A Statistical Study of Non-Linearity in the Leavitt Laws for Classical Cepheids

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Abstract

We present a detailed statistical analysis of possible non-linearities in the Period-Luminosity (P-L), Period-Wesenheit (P-W) and Period-Color (P-C) relations for Cepheid variables in the LMC at optical (VI) and near-infrared (JHK_s) wavelengths. We test for the presence of possible non-linearities and determine their statistical significance by applying a variety of robust statistical tests (F-test, Random-Walk, Testimator and the Davies test) to optical data from OGLE III and near-infrared data from LMC synoptic survey. We also use a global fit to Cepheid and Supernovae data using Chi-square minimization, a method used in determination of precise and accurate value of the Hubble constant in SH0ES project, to determine the impact of observed non-linearities in P-L relations on the distance scale. We do not find any significant difference in the slope of the global-fit solution using a linear or non-linear LMC P-L relation as calibrator, but the linear version provides a 2× better constraint on the slope and metallicity coefficient.

The data

Band	No. of stars FU	No. of stars FO
V	1849	1238
I	1842	1226
J	775	474
H	775	474
K_s	775	474

Table 1 : The Cepheid data selected for the present analysis. The optical data is taken from OGLE-III [1] catalog while the near-infrared data is taken from LMC synoptic survey [2]. The mean magnitudes for Cepheids in the optical and near-infrared bands are corrected for reddening using the Haschke maps [3] together with the extinction law of Cardelli et al. [4].

The Statistical Tests

F-test : We fit a single regression line over the entire period range under the null hypothesis while two-slope model is applied as an alternative hypothesis with an assumed break period. The F-statistic is calculated using equation:

$$F = \left(\frac{RSS_R - RSS_F}{RSS_F}\right)\left(\frac{\nu_F}{\nu_R - \nu_F}\right).$$

Here, RSS denotes the residual sums of squares, ν is the number of degrees of freedom, and subscripts R and F denote the one and two-slope models, respectively. We compare the observed value of F with the values obtained from the F-distribution under the null hypothesis at a given significance level.

Random walk : The non-parametric random walk test generates the distribution of the residuals from the data. If r_k are the residuals obtained from the linear fit to P-L relation, then partial sum of the residuals $C_j = \sum_{k=1}^{j} r_k$. If there is a departure from the linearity, then C_j will not be a simple random walk. The amplitude R = max(C_j) - min(C_j), is taken as the test-statistics. If the partial sums are a random walk, R will be small. To determine the significance level for the R-values, we use the permutation of residuals such as it destroys any possible trends. A large number of permutations are done to determine the probability as the theoretical distribution of the test statistic of C_j is not known.

Testimator : The entire sample is divided into n different non-overlapping, and hence completely independent subsets. We fit a linear regression to the first subset and determine a slope $\hat{\beta}$. This initial estimate (testimator) of the slope is taken as β_0 in the next subset, under the null hypothesis that the slope of the second subset is equal to the slope of the first subset, i.e. $\hat{\beta} = \beta_0$. The alternate hypothesis is that the two slopes are not equal. The t-statistic is used to compare the slopes and determine the probability that the initial guess of the testimator is true. If the null hypothesis is accepted, we derive the new testimator slope for the next subset using the previously determined β's such that:

$$\beta_w = k\hat{\beta} - (1-k)\beta_0, \text{ where, } k = (|t_{obs}|/t_c).$$

This value of the testimator is taken as β_0 for the next subset. This process of hypothesis testing is repeated n_g times or until the value k>1, suggesting rejection of the null hypothesis.

Segmented lines and the Davies test : This method first performs a linear piecewise regression considering the existence of the break. Thereafter, the Davies test is used to evaluate if the two segments are different enough to account for two separate linear behaviours. The method is implemented in the R package SEGMENTED. The regression is performed through the use of Generalized Linear Models (GLM).

All the test-statistics are discussed in detail in Bhardwaj et al. [5]. We also test our data for the assumption homoskedasticity and normality of residuals (q-q plots).

