Inference of Galaxy Population Statistics Using Photometric Redshift Probability Distribution Functions

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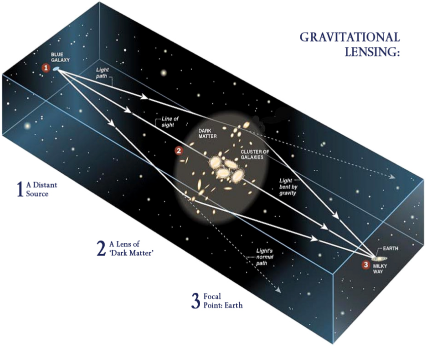
Background

Background

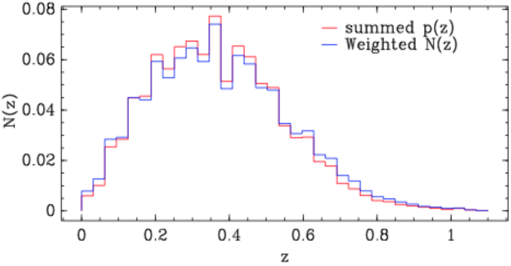
Weak gravitational lensing probes cosmology.



LSST

Background

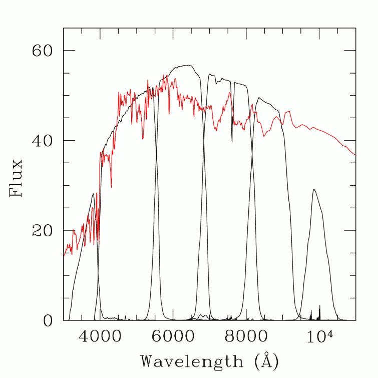
Weak gravitational lensing requires
the redshift distribution function $\mathcal{N}(z) = \frac{dN_g}{dz}$.



Sheldon+2012

Background

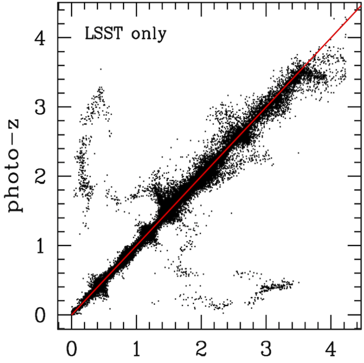
Photometric redshifts approximate spectroscopic redshifts.



LSST-DESC

Background

Photometric redshifts are problematic.



Jain+2015

Motivation

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Why use photo-z PDFs?

Photo-z PDFs include all redshift uncertainties and biases.

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How do we use photo-z PDFs?

A probabilistic object necessitates a probabilistic approach.

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Photo-z PDFs include all redshift uncertainties and biases.

How do we use photo-z PDFs?

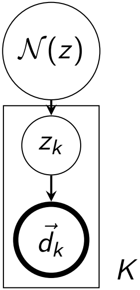
A probabilistic object necessitates a probabilistic approach.

What does the fully probabilistic approach get us?

It can yield the full posterior of parameters given data.

Probabilistic Graphical Models and Hierarchical Inference

Malz+2016 (in prep)



Probabilistic Graphical Models and Hierarchical Inference

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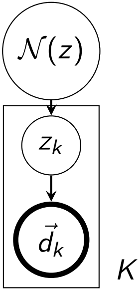

$$p(z|\mathcal{N}) = \frac{\mathcal{N}(z)}{\int \mathcal{N}(z)\ dz}$$

Probabilistic Graphical Models and Hierarchical Inference

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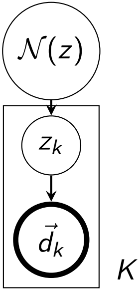

seek full posterior
$p(\mathcal{N}|\{\vec{d}_k\}_K)$

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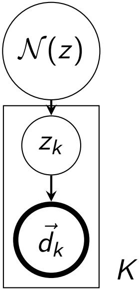

seek full posterior
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from individual posteriors
$\{p(z_k|\vec{d}_k)\}_K$

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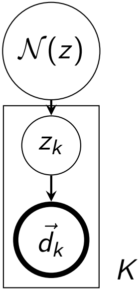

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(SDSS, LSST, ...)

