263e-09 ***
## 3    432 16.928  3   0.02408  0.2048    0.893
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(null, lm.1, lm.3)
```

```
## Analysis of Variance Table
##
## Model 1: log.per.cap.income ~ log.crimes
## Model 2: log.per.cap.income ~ log.crimes + region
## Model 3: log.per.cap.income ~ log.crimes * region
##   Res.Df    RSS Df Sum of Sq       F    Pr(>F)
## 1    438 17.271
## 2    435 14.949  3   2.32194 22.4823 1.523e-13 ***
```

	df	AIC	BIC
lm.1	6	-227.4746	-202.9539
lm.2	6	-172.1347	-147.6140

```
## 3    432 14.872  3   0.07678  0.7434    0.5266
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
data.frame(AIC=AIC(lm.1,lm.2),BIC=BIC(lm.1,lm.2))[,-3] %>%
  kbl(booktabs=T,col.names=c("df","AIC","BIC")) %>%
  kable_minimal(full_width=F)
```

Per Capita Income ~ All Predictors

Created new data sets, one without region and one with region to be used in the different varaible selection techniques.

We also removed pop, log.tot.income, and log.crimes from the dataset. Pop and log.tot.income were used in the calculation of log.per.cap.income thus removing them was necessary. We used the VIF calculation to check our logic and see if any other variables should be removed due to collinearity.

```
cdilogs.mod <- cdilogs %>%
  dplyr::select(-c(log.crimes, log.tot.income, pop))

cdilogs.mod.reg <-data.frame(cdilogs.mod,region=cdi$region)
```

```
vif <- as.data.frame(vif(lm(log.per.cap.income ~ ., data=cdilogs)))
vif
```

```
##                    vif(lm(log.per.cap.income ~ ., data = cdilogs))
## pop                                                       2.506851
## pop.18_34                                                 2.745454
## pop.65_plus                                               2.187081
## pct.hs.grad                                               4.050146
## pct.bach.deg                                              6.295664
## pct.below.pov                                             5.442001
## pct.unemp                                                 1.967483
## log.crimes                                              187.000319
## log.tot.income                                          125.673001
## log.land.area                                             1.372039
## log.doctors                                              17.324503
## log.hosp.beds                                             9.716302
## log.per.cap.crime                                        41.543524
```

All Subsets

Performed an all subsets regression on all the predictors (not including region). Then used BIC to pick the best model.

Once the best model was chosen we refit it in a standard linear regression to show the residual plots to check the validity of the model.

As a last check we looked at the marginal model plots to catch any possible missing interactions or transformations.
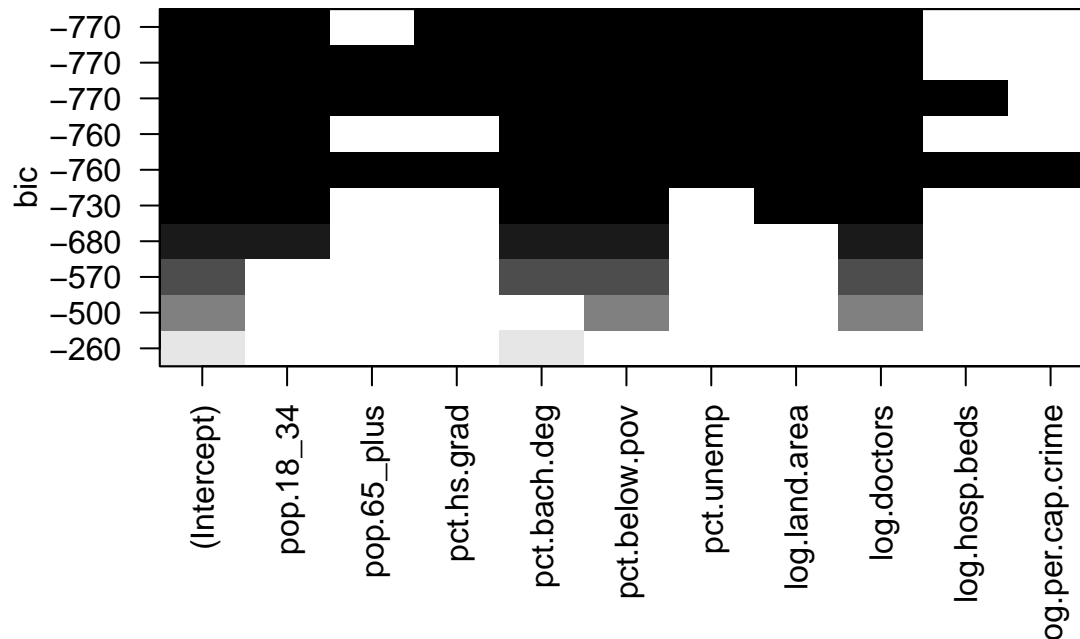
```
all.subs <- regsubsets(log.per.cap.income ~ ., data=cdilogs.mod, nvmax = 10)
outputs <- summary(all.subs)
outputs$adjr2
```

```
##  [1] 0.4570147 0.6923589 0.7406912 0.8005025 0.8237080 0.8369929 0.8426532
##  [8] 0.8439334 0.8441776 0.8441968
```

```
outputs$bic
```

```
##  [1] -257.5260 -502.4302 -572.5538 -682.8532 -732.1894 -761.5908 -772.0715
##  [8] -770.5990 -766.2235 -761.2151
```

```
plot(all.subs)
```



```
coef(all.subs, 7)
```

```
##   (Intercept)      pop.18_34    pct.hs.grad   pct.bach.deg pct.below.pov
##   3.314739762   -0.013900201   -0.004406396    0.015385301   -0.024278371
##     pct.unemp  log.land.area    log.doctors
##   0.010603691   -0.035674062    0.060676872
```

