Inferring the mass of the Dark Matter Halo from Globular Cluster 3D Kinematics

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NSERC CRSNG

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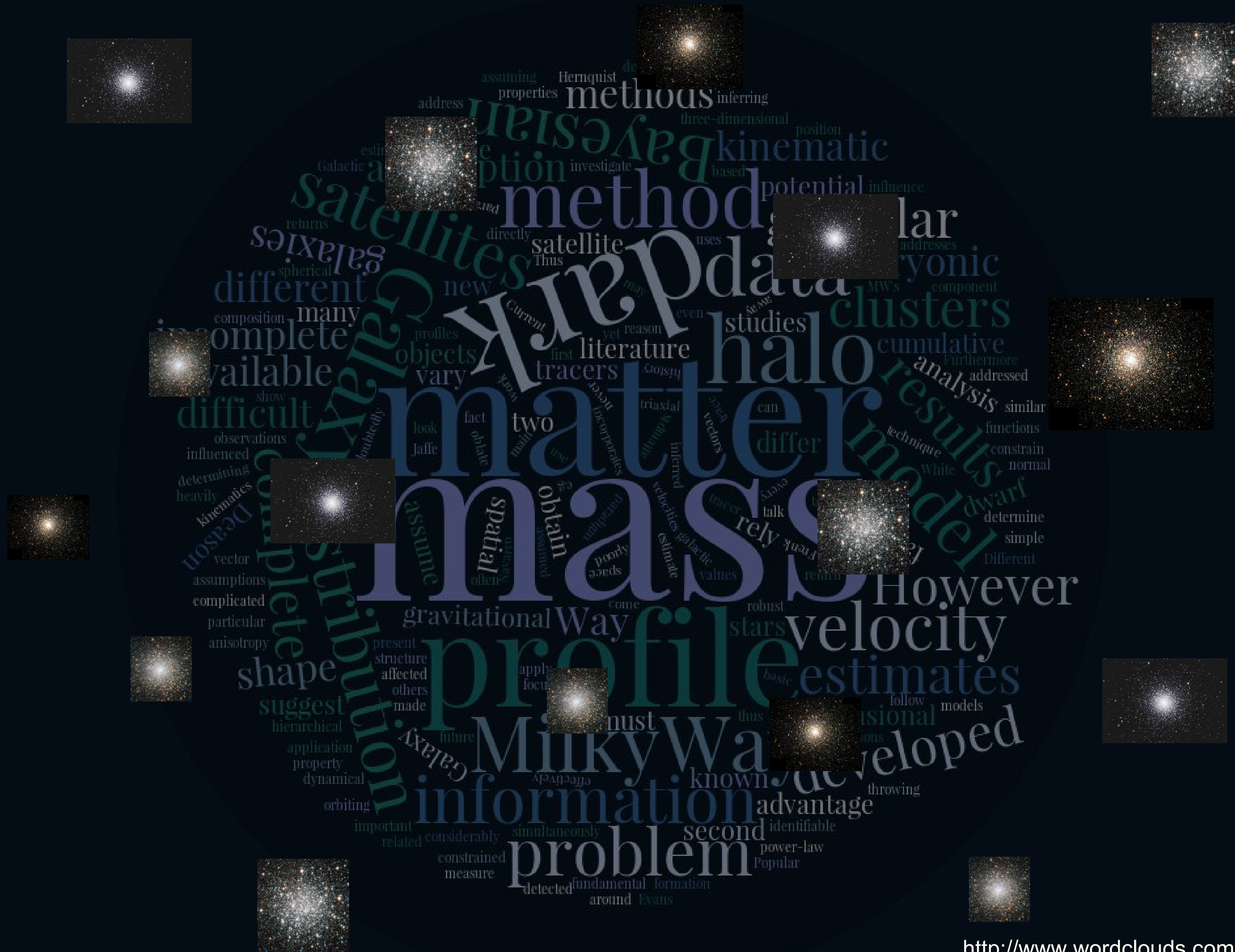
http://www.wordclouds.com/



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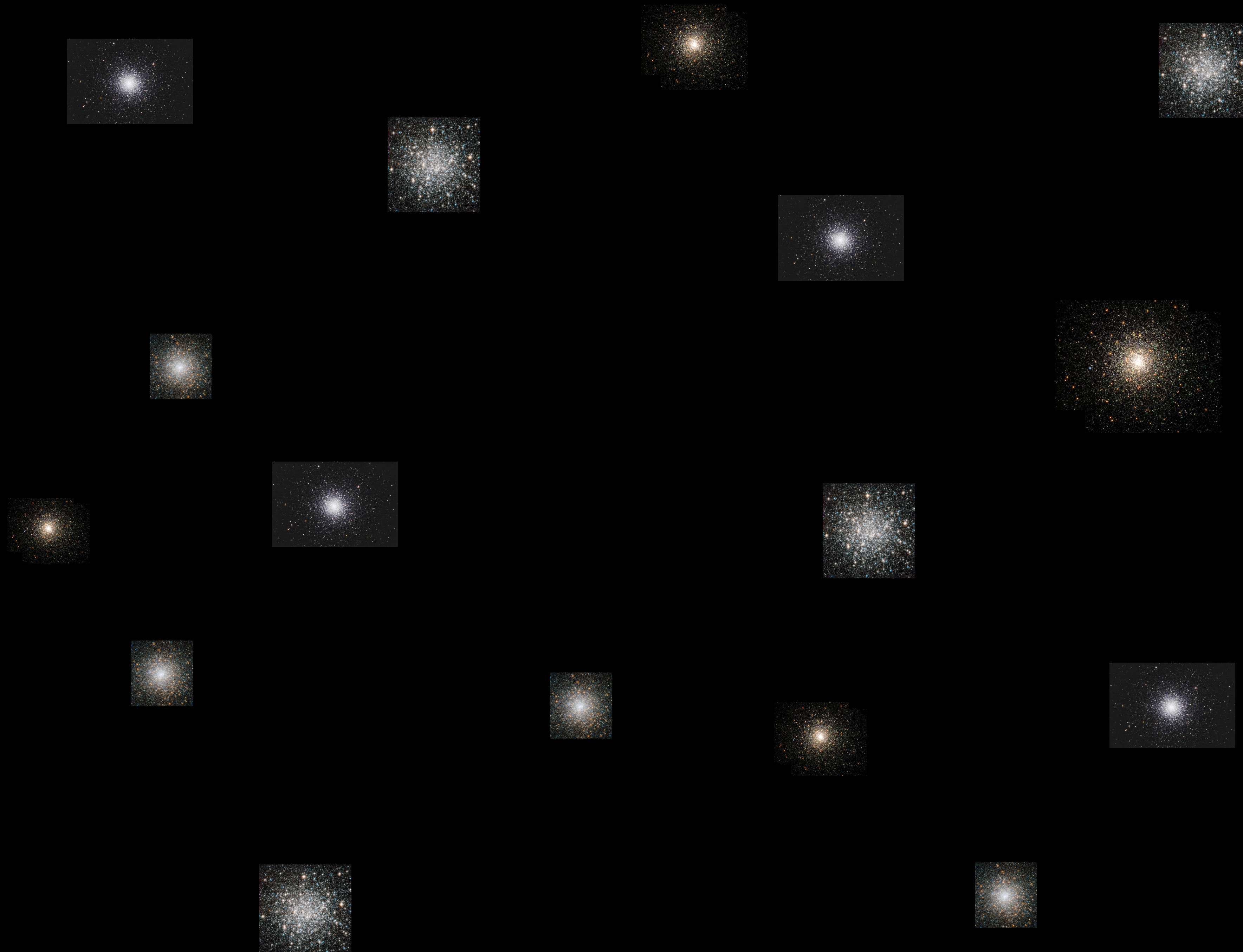


The Bayesian Paradigm

Thomas Bayes (1701-1761)

Bayes' Theorem

Posterior Distribution $\propto$ **Likelihood • Prior**

$$p\left(\boldsymbol{\theta}|\boldsymbol{y}\right) \propto p\left(\boldsymbol{y}|\boldsymbol{\theta}\right) p\left(\boldsymbol{\theta}\right)$$

model parameters **data**

Bayes' Theorem

Posterior Distribution $\propto$ **Likelihood • Prior**

$$p\left(\boldsymbol{\theta}\middle|\boldsymbol{y}\right) \propto p\left(\boldsymbol{y}\middle|\boldsymbol{\theta}\right) p\left(\boldsymbol{\theta}\right)$$

Probability distribution function of model parameters, given the data, the model, and prior assumptions.

