STATISTICS
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Discovery and Characterization of RR Lyrae from Sparsely Sampled Light Curves Using Parsimonious Models

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Time Domain Astronomy, Variable Stars, Galactic Halo

Period Estimation Methods

Parsimonious Model for RR Lyrae Variables

Simulations

Related and Ongoing Work

Texas A&M Collaboration

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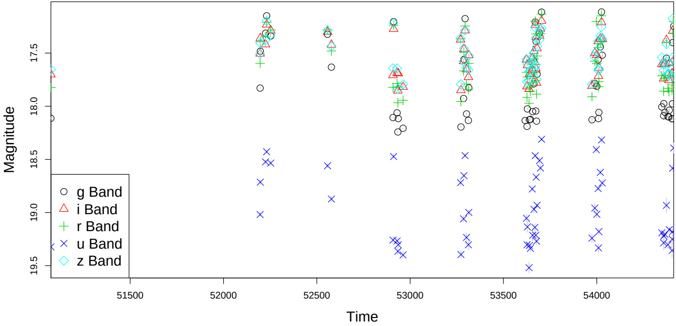
Outline

Periodic Variable Stars

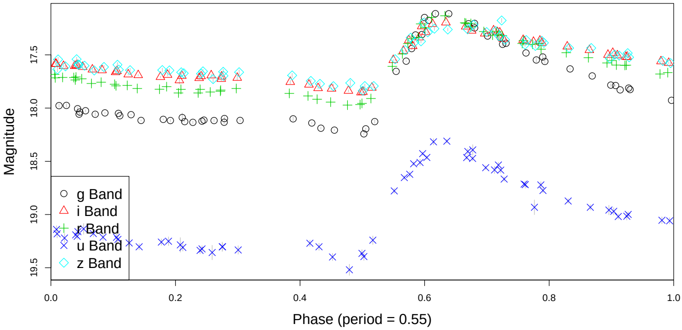
Periodic variables: Stars that repeat brightness variation over a fixed period.



- Star observed ≈ 250 times in 5 bands, u, g, r, i, z.
- Data for star is $D = \{t_{jb}, m_{jb}, \sigma_{jb}\}_{j=1}^{n_b}$ for $b = 1, \ldots, B$.
- Observe brightness m_{jb} at time t_{jb} with uncertainty σ_{jb} in band b.

Folded Light Curve of Periodic Variable

Folded light curve: Brightness versus time modulo period.

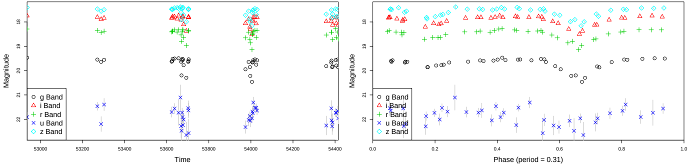


SDSS–III Stripe 82 [2]

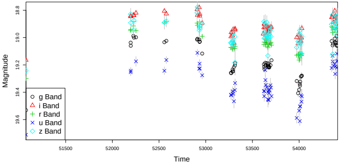
- Collected $\approx 60,000$ variables in Stripe 82 Region
- Variables belong to different **classes**
- Some periodic, some not

Two Examples

Eclipsing Binary



Quasar

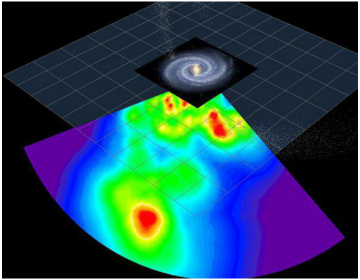


Example of Variable Star Science

- For RR Lyrae variable stars distance is a function of mean brightness in V:

$$d = 10^{(V - M_V + 5)/5}$$

- Discover structure in the Milky Way halo by finding, estimating distances to RR Lyrae



Sesar [10] using SDSS Stripe 82 Data

- Compare observed structure to cosmological simulations.

Mapping Galactic Halo with DES

Dark Energy Survey (DES)

- 10 photometric measurements in (g, i, r, z, Y) over five years
- depths to 24 mag in i
- 5000 square degrees
- few million variable stars

DES is deeper and wider than SDSS Stripe 82, but much more sparsely sampled.

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Sinusoid Based Models

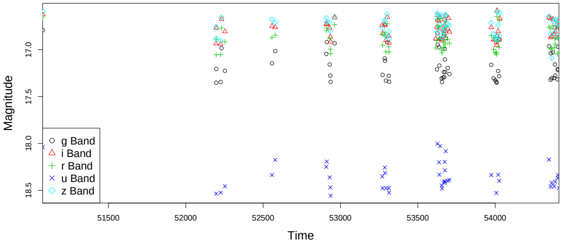
Method: Model light curve variation in each filter as sinusoid with K harmonics. [6, 14, 4, 8, 9]

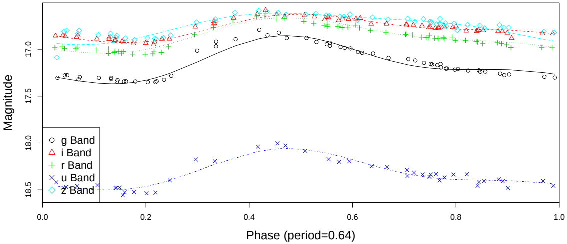
$$m_{jb} = \beta_b + \sum_{k=1}^{K} a_{bk} \sin(\omega t_{jb} + \phi_{bk}) + \epsilon_{jb}$$

where $\epsilon_{jb} \sim N(0, \sigma_{jb}^2)$.