```
best.allsubs <- lm(log.per.cap.income ~ pct.hs.grad + pct.bach.deg+ pct.below.pov+
                   pct.unemp+ log.land.area +log.doctors, data=cdilogs.mod)
summary(best.allsubs)
```
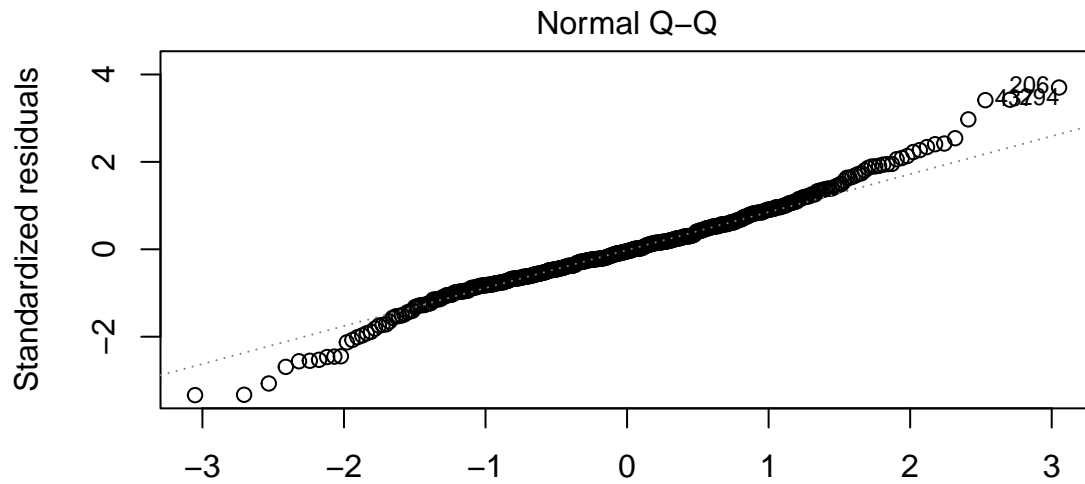
```
##
## Call:
```

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```
## lm(formula = log.per.cap.income ~ pct.hs.grad + pct.bach.deg +
##     pct.below.pov + pct.unemp + log.land.area + log.doctors,
##     data = cdilogs.mod)
##
## Residuals:
##      Min      1Q   Median      3Q      Max
## -0.31559 -0.05702 -0.00446  0.05381  0.34933
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3.002583   0.104586  28.709  < 2e-16 ***
## pct.hs.grad   -0.004979   0.001261  -3.950 9.12e-05 ***
## pct.bach.deg   0.011336   0.001010  11.228  < 2e-16 ***
## pct.below.pov -0.028776   0.001406 -20.470  < 2e-16 ***
## pct.unemp      0.013421   0.002524   5.317 1.70e-07 ***
## log.land.area -0.031293   0.005554  -5.635 3.16e-08 ***
## log.doctors    0.066681   0.004651  14.337  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0956 on 433 degrees of freedom
## Multiple R-squared:  0.7891, Adjusted R-squared:  0.7862
## F-statistic:   270 on 6 and 433 DF,  p-value: < 2.2e-16
```
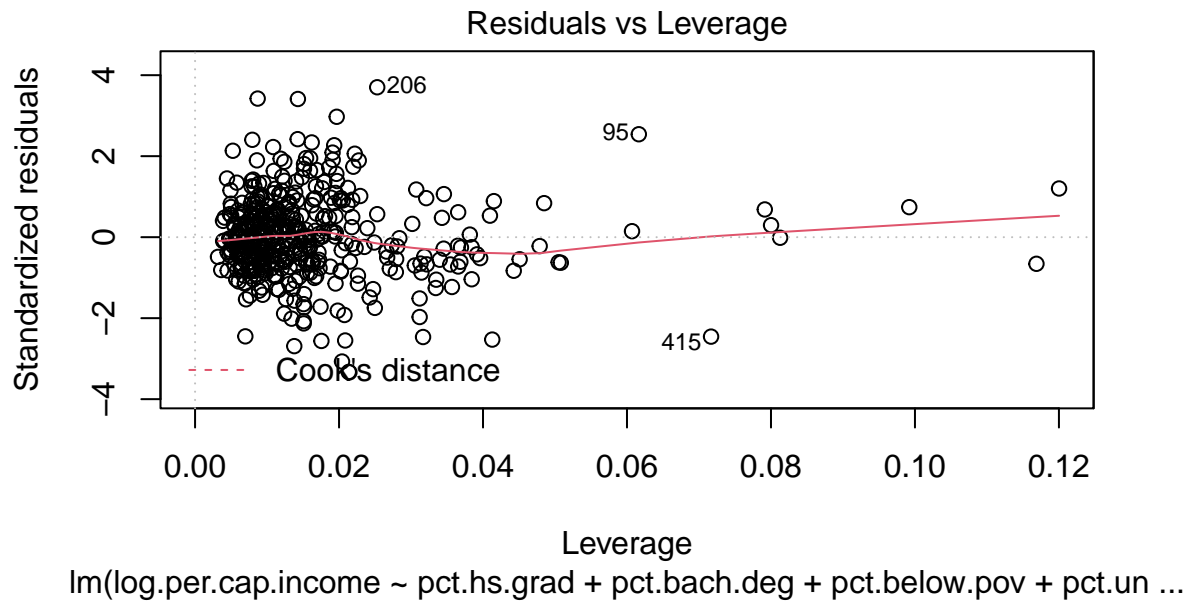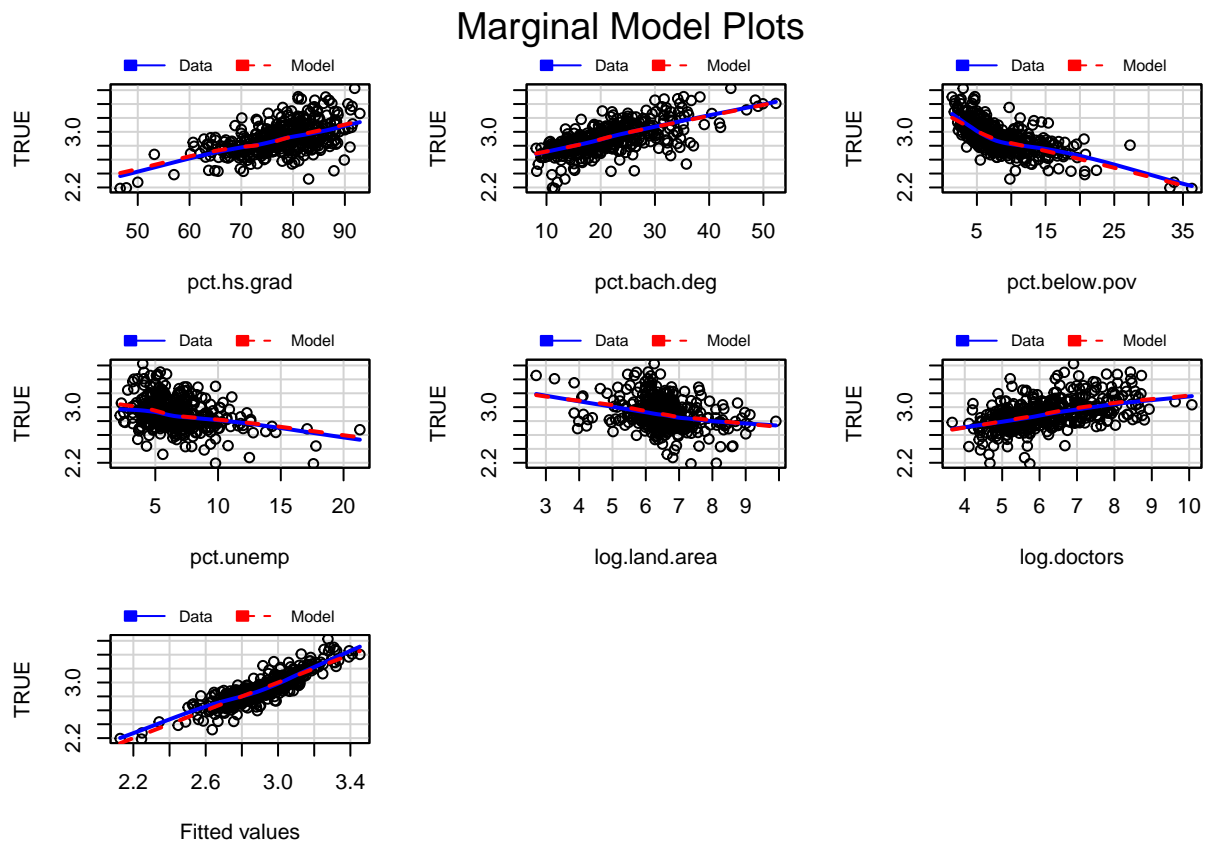
```
plot(best.allsubs)
```