Bayes' Theorem

Posterior Distribution $\propto$ **Likelihood • Prior**

$$p(\boldsymbol{\theta}|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{\theta})\, p(\boldsymbol{\theta})$$

model parameters

data

Assuming data *y* are independent:

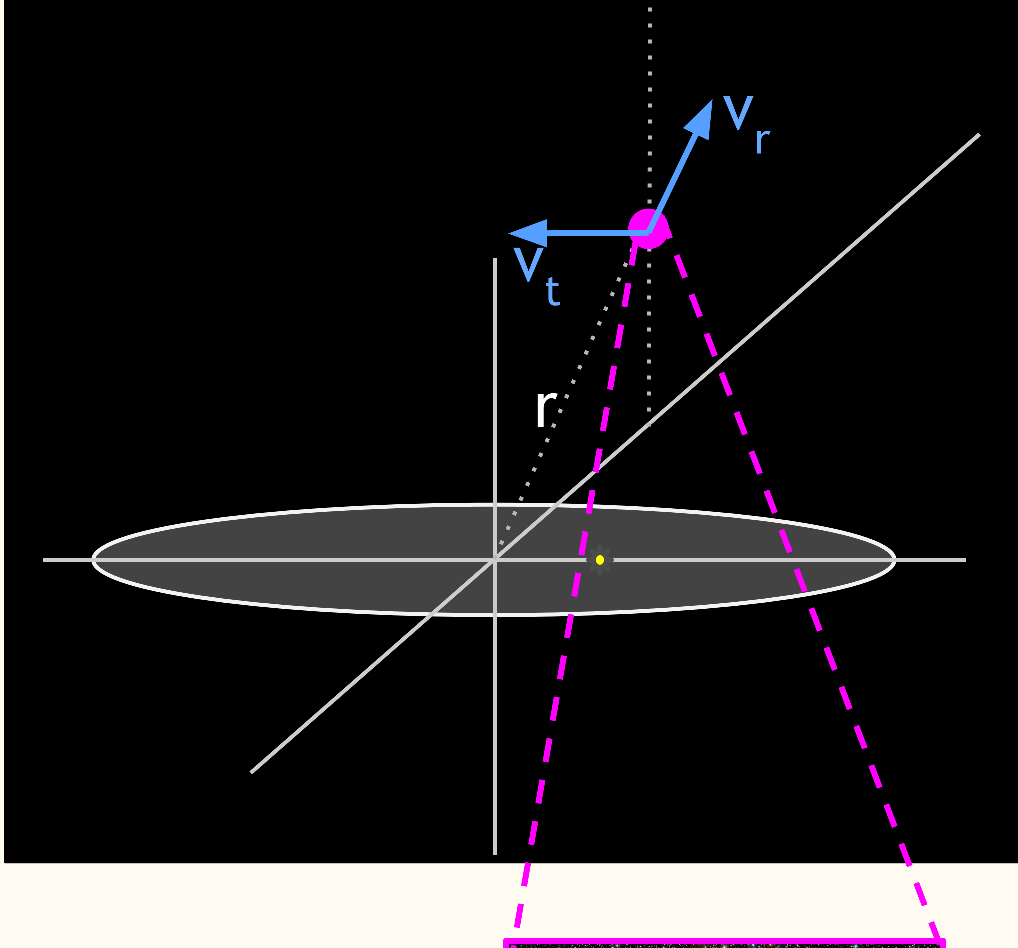
$$p(\boldsymbol{\theta}|\boldsymbol{y}) \propto \prod_i^n p(y_i|\boldsymbol{\theta})\, p(\boldsymbol{\theta})$$

The astrophysics part

The astrophysics part

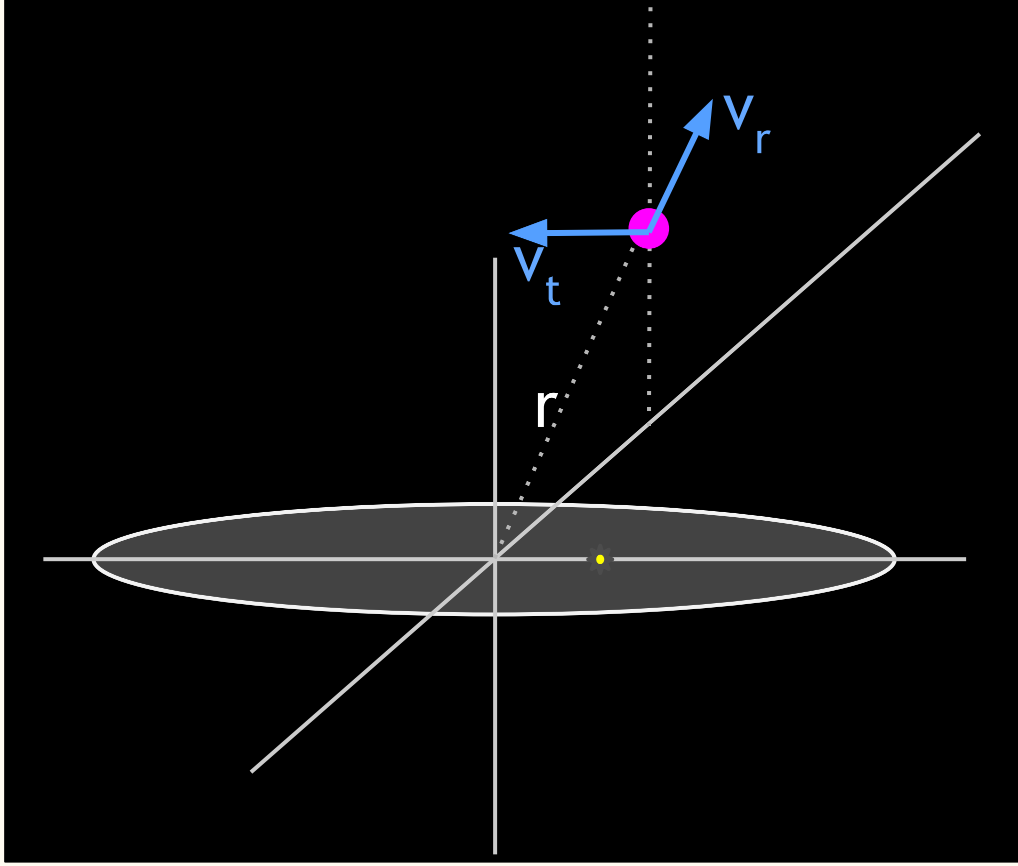
Probability Distribution Function

(Binney & Tremaine, Galactic Dynamics, 2008, Cuddeford 1991, Little & Tremaine 1987)



HST

The astrophysics part



Probability Distribution Function

(Binney & Tremaine, Galactic Dynamics, 2008, Cuddeford 1991, Little & Tremaine 1987)

$$f(\mathcal{E}, L) \propto L^{-2\beta} f(\mathcal{E})$$

Specific Energy:

$$\mathcal{E} = \frac{1}{2}(v_r^2 + v_t^2) - \Phi(r)$$

Specific Angular Momentum:

$$L = r v_t$$

Bayes' Theorem

Posterior Distribution $\propto$ **Likelihood • Prior**

$$p\left(\boldsymbol{\theta}|\boldsymbol{y}\right) \propto p\left(\boldsymbol{y}|\boldsymbol{\theta}\right) p\left(\boldsymbol{\theta}\right)$$

model parameters

data

Assuming data *y* are independent:

$$p\left(\boldsymbol{\theta}|\boldsymbol{y}\right) \propto \prod_{i}^{n} p\left(y_i|\boldsymbol{\theta}\right) p\left(\boldsymbol{\theta}\right)$$

The astrophysics part

Probability Distribution Function (DF)