Derivation
Malz+2016 (in prep)

seek full posterior $p(\mathcal{N}|\{\vec{d}_k\}_K)$

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$$\overbrace{p(\mathcal{N}|\{\vec{d}_k\}_K)}^{\text{posterior}} = \frac{\overbrace{p(\mathcal{N})}^{\color{blue}{\text{prior}}}}{\underbrace{p(\{\vec{d}_k\}_K)}_{\text{evidence}}} \times \qquad \overbrace{p(\{\vec{d}_k\}_K|\mathcal{N})}^{\text{likelihood}}$$

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$$\underbrace{\prod_{k=1}^{K}}_{\text{independence}} \underbrace{p(\vec{d}_k|\mathcal{N})}_{\text{likelihood}}$$

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$$\underbrace{\prod_{k=1}^{K}}_{\text{independence}} \underbrace{\int p(\vec{d}_k|z_k)p(z_k|\mathcal{N})dz_k}_{\text{hierarchical model}}$$

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need individual likelihoods $\{p(\vec{d}_k|z_k)\}_K$

from individual posteriors $\{p(z_k|\vec{d}_k)\}_K$

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from individual posteriors
$\{p(z_k|\vec{d}_k)\}_K$; interim prior $\mathcal{N}_0$

$$\overbrace{p(\vec{d}_k|z_k)}^{\text{likelihood}} = \overbrace{p(\vec{d}_k|z_k)}^{\text{likelihood}} \overbrace{\frac{p(z_k|\vec{d}_k,\mathcal{N}_0)}{p(z_k|\vec{d}_k,\mathcal{N}_0)}}^{1}$$

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$$\overbrace{p(\vec{d}_k|z_k)}^{\text{likelihood}} = \frac{\overbrace{p(z_k|\vec{d}_k,\mathcal{N}_0)}^{\text{interim posterior}}}{\underbrace{p(z_k|\mathcal{N}_0)}_{\propto\mathcal{N}_0}} \underbrace{\frac{\overbrace{p(\vec{d}_k|z_k)}^{\text{likelihood}}\, p(\vec{d}_k|\mathcal{N}_0)}{p(\vec{d}_k|z_k,\mathcal{N}_0)}}_{\text{independence}}$$

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given interim prior $\mathcal{N}_0$

have individual interim posteriors $\{p(z_k|\vec{d}_k,\mathcal{N}_0)\}_K$

Derivation

Malz+2016 (in prep)

$$p(\mathcal{N}|\{\vec{d}_k\}_K) \propto p(\mathcal{N}) \exp\left[-\int \mathcal{N}(z) dz\right] \times \prod_{k=1}^{K} \int p(z_k|\vec{d}_k, \mathcal{N}_0) \frac{p(z_k|\mathcal{N})}{p(z_k|\mathcal{N}_0)} dz_k$$

Sample this posterior!

Preliminary Results: Fiducial Test
Malz+2016 (in prep)

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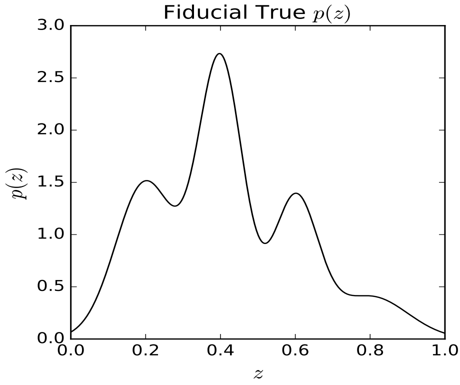
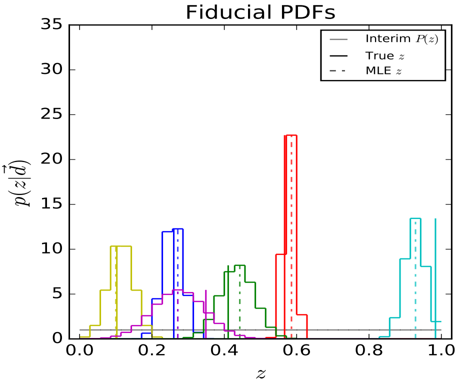


