Sparsity to the rescue of 3D weak lensing mass-mapping

Statistical Challenges in Modern Astronomy VI

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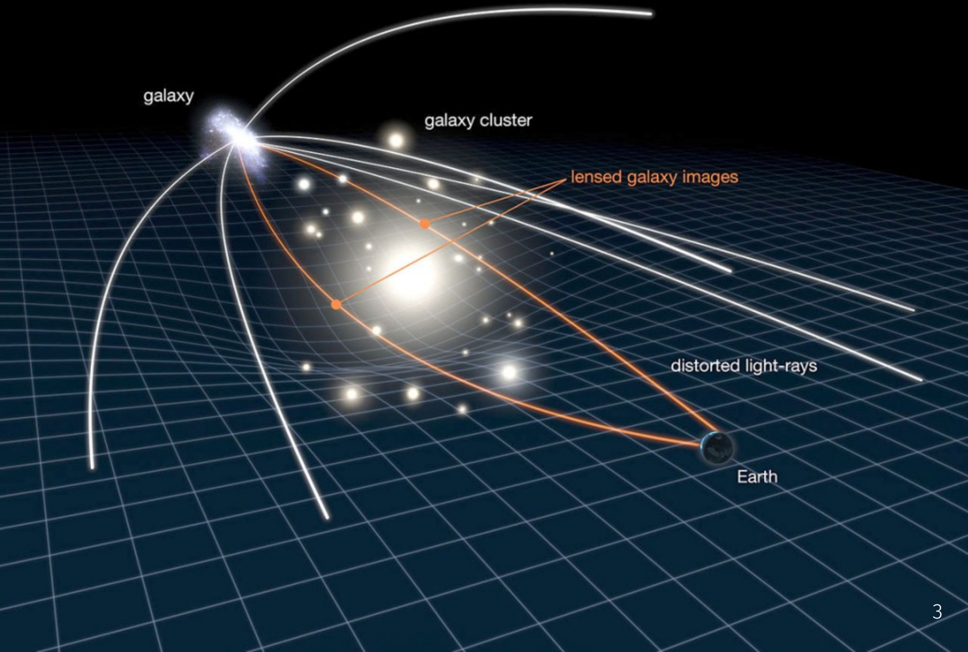
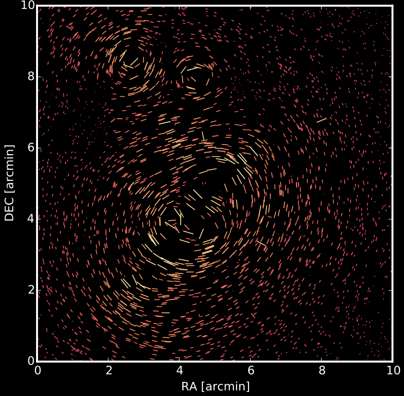
THE 3D MASS-MAPPING PROBLEM