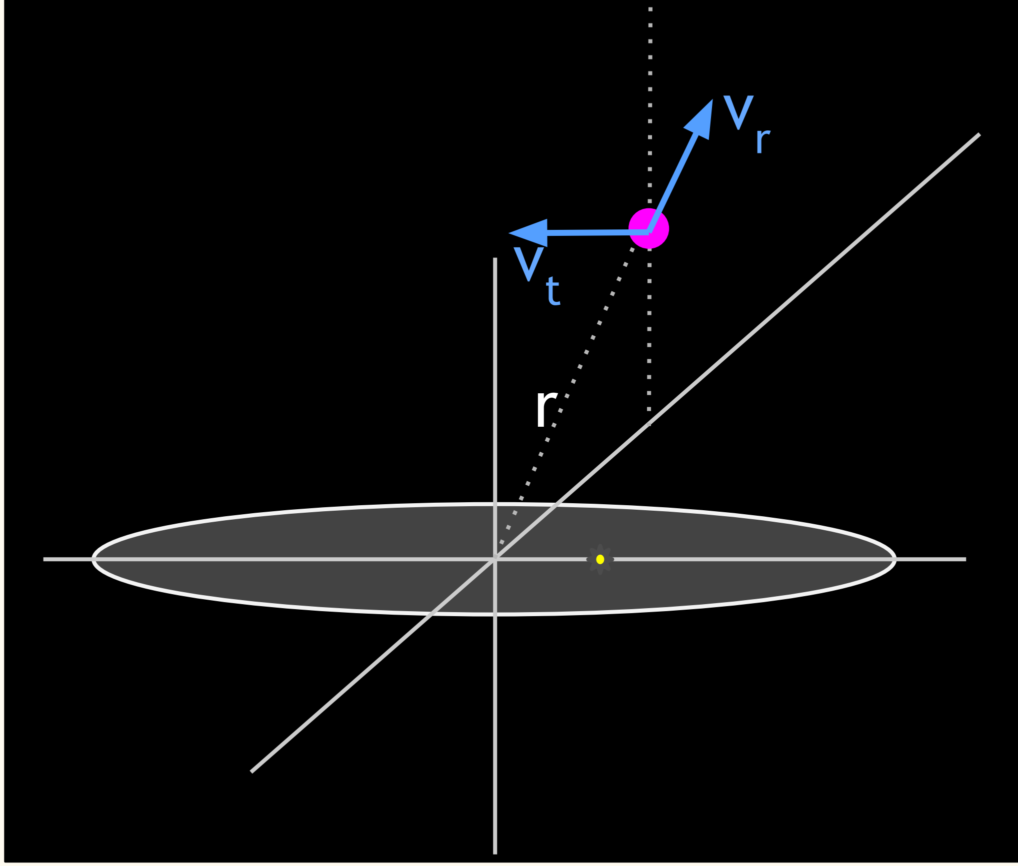
(Binney & Tremaine, Galactic Dynamics, 2008, Cuddeford 1991, Little & Tremaine 1987)

$$f(\mathcal{E}, L) \propto L^{-2\beta} f(\mathcal{E})$$

Assuming tracers are independent:

$$p(\boldsymbol{\theta}|\boldsymbol{\mathcal{E}}, L) \propto \prod_{i=1}^{n} f(\mathcal{E}_i, L_i|\boldsymbol{\theta}) p(\boldsymbol{\theta})$$

The astrophysics part



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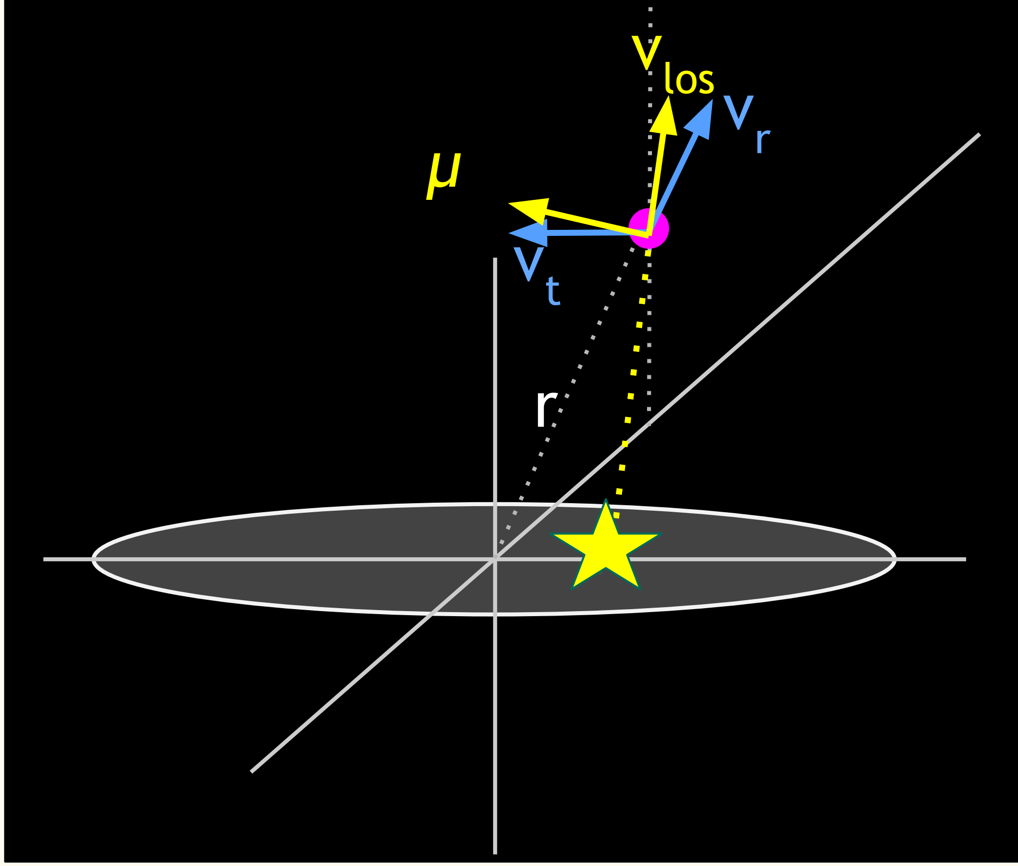
Galactocentric Reference Frame!

The astrophysics part



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Specific Energy:

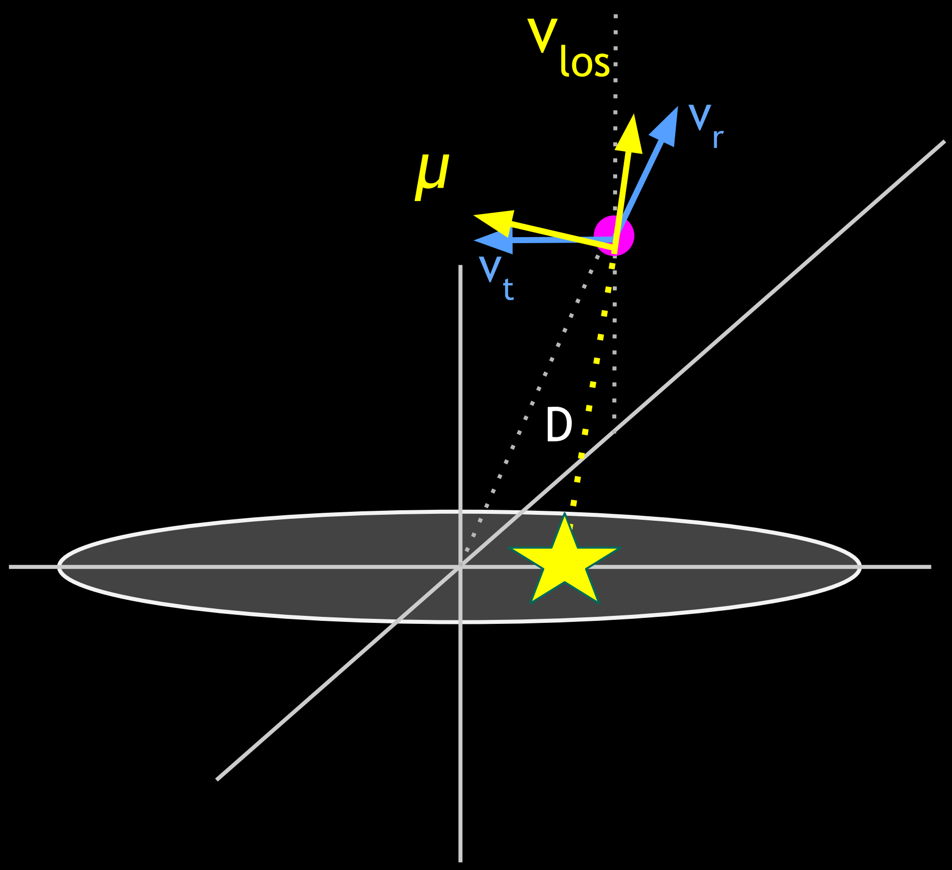
$$\mathcal{E} = \frac{1}{2}(v_r^2 + v_t^2) - \Phi(r)$$

Specific Angular Momentum:

$$L = r v_t$$

Helioocentric Reference Frame!