Residuals vs Fitted

Fitted values
lm(log.per.cap.income ~ pct.hs.grad + pct.bach.deg + pct.below.pov + pct.un ...

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Normal Q–Q

lm(log.per.cap.income ~ pct.hs.grad + pct.bach.deg + pct.below.pov + pct.un ...

Scale–Location

lm(log.per.cap.income ~ pct.hs.grad + pct.bach.deg + pct.below.pov + pct.un ...

Residuals vs Leverage
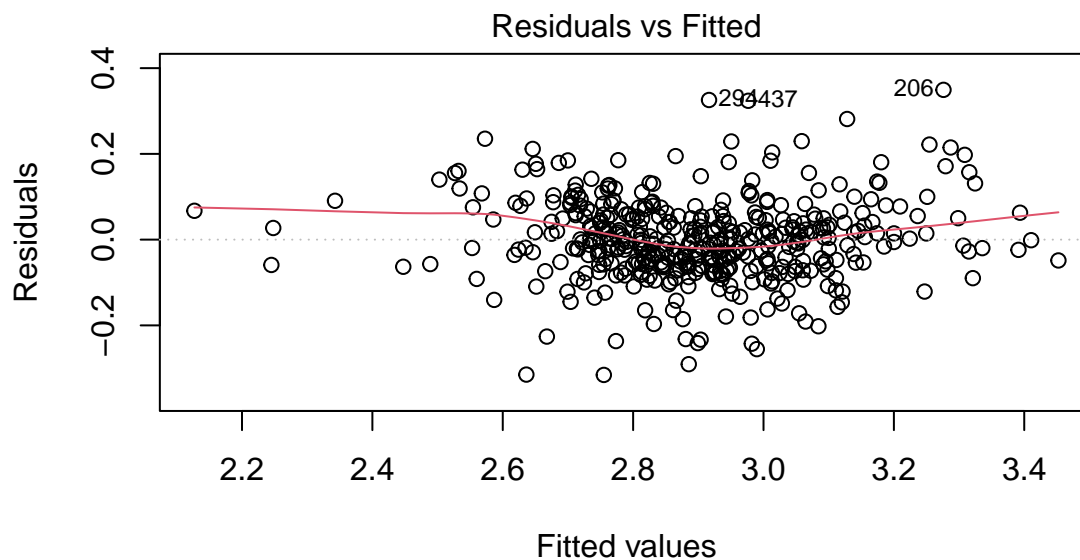
lm(log.per.cap.income ~ pct.hs.grad + pct.bach.deg + pct.below.pov + pct.un ...

```
mmps(best.allsubs)
```

Marginal Model Plots

Here we performed a linear regression using the best model found previously and added in region as an interaction term.

From the output we dropped log.land.area:region, pop.18_34:region, and log.doctors:region because none of the interactions were significant.