In simulations

Preliminary Results: Fiducial Test
Malz+2016 (in prep)



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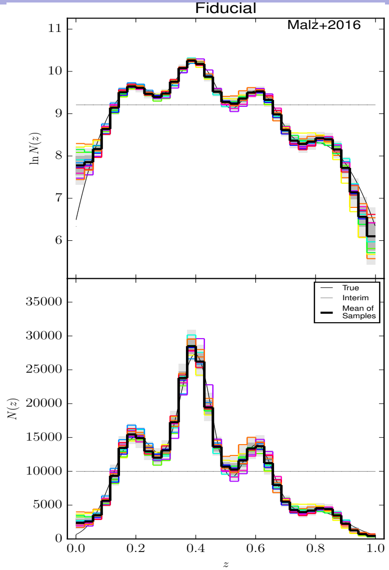

In simulations
the mean of the samples
recovers the true $\mathcal{N}(z)$
well

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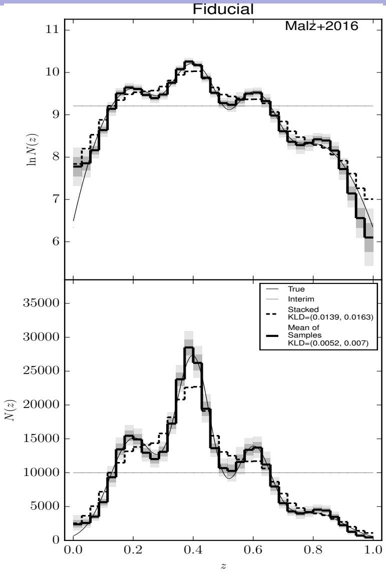
Malz+2016 (in prep)



In simulations
the mean of the samples
recovers the true $\mathcal{N}(z)$
well (and better than
popular alternatives).

Preliminary Results: Magnitude-cut BOSS Subsample
Malz+2016 (in prep)

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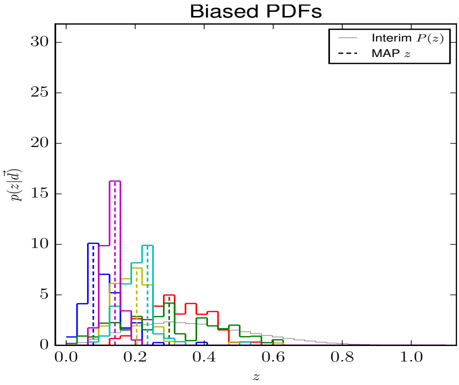
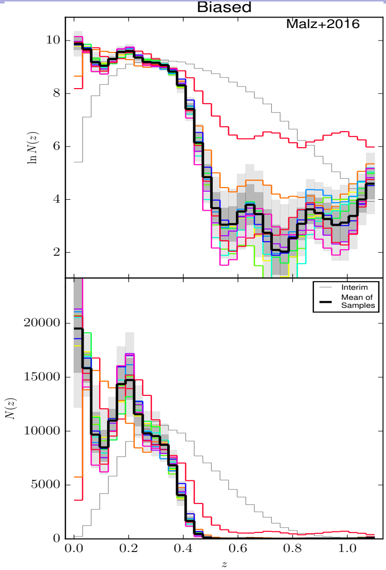


With real data

Preliminary Results: Magnitude-cut BOSS Subsample
Malz+2016 (in prep)



With real data
the mean of the samples
differs from the interim
prior.