Not really a problem...



Heliocentric

solar motion

rotation of disk

RA & Declination

distance

Galactocentric

Good guide: Johnson & Soderblom (1987), Astronomical Journal, 23:4.

… except that …
Globular Cluster 3D Kinematics are Incomplete

GC   ***r***   ***μ***   ***v*****los**

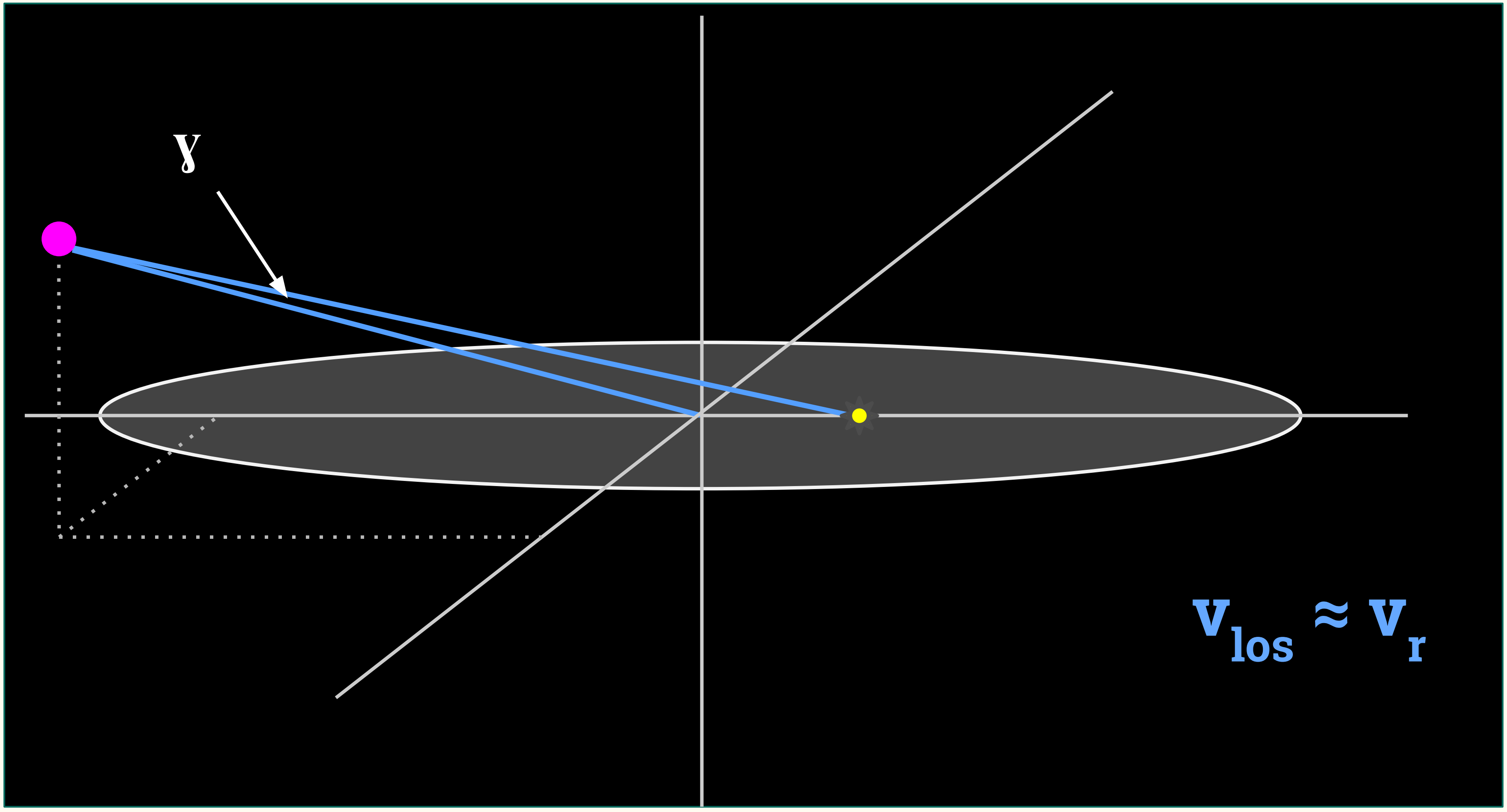
Eadie & Harris (2016), ***submitted to ApJ***

TABLE 1
KINEMATIC DATA OF GALACTIC GLOBULAR CLUSTERS

Object	r kpc	$\mu_\alpha \cos\delta$ (mas/year)	μ_δ (mas/year)	v_{los} km s^{-1}	$\cos\xi$	μ Reference	Included
NGC 104	7.4	7.26 ± 0.03	-1.25 ± 0.03	-18 ± 0.1	0.17	Cioni	✓
NGC 288	12.0	4.675 ± 0.219	-5.6 ± 0.35	-45.4 ± 0.2	0.75	Casseti	✓
NGC 362	9.4	4.873 ± 0.514	-2.727 ± 0.824	223.5 ± 0.5	0.61	Casseti	✓
Whiting 1	34.5	—	—	-130.6 ± 1.8	0.98	—	—
NGC 1261	18.1	—	—	68.2 ± 4.6	0.90	—	—
Pal 1	17.2	—	—	-82.8 ± 3.3	0.93	—	—
AM 1	124.6	—	—	116 ± 20	1.00	—	✓
Eridanus	95.0	—	—	-23.6 ± 2.1	1.00	—	✓
Pal 2	35.0	—	—	-133 ± 57	1.00	—	✓
NGC 1851	16.6	1.28 ± 0.68	2.39 ± 0.65	320.5 ± 0.6	0.89	Casseti	✓
NGC 1904	18.8	2.34 ± 0.69	-0.5 ± 0.75	205.8 ± 0.4	0.94	Wang	✓
NGC 2298	15.8	4.05 ± 1	-1.72 ± 0.98	148.9 ± 1.2	0.89	Casseti	✓
NGC 2419	89.9	—	—	-20.2 ± 0.5	1.00	—	✓
Ko 2	41.9	—	—	—	1.00	—	—
Pyxis	41.4	—	—	34.3 ± 1.9	0.98	—	✓
NGC 2808	11.1	0.58 ± 0.45	2.06 ± 0.46	101.6 ± 0.7	0.71	Casseti	✓
E 3	9.1	—	—	—	0.57	—	—
Pal 3	95.7	0.33 ± 0.23	0.3 ± 0.31	83.4 ± 8.4	1.00	Majewski & Cudworth	✓(μ not included)
NGC 3201	8.8	5.28 ± 0.32	-0.98 ± 0.33	494 ± 0.2	0.43	Casseti	✓
Pal 4	111.2	—	—	74.5 ± 2.1	1.00	—	✓
Ko 1	49.3	—	—	—	0.99	—	—

Our solution to include incomplete data:

1. Make approximation $v_r \approx v_{los}$ when appropriate



Our solution to include incomplete data:

1. Make approximation $v_r \approx v_{los}$ when appropriate
2. Sample missing v_t's as nuisance parameters in the Markov chain

BONUS: Helps to break mass- velocity anisotropy degeneracy

Simple Model!

Past work with this method:

Eadie, Harris, & Widrow (2015), ApJ 806, 54

Eadie, G. 2014, JSM Proceedings, ASA, Section 175.

Eadie, G. MSc Thesis, Queen's University (2013).