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Lastly we calculated the AIC and BIC for the base model and the model with the interactions.

```
allsubscdi <- cdilogs.mod.reg %>%
  dplyr::select(c(pct.hs.grad, pct.bach.deg,
                  pct.below.pov,  pct.unemp, log.land.area, log.doctors, region,
                  log.per.cap.income ))


all.subs.reg <- lm(log.per.cap.income ~ .*region, data=allsubscdi)
summary(all.subs.reg)
```

```
##
## Call:
## lm(formula = log.per.cap.income ~ . * region, data = allsubscdi)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.30870 -0.05222 -0.00779  0.04812  0.38824
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)             2.660788   0.318735   8.348 1.06e-15 ***
## pct.hs.grad             0.001377   0.004019   0.343 0.732091
## pct.bach.deg            0.004575   0.002884   1.586 0.113481
## pct.below.pov          -0.030093   0.004141  -7.267 1.86e-12 ***
## pct.unemp               0.021790   0.005827   3.739 0.000211 ***
## log.land.area          -0.049229   0.017992  -2.736 0.006484 **
## log.doctors             0.075264   0.010301   7.306 1.44e-12 ***
## regionNE                0.321322   0.400946   0.801 0.423357
## regionS                 0.278106   0.356333   0.780 0.435564
## regionW                 1.718705   0.484734   3.546 0.000437 ***
## pct.hs.grad:regionNE   -0.004202   0.005208  -0.807 0.420172
## pct.hs.grad:regionS    -0.006188   0.004442  -1.393 0.164385
## pct.hs.grad:regionW    -0.021368   0.005438  -3.929 9.99e-05 ***
## pct.bach.deg:regionNE   0.007756   0.004116   1.884 0.060201 .
## pct.bach.deg:regionS    0.006104   0.003271   1.866 0.062689 .
## pct.bach.deg:regionW    0.013758   0.003797   3.624 0.000327 ***
## pct.below.pov:regionNE -0.001940   0.005834  -0.332 0.739694
## pct.below.pov:regionS   0.005392   0.004594   1.174 0.241126
## pct.below.pov:regionW  -0.011175   0.006370  -1.754 0.080144 .
## pct.unemp:regionNE     -0.012784   0.008830  -1.448 0.148437
## pct.unemp:regionS      -0.016194   0.007723  -2.097 0.036618 *
## pct.unemp:regionW      -0.023618   0.008101  -2.916 0.003745 **
## log.land.area:regionNE -0.000923   0.023994  -0.038 0.969333
## log.land.area:regionS   0.024831   0.020654   1.202 0.229973
## log.land.area:regionW   0.027277   0.021694   1.257 0.209329
## log.doctors:regionNE   -0.007492   0.014961  -0.501 0.616822
## log.doctors:regionS    -0.006312   0.012964  -0.487 0.626599
## log.doctors:regionW    -0.036409   0.014798  -2.460 0.014289 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09124 on 412 degrees of freedom
## Multiple R-squared:  0.8172, Adjusted R-squared:  0.8052
```

```
## F-statistic: 68.22 on 27 and 412 DF,  p-value: < 2.2e-16
```

```
bestreg.allsubs <- update(all.subs.reg,. ~ . - region:log.land.area
                          -region:pop.18_34 - region:log.doctors)
summary(bestreg.allsubs)
```

```
##
## Call:
## lm(formula = log.per.cap.income ~ pct.hs.grad + pct.bach.deg +
##     pct.below.pov + pct.unemp + log.land.area + log.doctors +
##     region + pct.hs.grad:region + pct.bach.deg:region + pct.below.pov:region +
##     pct.unemp:region, data = allsubscdi)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.31312 -0.05090 -0.00701  0.04855  0.38446
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)           2.7322450  0.3030020   9.017  < 2e-16 ***
## pct.hs.grad          -0.0006368  0.0038273  -0.166 0.867935
## pct.bach.deg          0.0066250  0.0024782   2.673 0.007805 **
## pct.below.pov        -0.0289166  0.0040482  -7.143 4.08e-12 ***
## pct.unemp             0.0212730  0.0058071   3.663 0.000281 ***
## log.land.area        -0.0321779  0.0065327  -4.926 1.21e-06 ***
## log.doctors           0.0642046  0.0048446  13.253  < 2e-16 ***
## regionNE              0.2726560  0.3677538   0.741 0.458862
## regionS               0.2696895  0.3344733   0.806 0.420522
## regionW               1.3193081  0.4339614   3.040 0.002513 **
## pct.hs.grad:regionNE -0.0042558  0.0047117  -0.903 0.366927
## pct.hs.grad:regionS  -0.0040689  0.0042706  -0.953 0.341256
## pct.hs.grad:regionW  -0.0161196  0.0051053  -3.157 0.001707 **
## pct.bach.deg:regionNE 0.0078019  0.0031785   2.455 0.014512 *
## pct.bach.deg:regionS  0.0041325  0.0027834   1.485 0.138386
## pct.bach.deg:regionW  0.0086859  0.0031788   2.732 0.006554 **
## pct.below.pov:regionNE -0.0023178 0.0056493  -0.410 0.681805
## pct.below.pov:regionS  0.0045764  0.0044753   1.023 0.307098
## pct.below.pov:regionW -0.0091066  0.0062621  -1.454 0.146632
## pct.unemp:regionNE   -0.0127592  0.0088101  -1.448 0.148299
## pct.unemp:regionS    -0.0153619  0.0077114  -1.992 0.047010 *
## pct.unemp:regionW    -0.0199812  0.0080274  -2.489 0.013194 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09172 on 418 degrees of freedom
## Multiple R-squared:  0.8126, Adjusted R-squared:  0.8032
## F-statistic:  86.3 on 21 and 418 DF,  p-value: < 2.2e-16
```
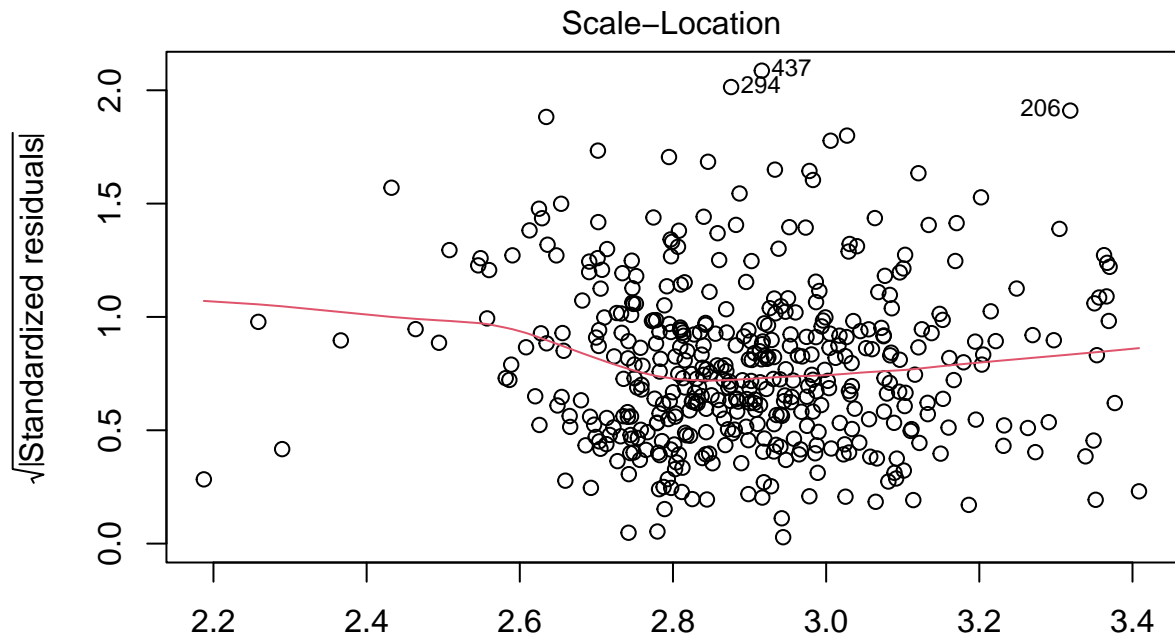
```
plot(bestreg.allsubs)
```