Results

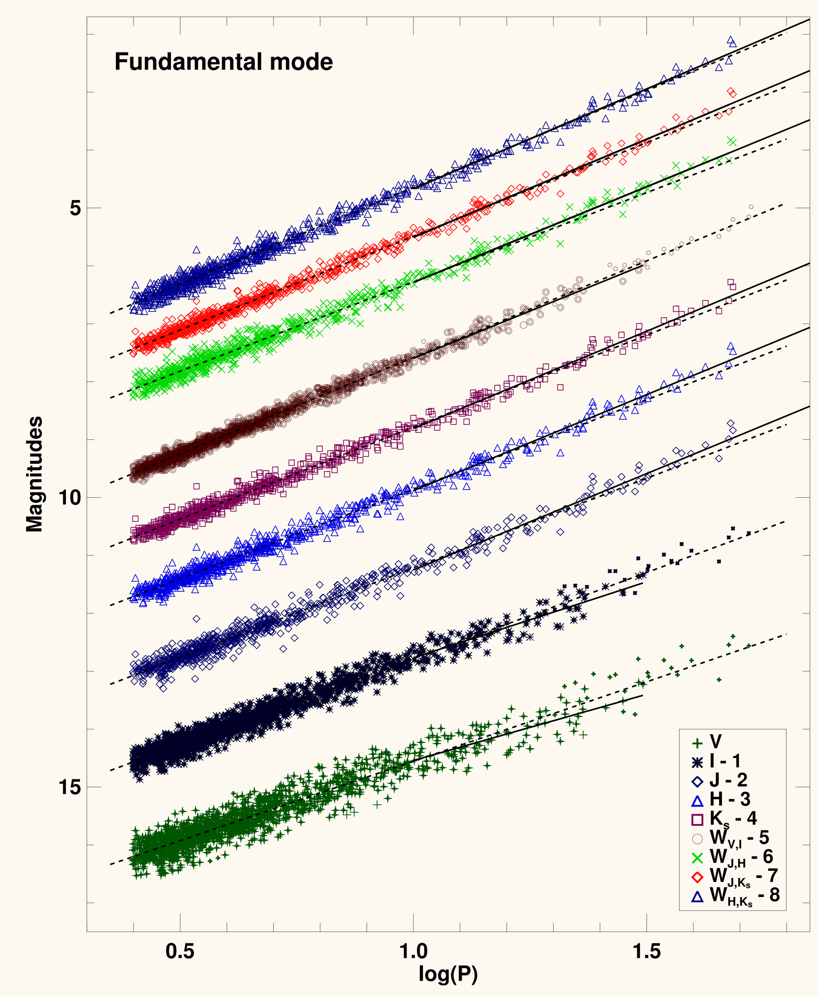

Figure 1: Optical and near-infrared P-L and P-W relations for LMC FU mode Cepheids.