GRAVITATIONAL LENSING BY A MASSIVE CLUSTER

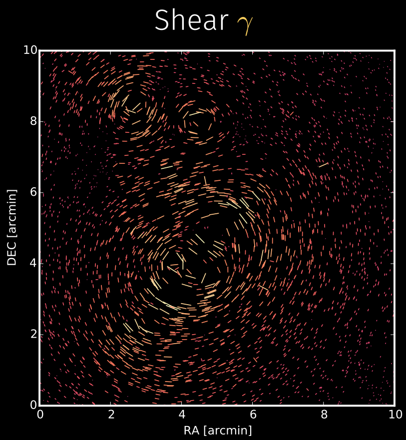


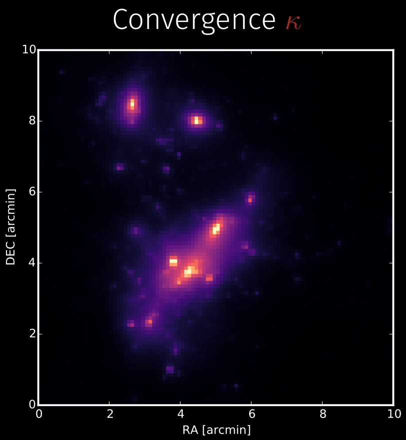
2D MASS MAPPING

Shear γ



2D MASS MAPPING





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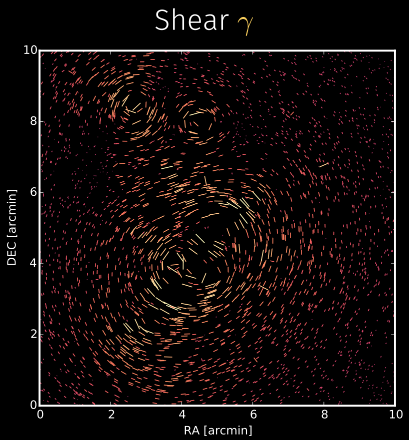
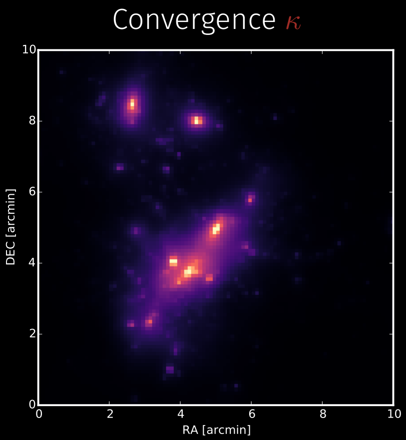
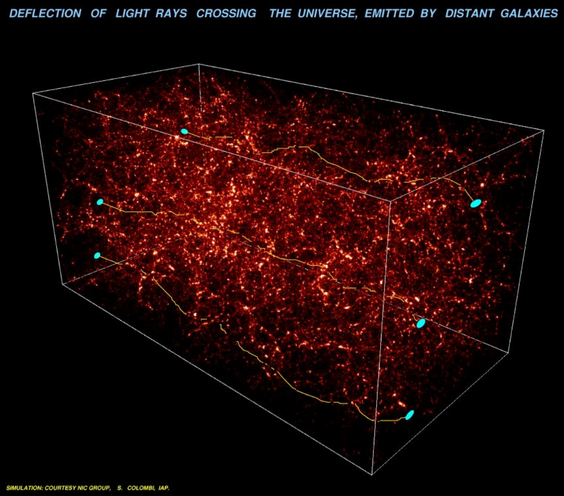




$$\gamma = \mathrm{P}\,\kappa$$

PROBING THE MATTER DISTRIBUTION IN 3D



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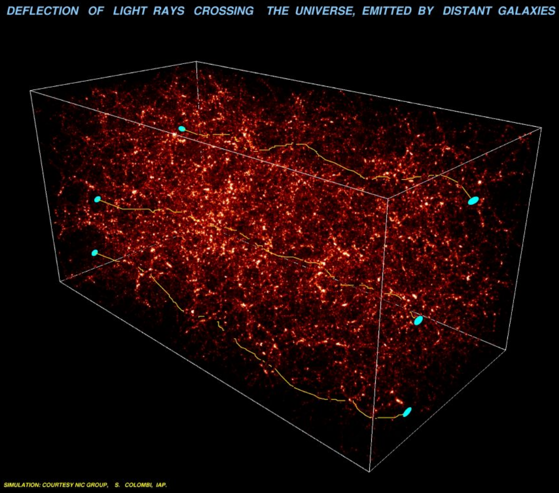
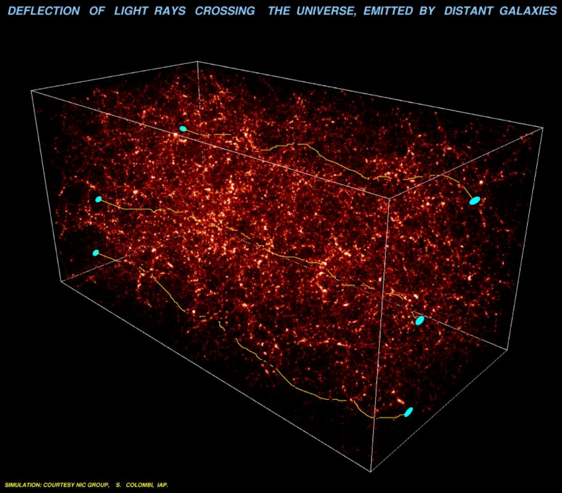


$$\gamma(z) = \mathsf{P} \int dz' \; Q(z, z') \; \delta(\boldsymbol{\theta}, z')$$