Residuals vs Fitted
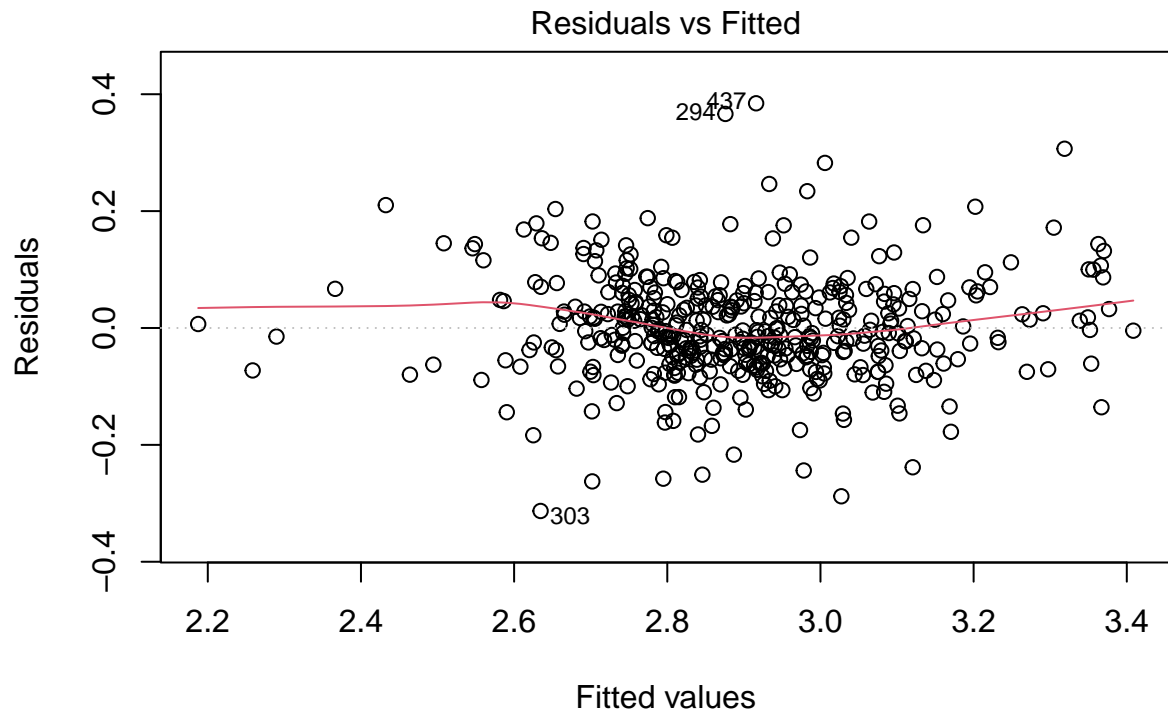
Fitted values
lm(log.per.cap.income ~ pct.hs.grad + pct.bach.deg + pct.below.pov + pct.un ...

Normal Q−Q
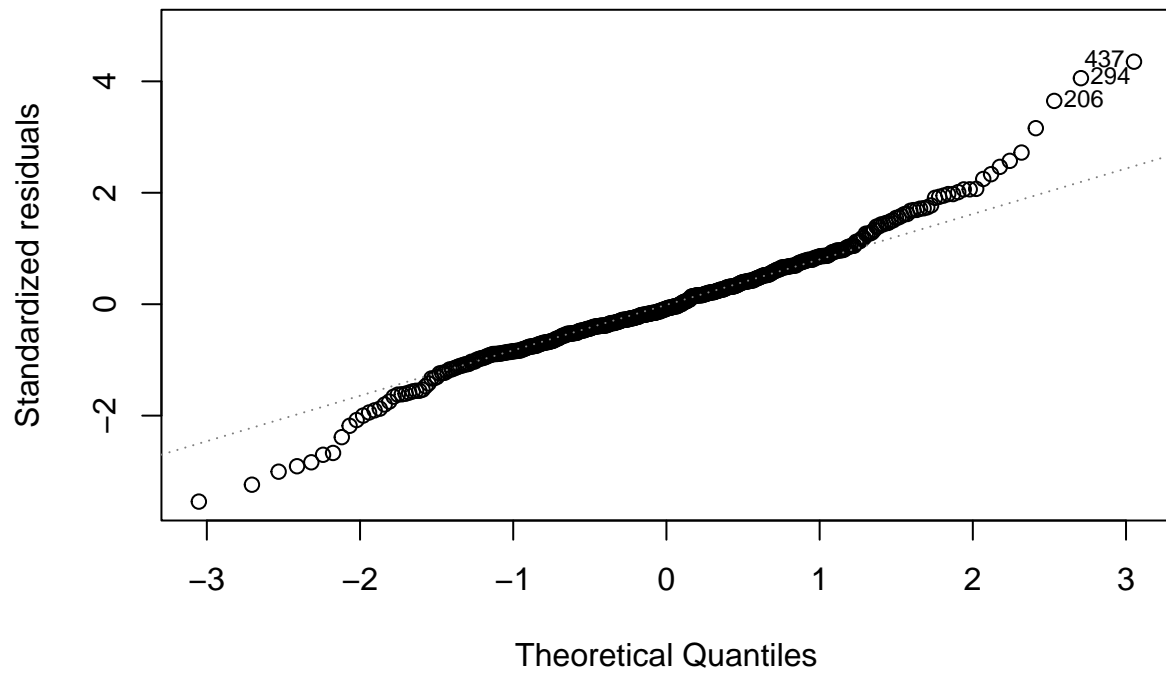
Theoretical Quantiles
lm(log.per.cap.income ~ pct.hs.grad + pct.bach.deg + pct.below.pov + pct.un ...