Recent Work

Model where the Dark Matter and Visible Matter have different radial profiles

(Evans et al 1997,
Deason et al 2011, 2012)

Gravitational Potential of Dark Matter Halo

$$\Phi = \frac{\Phi_o}{r^\gamma}$$

Density Profile of Tracers

$$\rho_t \propto \frac{1}{r^\alpha}$$

Recent Work

Model where the Dark Matter and Visible Matter have different radial profiles

(Evans et al 1997,
Deason et al 2011, 2012)

Cumulative Mass Profile

$$M(<r) = \frac{\gamma \Phi_o}{G} \left(\frac{r}{\mathrm{kpc}} \right)^{1-\gamma}$$

Isothermal Sphere: $\gamma \longrightarrow 0$

Keplerian Case: $\gamma \longrightarrow 1$

Distribution Function (DF)

$$f(\mathcal{E}, L) = \frac{L^{-2\beta}\mathcal{E}^{\frac{\beta(\gamma-2)}{\gamma}+\frac{\alpha}{\gamma}-\frac{3}{2}}}{\sqrt{8\pi^3 2^{-2\beta}}\Phi_o^{-\frac{2\beta}{\gamma}+\frac{\alpha}{\gamma}}} \frac{\Gamma\left(\frac{\alpha}{\gamma}-\frac{2\beta}{\gamma}+1\right)}{\Gamma\left(\frac{\beta(\gamma-2)}{\gamma}+\frac{\alpha}{\gamma}-\frac{1}{2}\right)}$$

(Evans et al 1997, Deason et al 2011, 2012)

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Velocity Anisotropy Parameter

$$\beta = 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2\sigma_r^2}$$

What do we want to know?

$$M(<r) = \frac{\gamma \Phi_o}{G} \left(\frac{r}{\mathrm{kpc}} \right)^{1-\gamma}$$

Φ_o the scale factor for the gravitational potential

γ the power-law slope of the gravitational potential

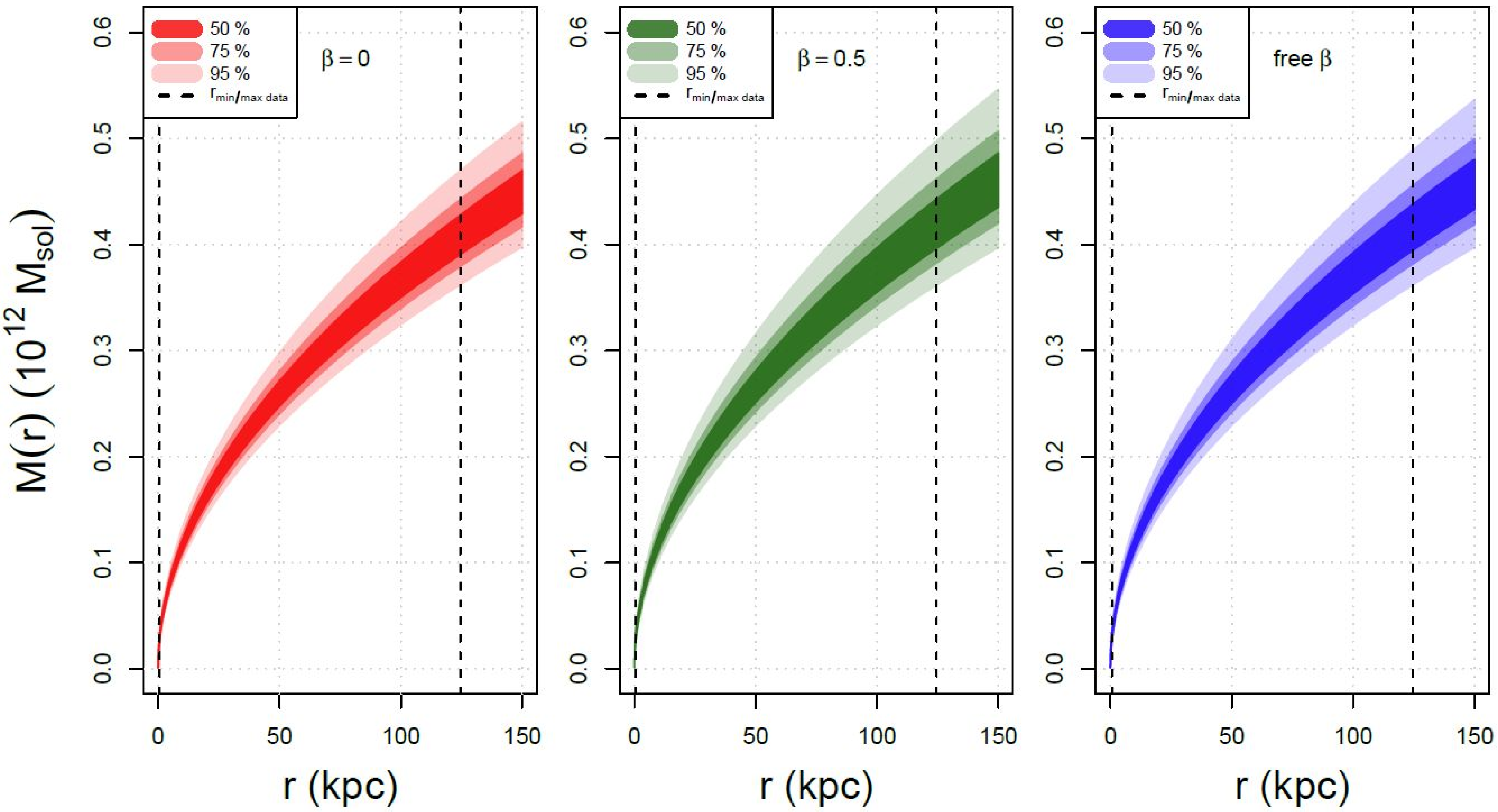
α the power-law slope of the satellite population

β the velocity anisotropy parameter

Group 1

Group	Scenario	Potential γ	Tracers α	Anisotropy β
(1)	I	0.5	3.5	0
	II			0.5
	III			free