PROBING THE MATTER DISTRIBUTION IN 3D



$$\gamma = \mathsf{P}\,\mathsf{Q}\,\delta$$

$$\underbrace{\gamma}_{\text{shear}} = \mathbf{P}\,\mathbf{Q} \underbrace{\delta}_{\text{overdensity}} + \underbrace{n}_{\text{noise}}$$

P and Q are the tangential and line of sight lensing operators

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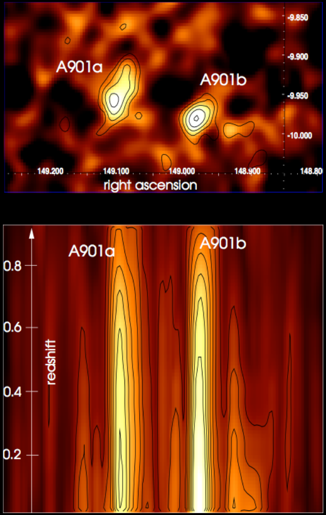
P and **Q** are the tangential and line of sight lensing operators

Difficulty of 3D mass-mapping

- **P** is non-invertible when the shear is irregularly sampled
- **Q** is ill-posed, direct inversion impossible

WIENER FILTER RECONSTRUCTION OF ABELL901/902



from Simon et al. (2012)

SPARSE REGULARISATION FOR 2D MAPPING

FOURIER INVERSION

$$\gamma_1 = \frac{1}{2}(\partial_1^2 - \partial_2^2)\,\Psi \qquad \gamma_2 = \partial_1\partial_2\,\Psi \qquad \kappa = \frac{1}{2}(\partial_1^2 + \partial_2^2)\,\Psi$$

- Explicit solution in Fourier space:

$$\hat{\kappa} = \frac{k_1^2 - k_2^2}{k_1^2 + k_2^2}\,\hat{\gamma}_1 + \frac{2k_1k_2}{k_1^2 + k_2^2}\,\hat{\gamma}_2$$

FOURIER INVERSION

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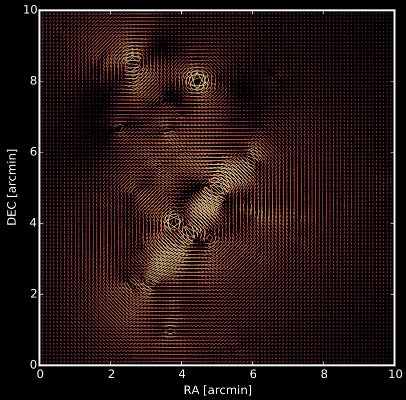
- Explicit solution in Fourier space:

$$\hat{\kappa} = \mathrm{P}^*\ \hat{\gamma}$$

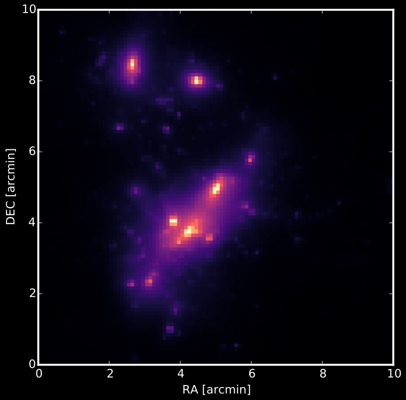
- Difficulties of the inversion:
    - Irregularly sampled or missing data.
    - Individual measurements extremely noise dominated.

IMPACT OF IRREGULAR SAMPLING

Shear γ



Convergence κ

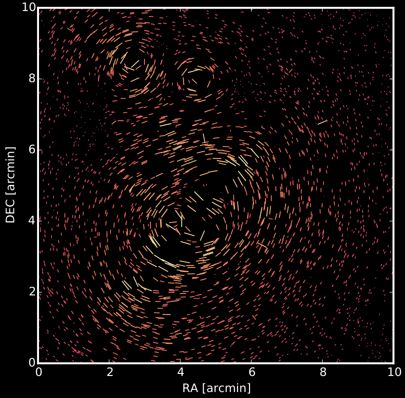


Regular shear sampling

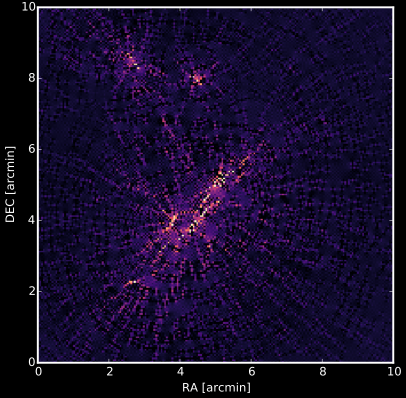
$$\kappa = \mathrm{F}^* \mathrm{P}^* \mathrm{F}\, \gamma$$