Scale–Location

√|Standardized residuals|

Fitted values
lm(log.per.cap.income ~ pct.hs.grad + pct.bach.deg + pct.below.pov + pct.un ...

Residuals vs Leverage

Standardized residuals

Cook's distance
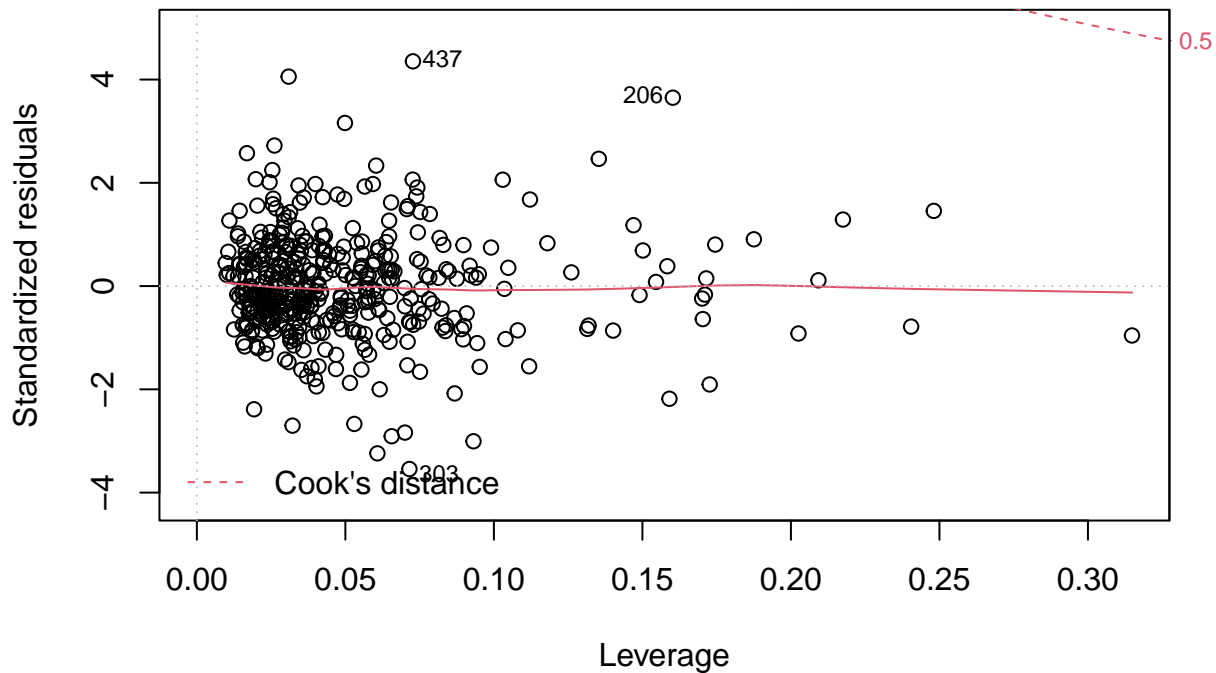
Leverage
lm(log.per.cap.income ~ pct.hs.grad + pct.bach.deg + pct.below.pov + pct.un ...

```
data.frame(AIC=AIC(best.allsubs,bestreg.allsubs),BIC=BIC(best.allsubs,bestreg.allsubs))[,-3] %>%
  kbl(booktabs=T,col.names=c("df","AIC","BIC")) %>%
  kable_minimal(full_width=F)
```

	df	AIC	BIC
best.allsubs	8	-808.2839	-775.5897
bestreg.allsubs	23	-830.2400	-736.2441