Eadie & Harris (2016), ***submitted to ApJ***



Group 1

Group	Scenario	Potential γ	Tracers α	Anisotropy β
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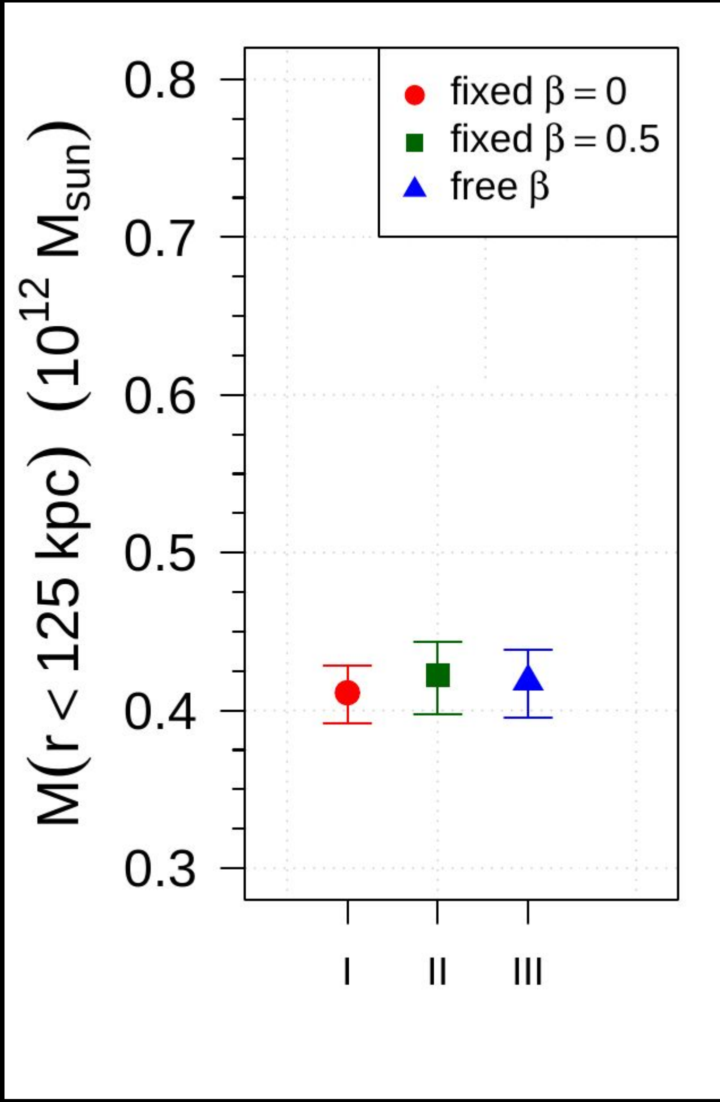
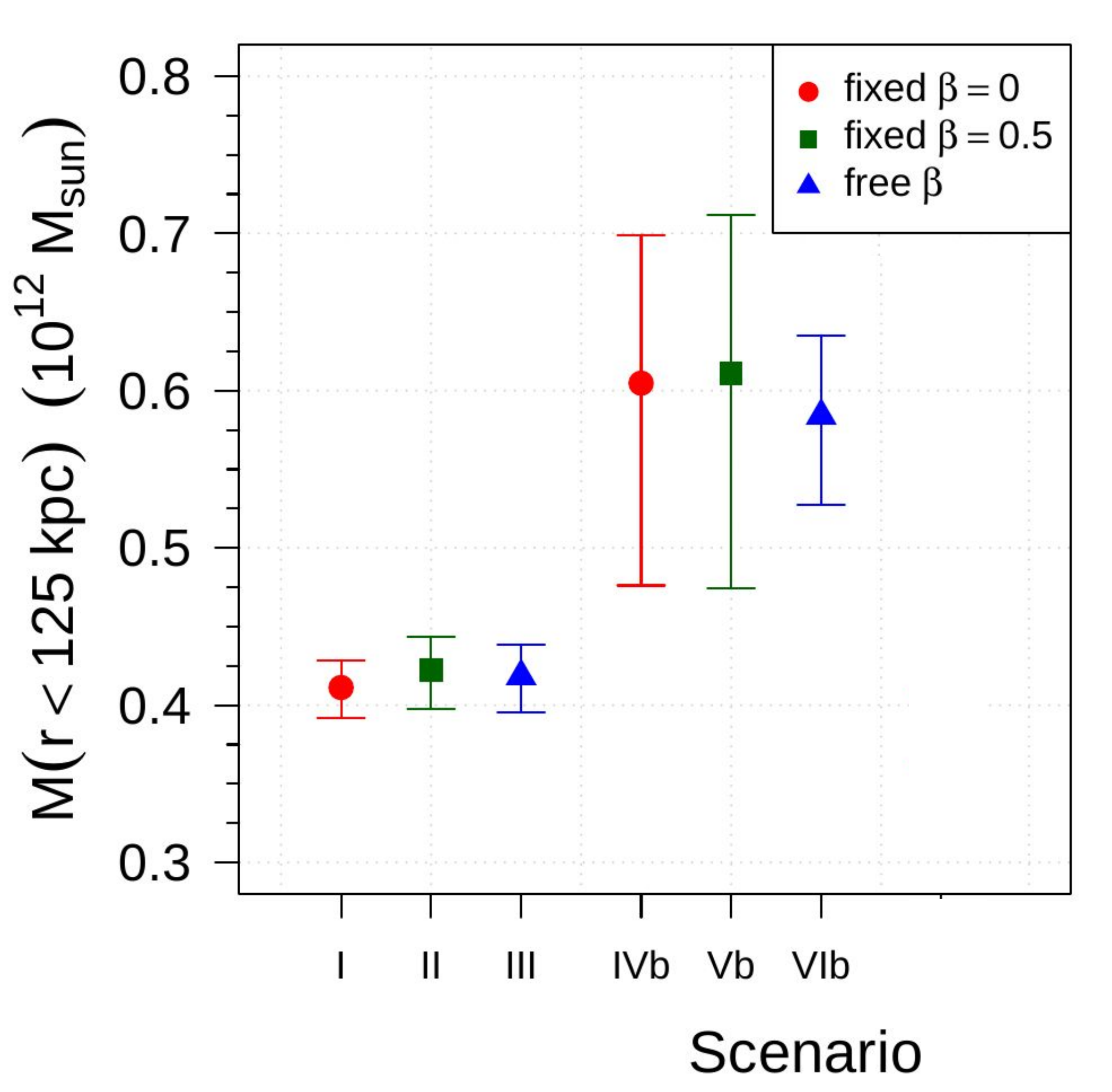
	II			0.5
	III			**free**



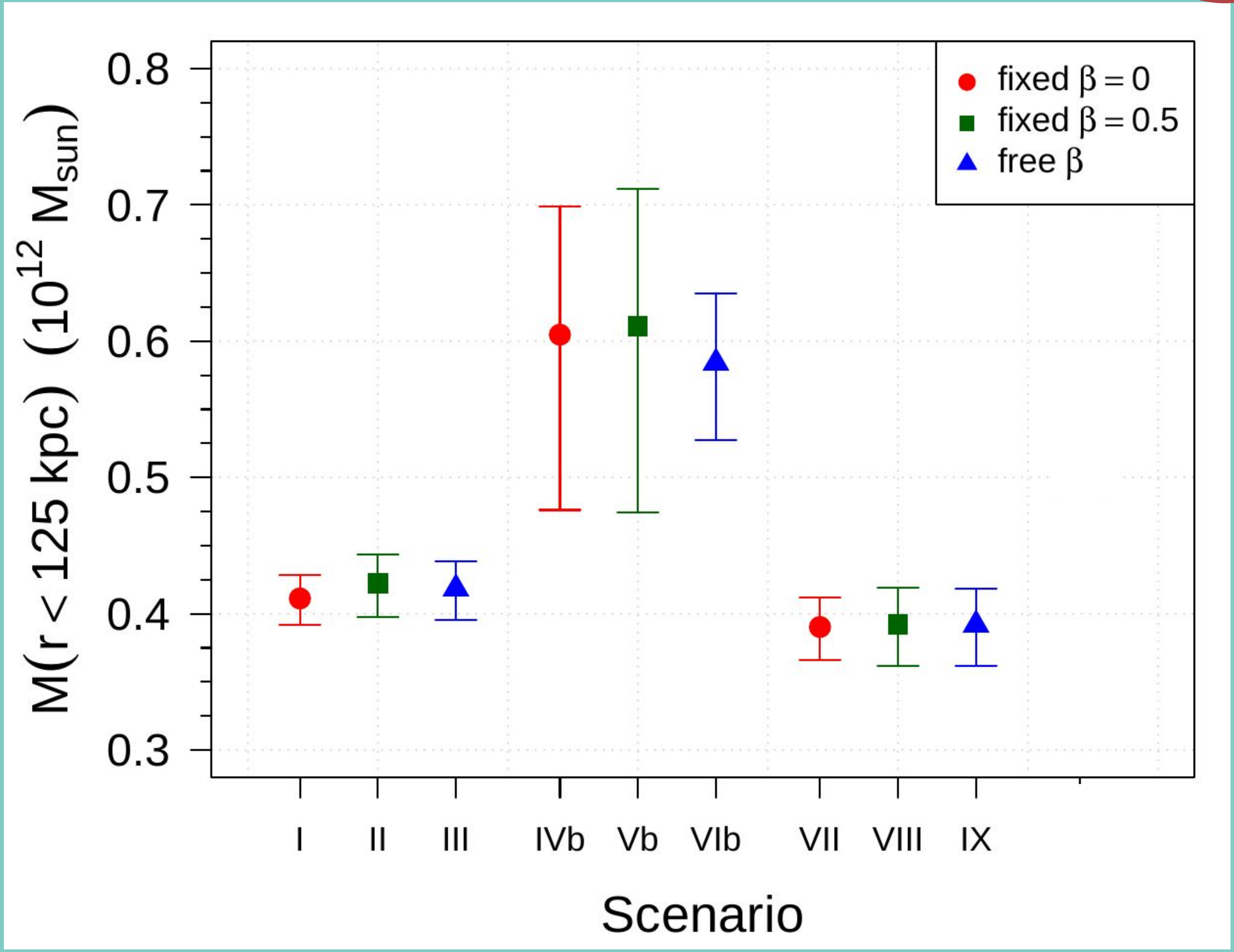
Group 2

Group	Scenario	Potential γ	Tracers α	Anisotropy β
(2)	IV V VI	**free**	3.5	0 0.5 **free**