IMPACT OF IRREGULAR SAMPLING

Shear γ



Convergence κ

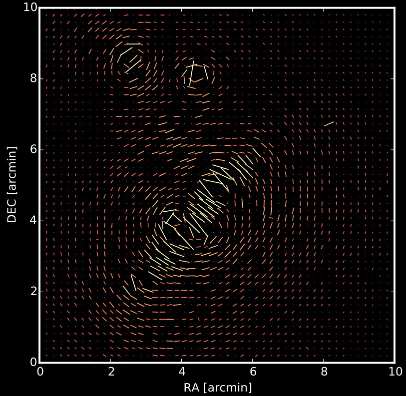


Irregular shear sampling

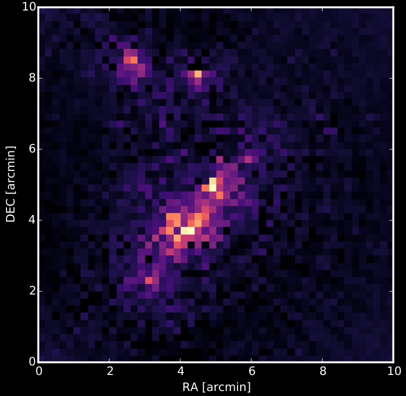
$$\kappa = F^* P^* F M \gamma$$

IMPACT OF IRREGULAR SAMPLING

Shear γ



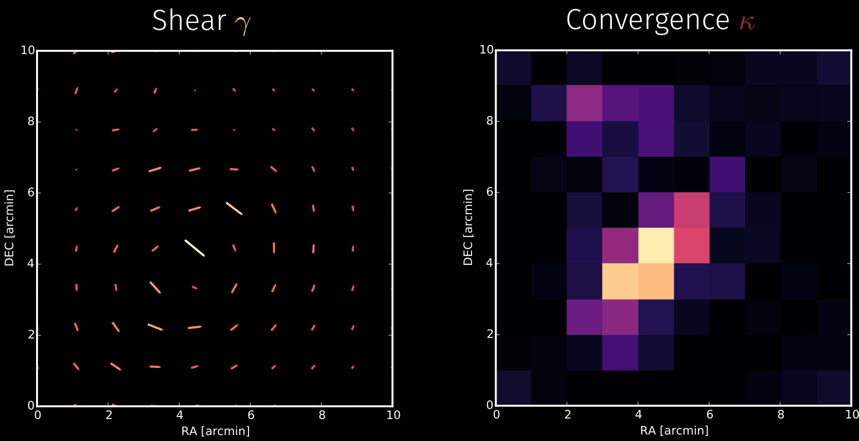
Convergence κ



Regularly binned shear (0.2 arcmin)

$$\kappa = F^* P^* F M \gamma$$

IMPACT OF IRREGULAR SAMPLING



Regularly binned shear (1 arcmin)

$$\kappa = \mathbf{F}^* \mathbf{P}^* \mathbf{F} \mathbf{M} \, \gamma$$

From a regularly sampled convergence map κ we compute the shear at each galaxy position: no binning.

$$\gamma = \mathbf{TPF}\,\kappa$$

Non Equispaced Discrete Fourier Transform (NDFT)

Fourier transform from a regular grid to an irregular grid x_l:

$$f = \mathbf{T}\hat{f} \quad \text{with} \quad T_{lk} = \frac{1}{\sqrt{N}} e^{2\pi i k x_l}$$

$\mathbf{T}$ is no longer directly invertible $\Rightarrow$ Linear inverse problem.