Stepwise

Next we repeated the same process with stepwise regression, starting without region and adding it in later. We did both an AIC and BIC model to compare the results to the all subsets model.

```
stepwise_AIC <- stepAIC(lm(log.per.cap.income ~ .,data=cdilogs.mod),direction="both",k=2,
                    trace = FALSE)
summary(stepwise_AIC)
```

```
##
## Call:
## lm(formula = log.per.cap.income ~ pop.18_34 + pop.65_plus + pct.hs.grad +
##     pct.bach.deg + pct.below.pov + pct.unemp + log.land.area +
##     log.doctors, data = cdilogs.mod)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35756 -0.04551 -0.00543  0.04844  0.27399
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3.4082113  0.1025858  33.223  < 2e-16 ***
## pop.18_34     -0.0153488  0.0012988 -11.818  < 2e-16 ***
## pop.65_plus   -0.0027664  0.0012978  -2.132   0.0336 *
## pct.hs.grad   -0.0046579  0.0010843  -4.296 2.15e-05 ***
## pct.bach.deg   0.0152149  0.0009242  16.462  < 2e-16 ***
## pct.below.pov -0.0246144  0.0012631 -19.488  < 2e-16 ***
## pct.unemp      0.0107688  0.0021696   4.963 9.99e-07 ***
## log.land.area -0.0364935  0.0047728  -7.646 1.36e-13 ***
## log.doctors    0.0626053  0.0041029  15.259  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.08167 on 431 degrees of freedom
## Multiple R-squared:  0.8468, Adjusted R-squared:  0.8439
## F-statistic: 297.7 on 8 and 431 DF,  p-value: < 2.2e-16
```

```
stepwise_BIC <- stepAIC(lm(log.per.cap.income ~ .,data=cdilogs.mod),direction="both",
                    k=log(nrow(cdilogs.mod)), trace = FALSE)
summary(stepwise_BIC)
```

```
##
## Call:
## lm(formula = log.per.cap.income ~ pop.18_34 + pct.hs.grad + pct.bach.deg +
##     pct.below.pov + pct.unemp + log.land.area + log.doctors,
##     data = cdilogs.mod)
##
```

```
## Residuals:
##      Min      1Q  Median      3Q     Max
## -0.34147 -0.04886 -0.00538  0.04818  0.26969
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.3147398  0.0931210  35.596  < 2e-16 ***
## pop.18_34    -0.0139002  0.0011113 -12.508  < 2e-16 ***
## pct.hs.grad  -0.0044064  0.0010823  -4.071 5.56e-05 ***
## pct.bach.deg  0.0153853  0.0009246  16.641  < 2e-16 ***
## pct.below.pov -0.0242784  0.0012583 -19.294  < 2e-16 ***
## pct.unemp     0.0106037  0.0021771   4.871 1.56e-06 ***
## log.land.area -0.0356741  0.0047767  -7.468 4.53e-13 ***
## log.doctors   0.0606769  0.0040183  15.100  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.082 on 432 degrees of freedom
## Multiple R-squared:  0.8452, Adjusted R-squared:  0.8427
## F-statistic: 336.9 on 7 and 432 DF,  p-value: < 2.2e-16
```

```r
stepwisereg_AIC <- stepAIC(lm(log.per.cap.income ~ .*region,data=allsubscdi),direction="both",
                      k=2, trace = FALSE)
summary(stepwisereg_AIC)
```

```
##
## Call:
## lm(formula = log.per.cap.income ~ pct.hs.grad + pct.bach.deg +
##     pct.below.pov + pct.unemp + log.land.area + log.doctors +
##     region + pct.hs.grad:region + pct.bach.deg:region + pct.below.pov:region +
##     pct.unemp:region + log.doctors:region, data = allsubscdi)
##
## Residuals:
##      Min      1Q  Median      3Q     Max
## -0.30870 -0.05143 -0.00880  0.04925  0.38721
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)            2.6169464  0.3162672   8.274 1.78e-15 ***
## pct.hs.grad            0.0005046  0.0039373   0.128 0.898084
## pct.bach.deg           0.0048970  0.0028700   1.706 0.088704 .
## pct.below.pov         -0.0300625  0.0041432  -7.256 1.98e-12 ***
## pct.unemp              0.0209967  0.0057832   3.631 0.000318 ***
## log.land.area         -0.0312781  0.0065154  -4.801 2.21e-06 ***
## log.doctors            0.0750128  0.0103045   7.280 1.69e-12 ***
## regionNE               0.3403227  0.3982715   0.854 0.393321
## regionS                0.3613728  0.3496537   1.034 0.301964
## regionW                1.8283250  0.4763204   3.838 0.000143 ***
## pct.hs.grad:regionNE  -0.0050473  0.0049370  -1.022 0.307215
## pct.hs.grad:regionS   -0.0052400  0.0043633  -1.201 0.230467
## pct.hs.grad:regionW   -0.0203938  0.0053744  -3.795 0.000170 ***
## pct.bach.deg:regionNE  0.0089349  0.0039266   2.275 0.023387 *
## pct.bach.deg:regionS   0.0055619  0.0032415   1.716 0.086937 .
## pct.bach.deg:regionW   0.0127607  0.0036991   3.450 0.000619 ***
```

```
## pct.below.pov:regionNE -0.0015959   0.0058296   -0.274 0.784405
## pct.below.pov:regionS    0.0054667   0.0045948    1.190 0.234820
## pct.below.pov:regionW   -0.0107409   0.0063541   -1.690 0.091705 .
## pct.unemp:regionNE       -0.0121377  0.0088015   -1.379 0.168625
## pct.unemp:regionS        -0.0151578  0.0076783   -1.974 0.049033 *
## pct.unemp:regionW        -0.0228089  0.0080709   -2.826 0.004940 **
## log.doctors:regionNE     -0.0063761  0.0149549   -0.426 0.670071
## log.doctors:regionS      -0.0059443  0.0129673   -0.458 0.646900
## log.doctors:regionW      -0.0352067  0.0147637   -2.385 0.017543 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09129 on 415 degrees of freedom
## Multiple R-squared:  0.8157, Adjusted R-squared:  0.805
## F-statistic: 76.52 on 24 and 415 DF,  p-value: < 2.2e-16
```

```
stepwisereg_BIC <- stepAIC(lm(log.per.cap.income ~ .*region,data=allsubscdi),
                        direction="both",k=log(nrow(allsubscdi)), trace = FALSE)
summary(stepwisereg_BIC)
```

```
##
## Call:
## lm(formula = log.per.cap.income ~ pct.hs.grad + pct.bach.deg +
##     pct.below.pov + pct.unemp + log.land.area + log.doctors +
##     region + pct.bach.deg:region, data = allsubscdi)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.33309 -0.05200 -0.00590  0.04898  0.35443
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           3.170626   0.119109  26.619  < 2e-16 ***
## pct.hs.grad          -0.006084   0.001345  -4.525 7.84e-06 ***
## pct.bach.deg          0.008093   0.001465   5.525 5.74e-08 ***
## pct.below.pov        -0.027787   0.001496 -18.575  < 2e-16 ***
## pct.unemp             0.010820   0.002706   3.999 7.50e-05 ***
## log.land.area        -0.028486   0.006395  -4.454 1.08e-05 ***
## log.doctors           0.064023   0.004634  13.815  < 2e-16 ***
## regionNE             -0.180452   0.039225  -4.600 5.57e-06 ***
## regionS              -0.099337   0.034709  -2.862  0.00442 **
## regionW              -0.119614   0.047127  -2.538  0.01150 *
## pct.bach.deg:regionNE 0.008450   0.001778   4.752 2.76e-06 ***
## pct.bach.deg:regionS  0.003516   0.001557   2.258  0.02442 *
## pct.bach.deg:regionW  0.005557   0.001987   2.797  0.00539 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09308 on 427 degrees of freedom
## Multiple R-squared:  0.8028, Adjusted R-squared:  0.7973
## F-statistic: 144.9 on 12 and 427 DF,  p-value: < 2.2e-16
```
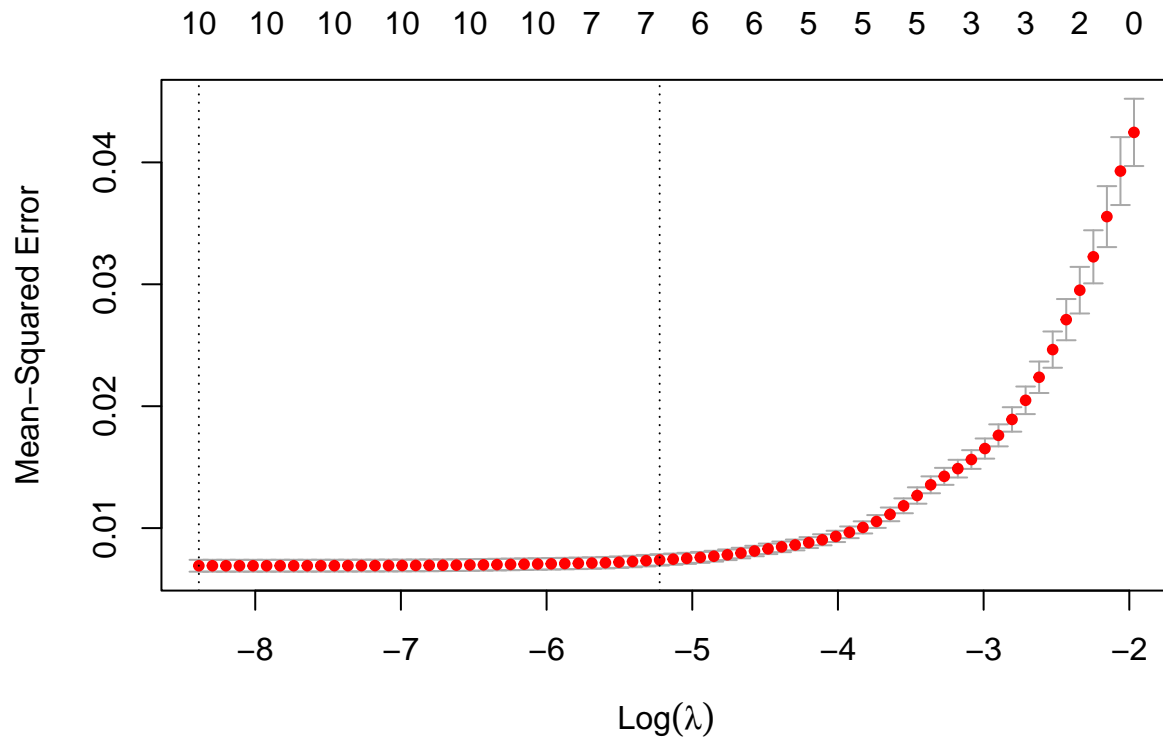
Lasso