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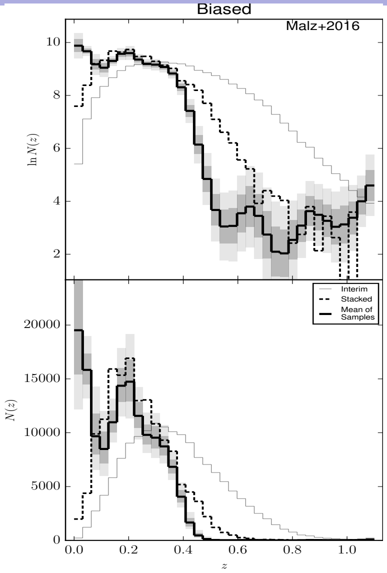
Malz+2016 (in prep)



With real data the mean of the samples differs from the interim prior.

Conclusion

https://github.com/aimalz/prob-z/nz

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Summary

- Cosmology needs a trustworthy $\mathcal{N}(z)$.
- Photo-z PDFs propagate all uncertainties and errors.
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Conclusion

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Recommendations

- Stacking photo-z PDFs yields a biased estimate of $\mathcal{N}(z)$.
- The mean of samples is an accurate point estimator.
- **It's best to sample the full posterior.**

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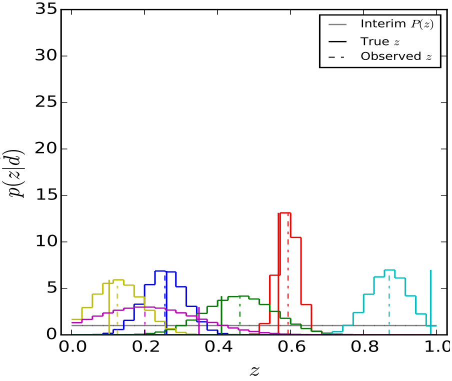
Code is now publicly available on GitHub!

Preliminary Results: Noisier Mock Data

Malz+2016 (in prep)



In simulations with lower S/N data

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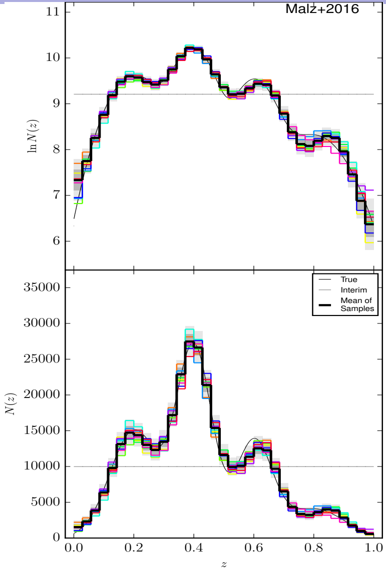
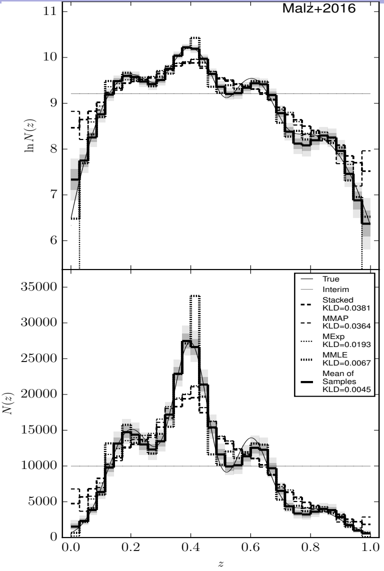


In simulations with lower S/N data
the marginalized MLE
and mean of the samples
recover the true $\mathcal{N}(z)$

Preliminary Results: Noisier Mock Data

Malz+2016 (in prep)



In simulations with lower S/N data the marginalized MLE and mean of the samples recover the true $\mathcal{N}(z)$ (with broader error bars).