- Extremely analogous to radio-interferometry and MRI

$$\underset{\kappa}{\arg\min}\ \frac{1}{2}\parallel \gamma - \mathsf{TPF}\,\kappa \parallel_2^2 \ + \ \lambda \parallel \Phi^*\kappa \parallel_1$$

SPARSE REGULARISATION – SIMPLIFIED PROBLEM

Data fidelity

$$\underset{\kappa}{\arg\min} \; \overbrace{\frac{1}{2} \parallel \gamma - \mathrm{TPF}\,\kappa \parallel_2^2}^{} \; + \; \lambda \parallel \Phi^* \kappa \parallel_1$$

SPARSE REGULARISATION - SIMPLIFIED PROBLEM

$$\arg\min_{\kappa} \underbrace{\frac{1}{2} \| \gamma - \mathrm{TPF}\, \kappa \|_2^2}_{\text{Data fidelity}} + \underbrace{\lambda \| \Phi^* \kappa \|_1}_{\text{Sparsity prior}}$$

SPARSE REGULARISATION – SIMPLIFIED PROBLEM

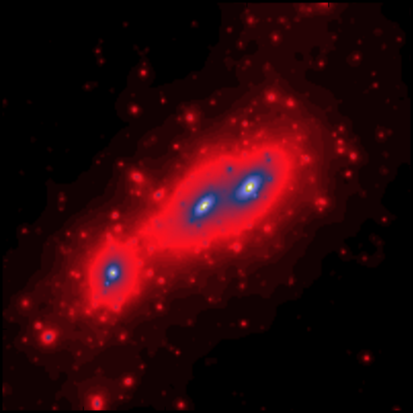
Data fidelity

$$\arg\min_{\kappa} \; \underbrace{\frac{1}{2} \parallel \gamma - \mathbf{TPF}\, \kappa \parallel_2^2}_{\text{Data fidelity}} + \underbrace{\lambda \parallel \Phi^* \kappa \parallel_1}_{\text{Sparsity prior}}$$