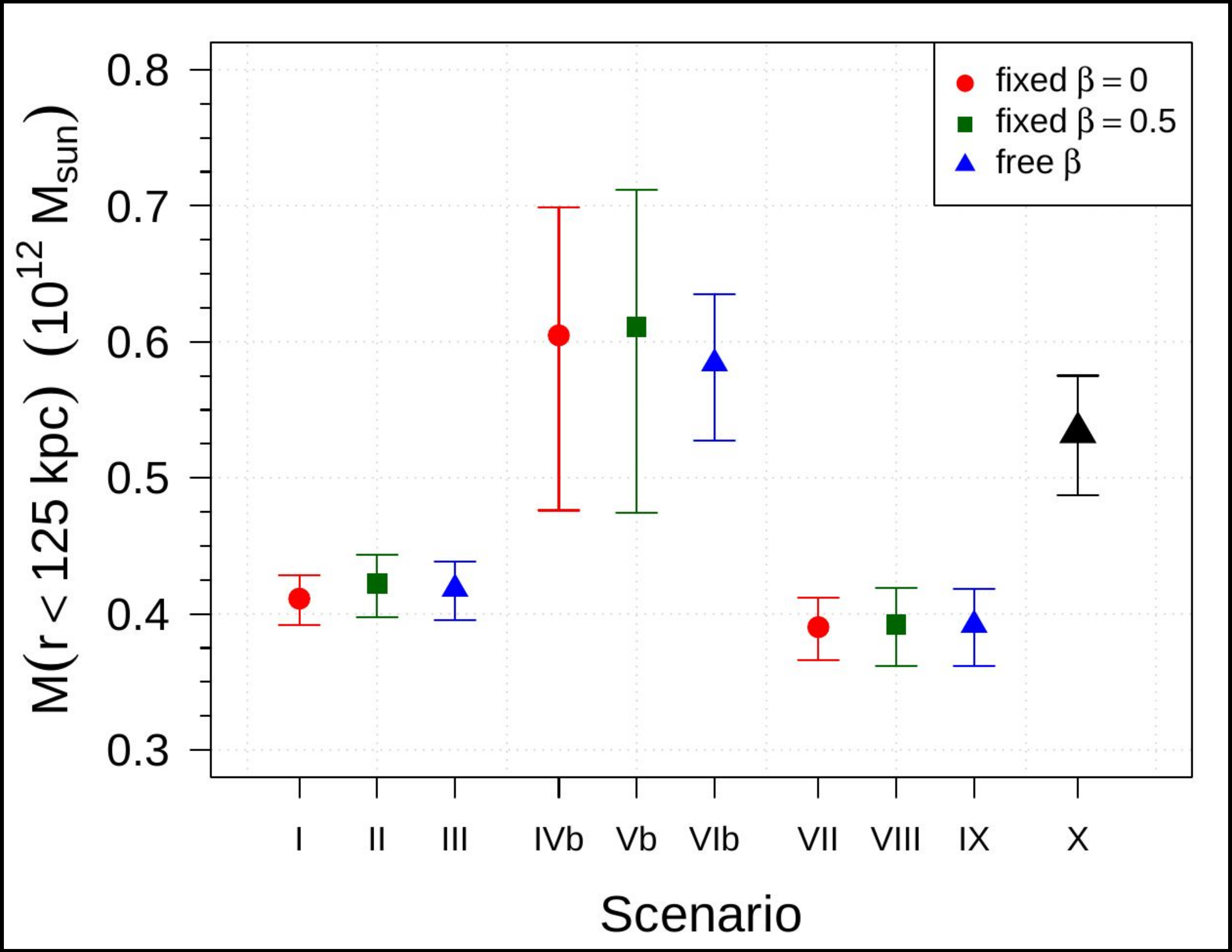
Group 3

Group	Scenario	Potential γ	Tracers α	Anisotropy β
(3)	VII	0.5	**free**	0
	VIII			0.5
	IX			**free**



Scenario X

Group	Scenario	Potential γ	Tracers α	Anisotropy β
(4)	X	free	free	free

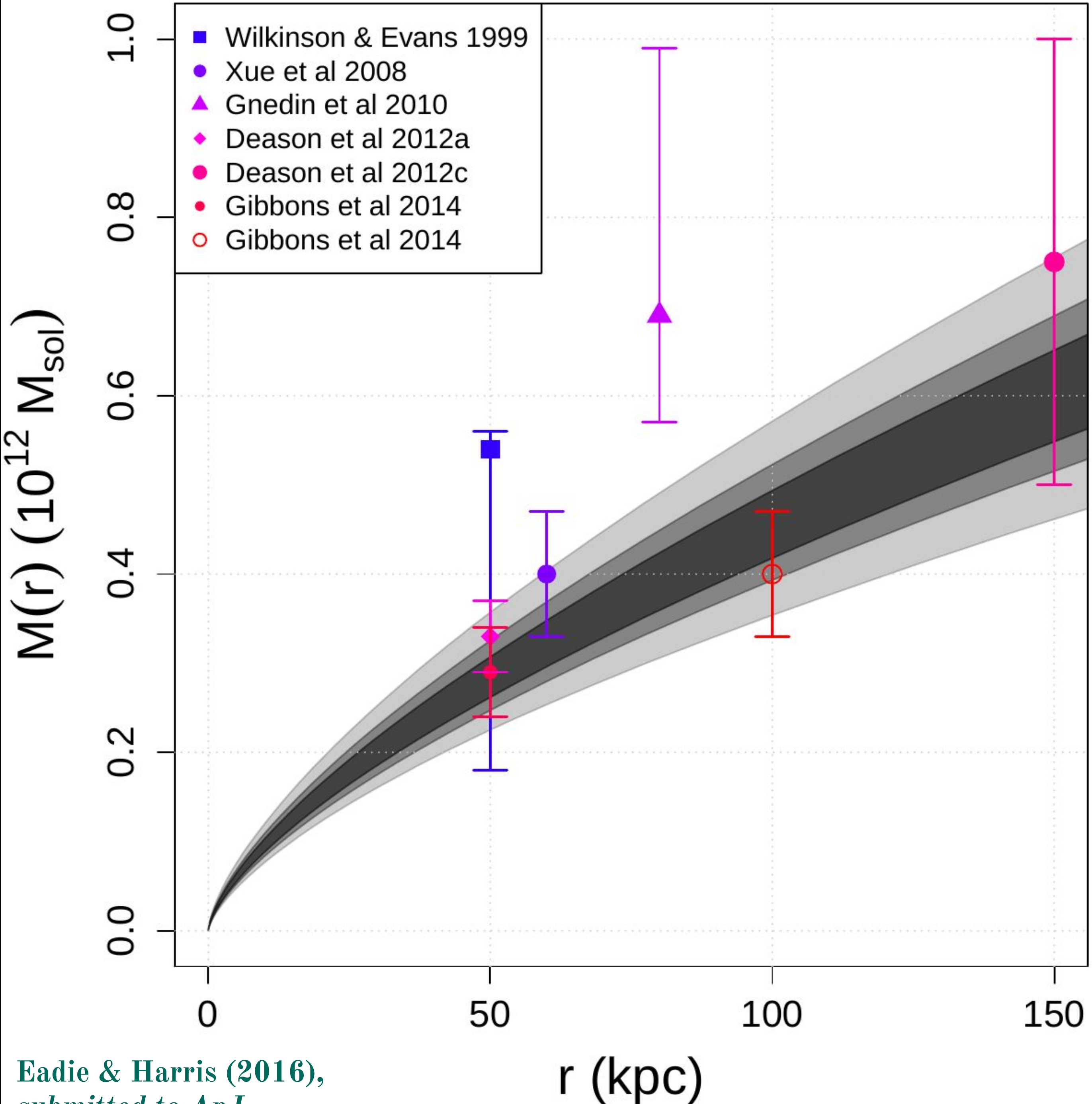


Eadie & Harris (2016), *submitted to ApJ*

Scenario X's cumulative mass profile

Virial Mass:
7.04×10^{11} M_{sun}

50% cred.
(6.23, 7.76)



Eadie & Harris (2016), ***submitted to ApJ***

Latest Results

Hierarchical Bayes --- including measurement uncertainties

Setting up the Hierarchical Bayesian Model

- **Measurements** *are inherently uncertain*

  y

- **True values** *of positions and velocities are unknown*

  ϑ

- **Uncertainties** *are well understood*

  Δ

Data drawn from a distribution

Parameter (true value, unknown)

$$r \sim N(r, \Delta r)$$

Data

Uncertainty (fixed)

Probability distribution

Parameter (true value, unknown)

$$p\,(\mathrm{r} \mid \mathrm{r}, \Delta\mathrm{r})$$

Uncertainty (fixed)

Data

Hierarchical Bayesian Model

$$p(r|r, \Delta r)$$

Data

Parameter

Uncertainty (fixed)

Hierarchical Bayesian Model

Data

$p(r|r, \Delta r)$

$p(v_{los}|v_{los}, \Delta v_{los})$

Parameter

Uncertainty (fixed)