- $(2K + 1)B + 1$ parameters. Pure sine $3B + 1$ parameters.
- Maximum likelihood has closed form solution at fixed ω
  - Maximization strategy: Grid search on ω.

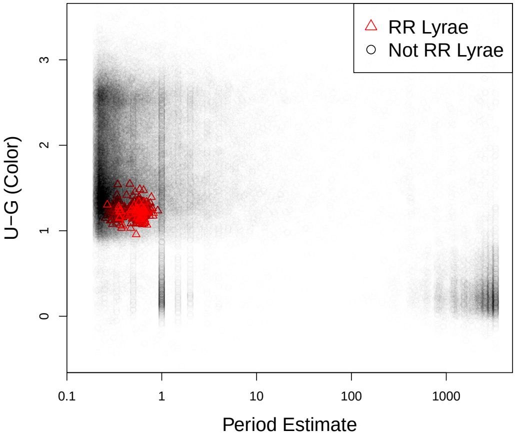
Example of Maximum Likelihood Fit with $K = 2$



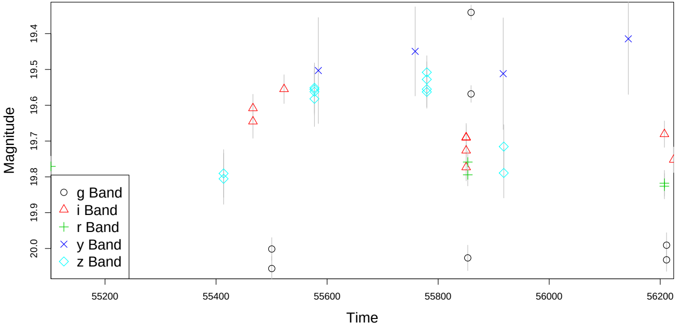


Total of 26 parameters, model estimates period well.

Output from Model Useful for Classification



Problem: Sparsely Sampled Light Curves



Pan–STARRS light curve. Period estimation difficult for this quality data.

Outline

Parsimonious RR Lyrae Model

global parameters fit once for all RR Lyrae

magnitude offset band b | extinction law band b | **RR Lyrae shape**

$$m_b(t) = \mu + M_b + E[B-V]R_b + a\gamma_b(\omega t + \phi)$$

distance modulus | amount of dust | amplitude | frequency | phase

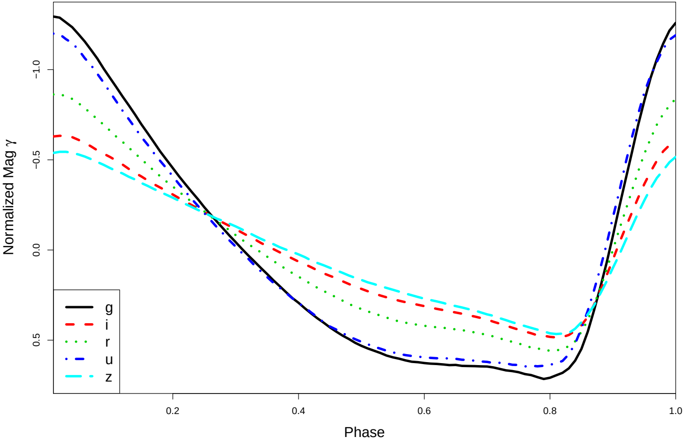
individual parameters fit for each RR Lyrae

- Data $D = \{\{t_{jb}, m_{jb}, \sigma_{jb}\}_{j=1}^{n_b}\}_{b=1}^{B}$
- Normal measurement error model:

$$m_{jb} = m_b(t_{jb}) + \epsilon_{jb}$$

where $\epsilon_{jb} \sim N(0, \sigma_{jb}^2)$.

γ_b Estimated from SDSS Stripe 82 RR Lyrae



Parameter Estimation

$$RSS(\omega, \mu, E[B-V], a, \phi)$$
$$\equiv \sum_{b=1}^{B} \sum_{j=1}^{n_b} \left(\frac{m_{jb} - \mu - M_b - E[B-V]R_b - a\gamma_b(\omega t_{jb} + \phi)}{\sigma_{jb}} \right)^2$$

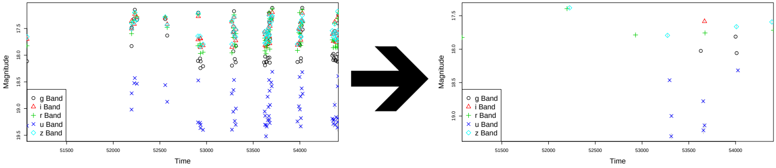
Estimate parameters with maximum likelihood (χ^2 minimization):

- Likelihood is highly multimodal in ω, grid search.
- Model is linear in μ,$E[B-V]$, and a, closed form updates.
- Warm start Newton–Raphson updates for ϕ.