As a last check, we performed lasso regression with cross validation and the 1se lambda.

```r
y <- cdilogs.mod$log.per.cap.income
x <- data.matrix(cdilogs.mod[,-11])

lasso.cdi <- cv.glmnet(x,y, alpha = 1)
plot(lasso.cdi)
```



```r
c(lambda.1se=lasso.cdi$lambda.1se,lambda.min=lasso.cdi$lambda.min)
```

```
##    lambda.1se    lambda.min
## 0.0053867116 0.0002278171
```

```r
cbind(coef(lasso.cdi),
      coef(lasso.cdi,s=lasso.cdi$lambda.1se),
      coef(lasso.cdi,s=lasso.cdi$lambda.min))
```

```
## 11 x 3 sparse Matrix of class "dgCMatrix"
##                          s1            s1          s1
## (Intercept)    2.975312e+00  2.975312e+00  3.346845125
## pop.18_34     -1.219667e-02 -1.219667e-02 -0.015401696
## pop.65_plus        .             .        -0.002842179
## pct.hs.grad   -7.659009e-06 -7.659009e-06 -0.004512585
## pct.bach.deg   1.195019e-02  1.195019e-02  0.015585535
## pct.below.pov -2.018164e-02 -2.018164e-02 -0.025186170
## pct.unemp      7.083252e-03  7.083252e-03  0.011038957
```

```
## log.land.area     -3.343760e-02 -3.343760e-02 -0.035724991
## log.doctors        5.985236e-02  5.985236e-02  0.049208996
## log.hosp.beds      .             .             0.013308025
## log.per.cap.crime  .             .             0.009115038
```

```
bestmodel.lasso <- glmnet(x,y, alpha = 1, lambda = lasso.cdi$lambda.1se)
coef(bestmodel.lasso)
```

```
## 11 x 1 sparse Matrix of class "dgCMatrix"
##                             s0
## (Intercept)       2.977231e+00
## pop.18_34        -1.218624e-02
## pop.65_plus       .
## pct.hs.grad      -3.526294e-05
## pct.bach.deg      1.195202e-02
## pct.below.pov    -2.020428e-02
## pct.unemp         7.060445e-03
## log.land.area    -3.341078e-02
## log.doctors       5.986190e-02
## log.hosp.beds     .
## log.per.cap.crime .
```