Hierarchical Bayesian Model

$$p(r|r, \Delta r)$$

$$p(v_{los}|v_{los}, \Delta v_{los})$$

$$p(\mu_\delta|\mu_\delta, \Delta\mu_\delta)$$

$$p(\mu_\alpha \cos\delta|\mu_\alpha \cos\delta, \Delta\mu_\alpha \cos\delta)$$

Data

Parameter

Uncertainty (fixed)

Hierarchical Bayesian Model

distance

$p(r|r, \Delta r)$

line-of-sight velocity

$p(v_{los}|v_{los}, \Delta v_{los})$

proper motion (DEC)

$p(\mu_\delta|\mu_\delta, \Delta\mu_\delta)$

proper motion (RA)

$p(\mu_\alpha \cos\delta|\mu_\alpha \cos\delta, \Delta\mu_\alpha \cos\delta)$

Hierarchical Bayesian Model

$$\mathcal{L} = p(r|r, \Delta r)p(v_{los}|v_{los}, \Delta v_{los})p(\mu_\delta|\mu_\delta, \Delta\mu_\delta)p(\mu_\alpha \cos\delta|\mu_\alpha \cos\delta, \Delta\mu_\alpha \cos\delta)$$

Hierarchical Bayesian Model

$$\mathcal{L} = p(r|r, \Delta r)p(v_{los}|v_{los}, \Delta v_{los})p(\mu_\delta|\mu_\delta, \Delta\mu_\delta)p(\mu_\alpha \cos\delta|\mu_\alpha \cos\delta, \Delta\mu_\alpha \cos\delta)$$

$$p(\boldsymbol{\theta}|\boldsymbol{y}, \Delta) \propto \prod_i^N \mathcal{L}(\boldsymbol{y_i}, \Delta_i|\vartheta_i)p(h(\vartheta_i)|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

Hierarchical Bayesian Model

$$f(\mathcal{E}, L) = \frac{L^{-2\beta}\mathcal{E}^{\frac{\beta(\gamma-2)}{\gamma}+\frac{\alpha}{\gamma}-\frac{3}{2}}}{\sqrt{8\pi^3 2^{-2\beta}}\Phi_o^{-\frac{2\beta}{\gamma}+\frac{\alpha}{\gamma}}} \frac{\Gamma\left(\frac{\alpha}{\gamma}-\frac{2\beta}{\gamma}+1\right)}{\Gamma\left(\frac{\beta(\gamma-2)}{\gamma}+\frac{\alpha}{\gamma}-\frac{1}{2}\right)}$$

$$p(\boldsymbol{\theta}|\boldsymbol{y}, \Delta) \propto \prod_{i}^{N} \mathcal{L}(\boldsymbol{y_i}, \Delta_i|\vartheta_i)\, p(h(\vartheta_i)|\boldsymbol{\theta})\, p(\boldsymbol{\theta})$$

Hierarchical Bayesian Model

Hyperpriors on model parameters

$$p(\boldsymbol{\theta}|\boldsymbol{y},\Delta) \propto \prod_{i}^{N} \mathcal{L}(\boldsymbol{y_i},\Delta_i|\vartheta_i)p(h(\vartheta_i)|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

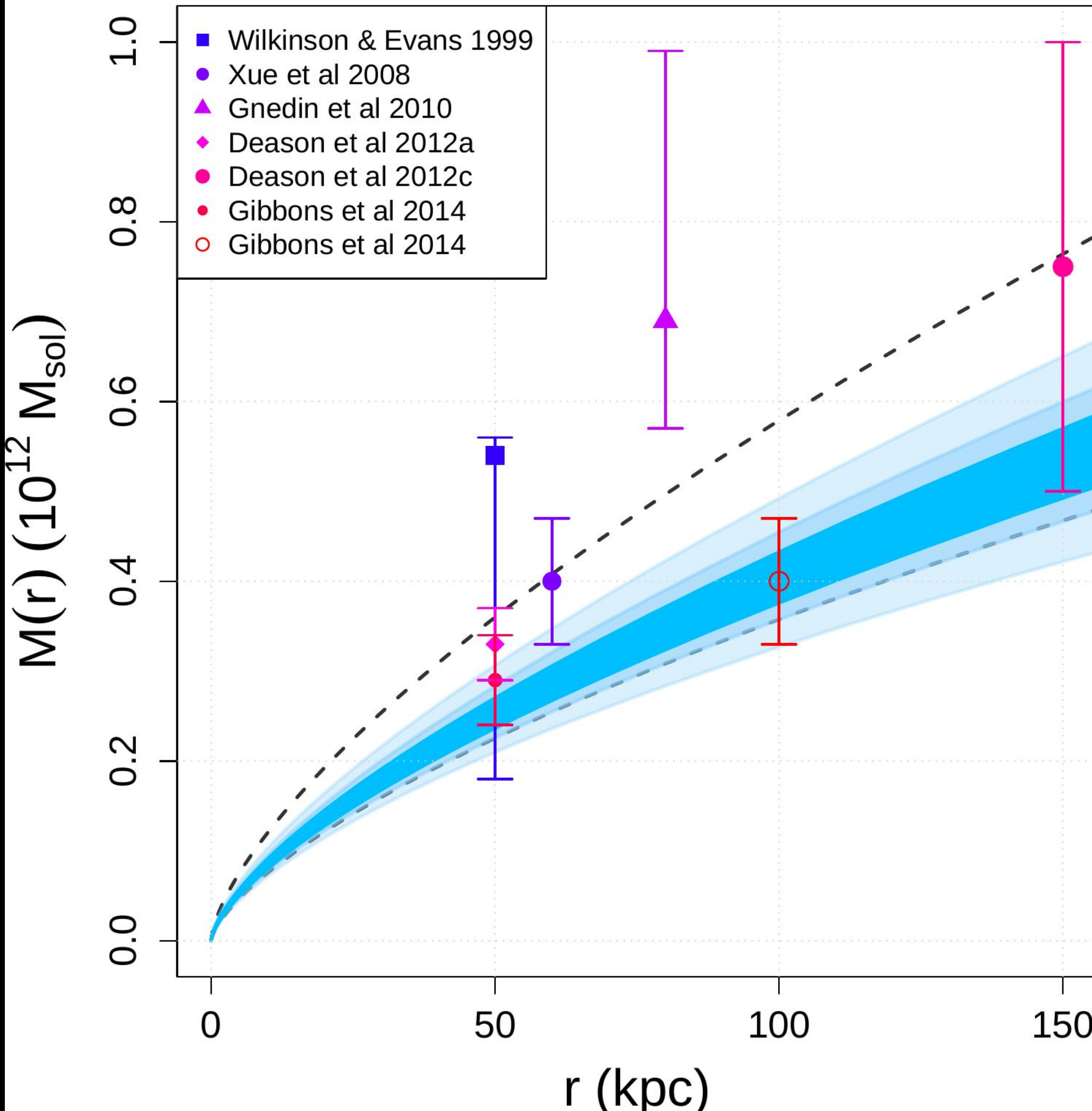
Eadie, Harris, and Springford (2016) ***in prep***
Eadie, Harris, Widrow, and Springford (2015) IAU Proceedings
Eadie, Harris, & Springford (2015) JSM Proceedings

Scenario X (new)

same data (89 GCs) but *including uncertainties*

Virial Mass:
5.98×10^{11} M_{sun}

50% cred.
(5.39, 6.52)



Eadie, Harris, and Springford (2016), *in prep*

Conclusions

- **Mass profile credible regions** easy to compare to other studies
- Includes **incomplete and complete data** simultaneously
- **Mass of the Milky Way** is "light" under power-law model

$M_{vir} = (4.40 - 7.76) \times 10^{11}\ M_{sun}$ (95% credible region)

Future Work & Challenges

- Other galaxies where 3D motions are never known
- Implement more realistic, physical models
- First GAIA data release coming soon!

Thank you!

@gweneadie

#figureskatingastrophysicist

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NSERC CRSNG

Eadie, Harris, & Springford, *in prep*.
Eadie & Harris (2016), submitted to ApJ.
Eadie, Harris, Widrow (2015), ApJ 806, 54.
Eadie, Harris, Widrow & Springford (2015), IAU Proceedings, Symposium 317.
Eadie, Harris, Springford (2015), JSM Proceedings, Section on Physical & Engineering Sciences.