Outline

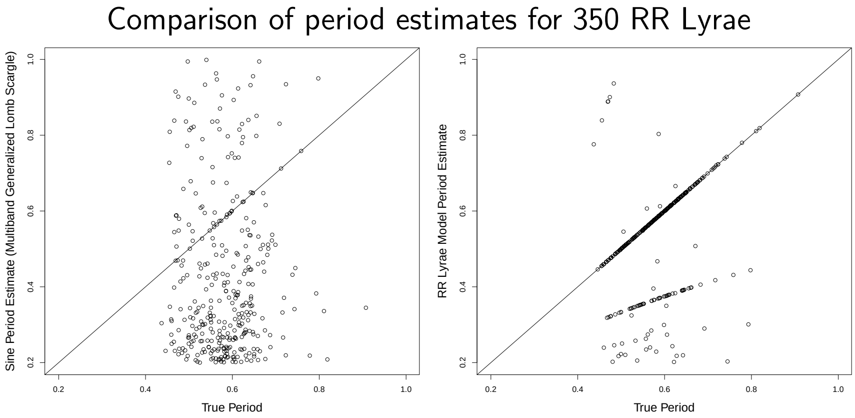
Simulation

Downsample Stripe 82 variables to 20 observations across all bands:



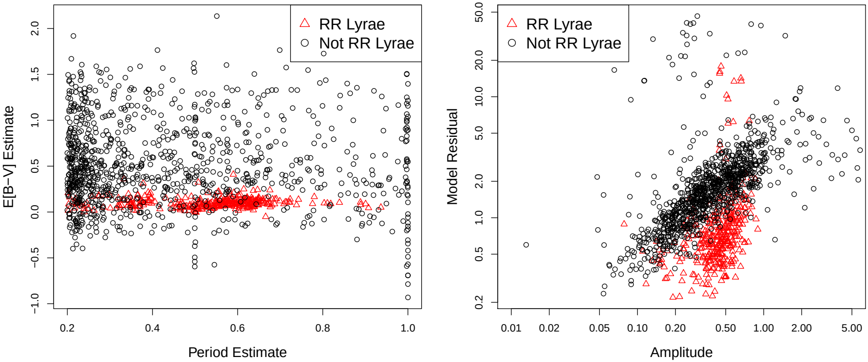
- Can we estimate periods correctly for RR Lyrae?
- Can we separate RR Lyrae from non–RR Lyrae?
- Can we estimate distances accurately?
- Can we reproduce halo maps of Sesar 2010?

Simulation Results for Period Estimation



Simulation Results for Classification

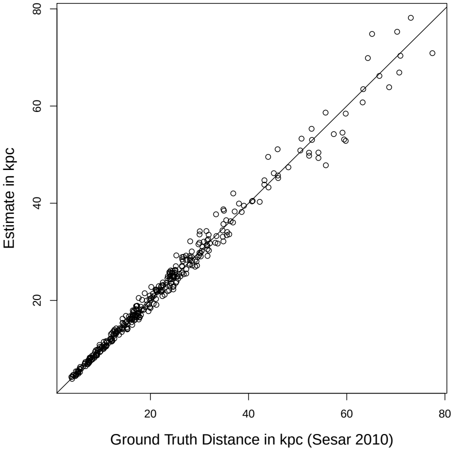
Downsampled 1000 Not–RR Lyrae and 350 RR Lyrae



- Visually good separation with small number of features.
- Potential to use model output as feature input to classifier.

Simulation Results for Distance Estimation

- Use model to estimate distance modulus (μ) for RR Lyrae.
- Convert μ to distance (d) in parsecs: $d = 10^{\mu/5+1}$



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Related Work

Period Estimation Algorithms / Variable Star Models:

- Sinusoid Based Methods
  - Lomb Scargle (LS) [4, 8]
  - Generalized LS (GLS) [14]
  - Multiband Extensions [12, 5].
  - AoV [9]
- "Non–parametric" methods
  - Phase Dispersion Minimization [11]
  - Supersmoother [10]
- Template Based Methods
  - RR Lyrae templates by Sesar [10], Kovacs [3]
  - Cepheid templates by Pejcha [7], Yoachim [13]

Finding RR Lyrae in Sparsely Sampled Light Curves:

- Hernitschek [1] uses structure functions to find RRL / QSO.

Ongoing Work

- Model refinements:
  - M_b dependence on period, metallicity.
  - Bands other than g, i, u, r, z
  - Parameterize γ_b to account for shape differences across RRL
- Propagate uncertainty to 3D halo density estimate
  - Treat as Inference Problem: Bayesian posterior or frequentist confidence bands. Hierarchical model?
  - Treat as Prediction Problem: Evaluate distance / density estimates using Stripe 82 data or follow–up observations.
- Milky Way Halo maps using DES data

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