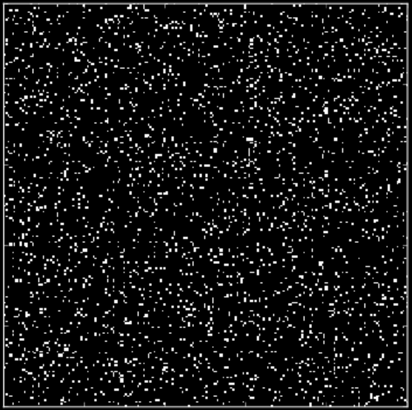
Sparsity prior

Wavelet based

Illustration - Noiseless Recovery



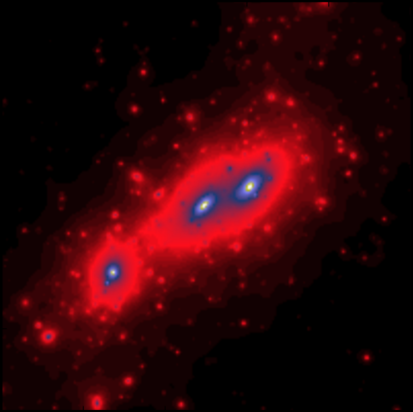
Input map 10×10 arcmin



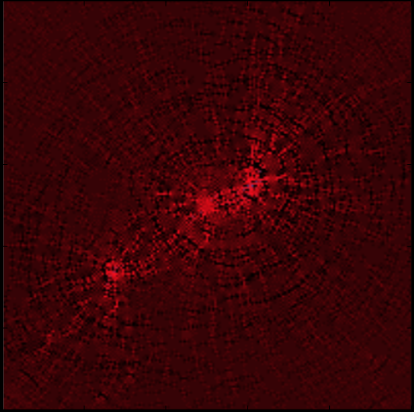
Source distribution

30 gal/arcmin^2 93% of missing data

ILLUSTRATION – NOISELESS RECOVERY



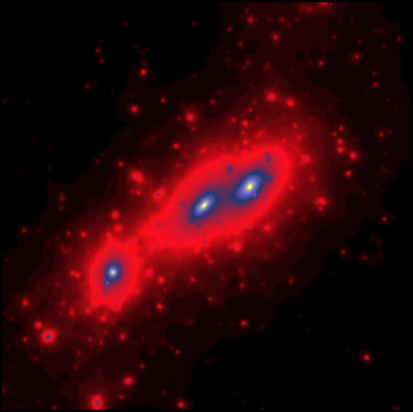
Input map 10 $\times$ 10 arcmin



Without regularisation

30 gal/arcmin^2 93% of missing data

ILLUSTRATION - NOISELESS RECOVERY



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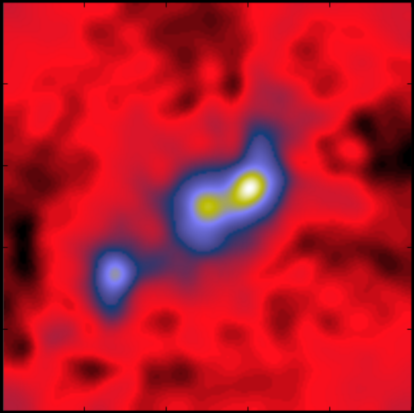
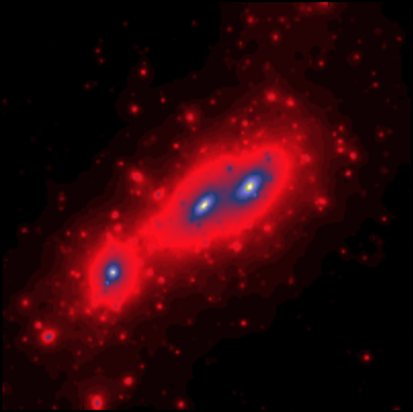


Binning + Gaussian Smoothing

30 gal/arcmin^2 93% of missing data

ILLUSTRATION - NOISELESS RECOVERY



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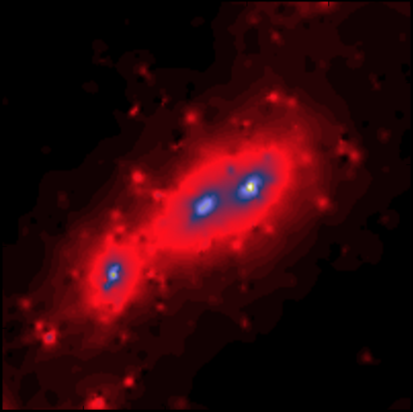

Sparse regularisation

30 gal/arcmin^2 93% of missing data

SPARSE REGULARISATION - FULL 2D PROBLEM

Lanusse, Starck, Leonard, Pires (2016)

$$\underset{\boldsymbol{\kappa},\tilde{\boldsymbol{\mathcal{F}}}}{\arg\min} \frac{1}{2} \parallel \mathcal{C}_\kappa^{-1} \left[(1 - \mathrm{ZTF}^* \boldsymbol{\kappa}) g - \mathrm{ZTPF}^* \boldsymbol{\kappa}\right] \parallel_2^2$$
$$+ \frac{1}{2} \parallel \mathcal{C}_\kappa^{-1} \left[(1 - \mathrm{ZTF}^* \boldsymbol{\kappa}) F - \mathrm{ZTF}^* \tilde{\boldsymbol{\mathcal{F}}}\right] \parallel_2^2$$
$$+ \lambda \parallel w \circ \boldsymbol{\Phi}^t \boldsymbol{\kappa} \parallel_1 + i_{\mathrm{Im}(\mathrm{R})} \left(\begin{bmatrix} \boldsymbol{\kappa} \\ \tilde{\boldsymbol{\mathcal{F}}} \end{bmatrix} \right)$$

- Includes reduced shear, reduced flexion, individual redshifts
- Optimally combines shear and flexion
- Tuning of the sparsity constraint based on local noise levels
- Solved using a primal-dual proximal algorithm adapted from (Vu, 2013)

EXTENSION TO THE 3D PROBLEM

SPARSE REGULARISATION OF THE FULL INVERSE PROBLEM

Leonard, **Lanusse**, Starck (2015)
Leonard, **Lanusse**, Starck (2014)
Leonard, Dupe, Starck (2012)

$$\hat{\delta} = \arg\min_{\delta} \frac{1}{2} \parallel \gamma - \mathbf{T}\,\mathbf{P}\,\mathbf{F}\;\;\mathbf{Q}\;\;\delta \parallel_2^2 + \lambda \parallel \Phi^* \delta \parallel_1$$

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Typically solved using FISTA (Fast Iterative Soft Thresholding)

Leonard, **Lanusse**, Starck (2015)
Leonard, **Lanusse**, Starck (2014)
Leonard, Dupe, Starck (2012)

Badly conditioned

$$\hat{\delta} = \arg\min_{\delta} \frac{1}{2} \parallel \gamma - \mathbf{T}\,\mathbf{P}\,\mathbf{F}\,\mathbf{Q}\,\delta \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^* \delta \parallel_1$$

Typically solved using FISTA (Fast Iterative Soft Thresholding)

Lanusse, Starck (2016), in prep.

We use the SVD of **Q** to build a preconditioning matrix **N**:

$$\mathbf{Q} = \mathbf{U}\mathbf{S}\mathbf{V}^* \qquad \mathbf{N} = \mathbf{V}\,\overline{S^{-1}}\,\mathbf{V}^*$$

and we solve the problem:

$$\delta = \mathbf{N} \quad \arg\min_{\delta'} \frac{1}{2} \parallel \gamma - \mathbf{P}\,\mathbf{Q}'\,\delta' \parallel_2^2 + \lambda \parallel \Phi^* \mathbf{N} \delta' \parallel_1$$

Lanusse, Starck (2016), in prep.

We use the SVD of Q to build a preconditioning matrix N:

$$\mathsf{Q} = \mathsf{USV}^* \qquad \mathsf{N} = \mathsf{V}\ \overline{S^{-1}}\ \mathsf{V}^*$$

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- Convergence rate improved by orders of magnitudes

PRECONDITIONING OF THE PROBLEM

Lanusse, Starck (2016), in prep.

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$$\mathbf{Q} = \mathbf{U}\mathbf{S}\mathbf{V}^* \qquad \mathbf{N} = \mathbf{V}\, \overline{S^{-1}}\, \mathbf{V}^*$$

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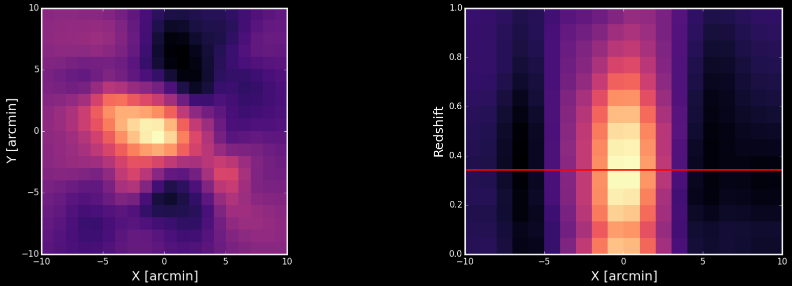
- Convergence rate improved by orders of magnitudes
- Sparsity constraint more complex to compute
  - Fast Spectral Projected Gradient algorithm
  - GP-GPU parallel implementation

Illustration on a cluster from the MICE N-body simulations

$10^{15}h^{-1}M_{\odot}$ cluster at z=0.37



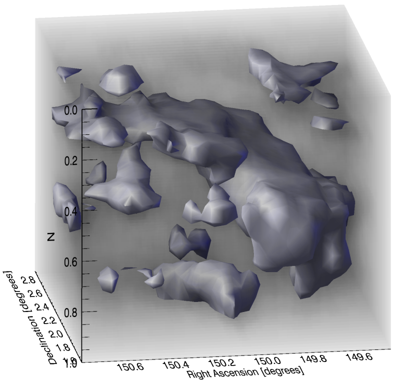
Reconstruction using transverse Wiener filter from Simon et al. (2011)

$10^{15} h^{-1} M_{\odot}$ cluster at z=0.37

Reconstruction using GLIMPSE

PRELIMINARY RESULTS ON COSMOS

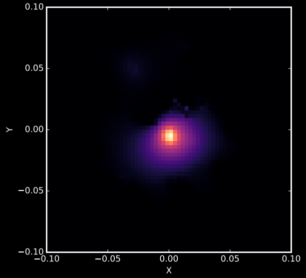
RECONSTRUCTION FROM MASSEY ET AL. (2007)

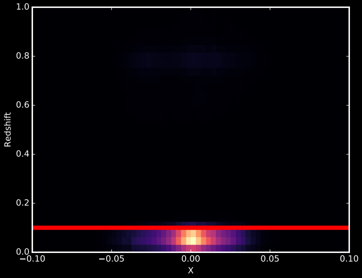


RECONSTRUCTED 3D DENSITY CONTRAST

Lanusse, Starck (2016), in prep.

ZOOM IN ON A CLUSTER AT REDSHIFT 0.12





CONCLUSION

- Incorporates developments introduced in Lanusse et al. (2016):
  - Handles irregularly distributed galaxies (**no binning**)
  - Include **individual redshift pdfs**
  - Accounts for **reduced shear**

- Tractable optimisation thanks to new algorithm

- Promising preliminary results on real data.

- Open-source software. The 2D version already available at:

https://github.com/CosmoStat/Glimpse