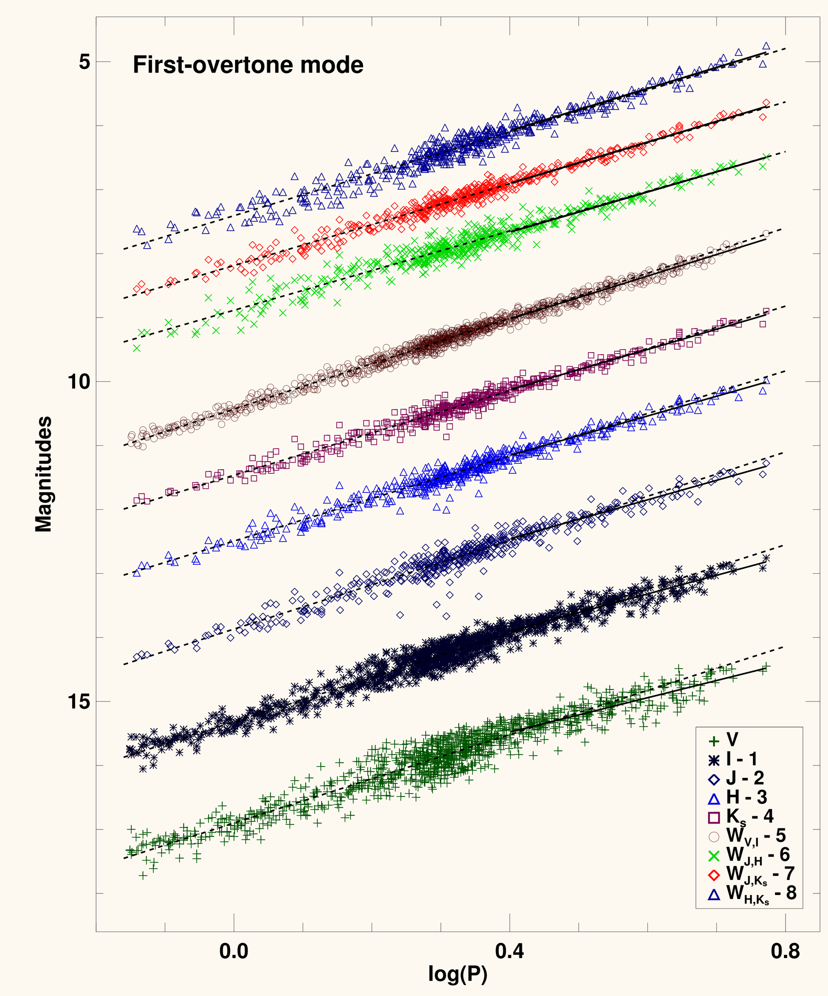

Figure 2: Optical and near-infrared P-L and P-W relations for LMC FO mode Cepheids.

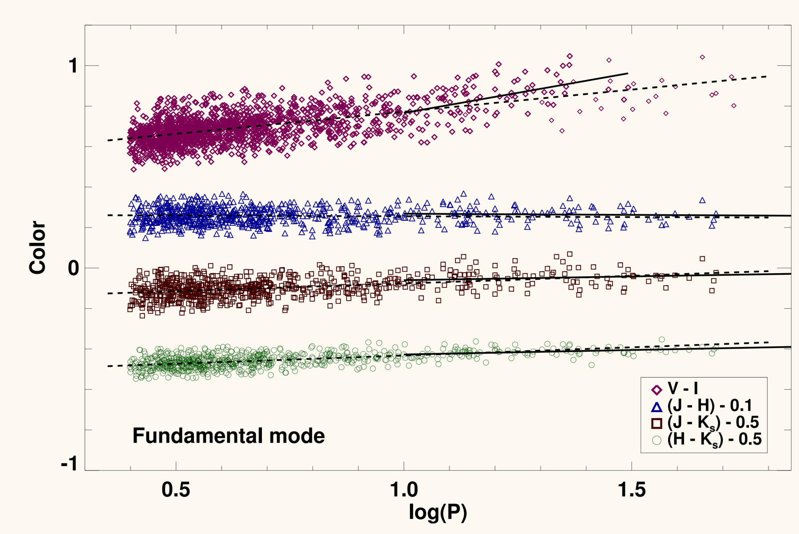

Figure 3: Optical and near-infrared P-C relations for LMC FU mode Cepheids.

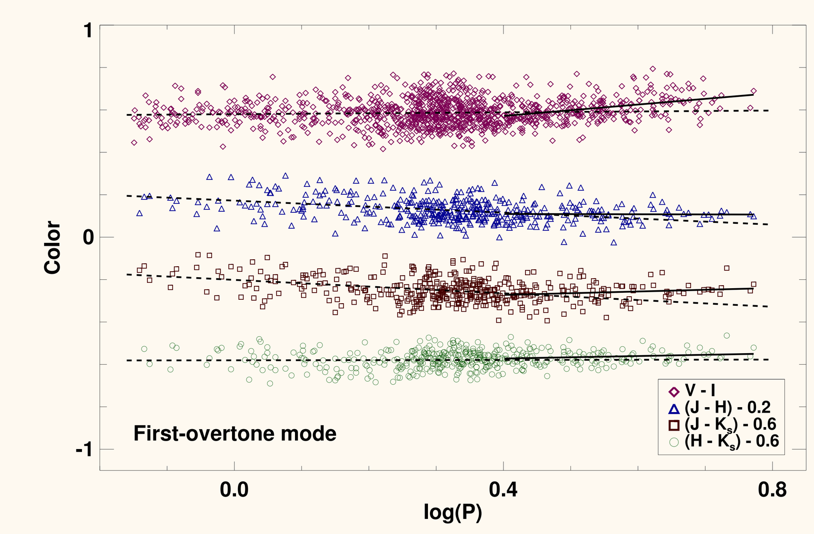

Figure 4: Optical and near-infrared P-C relations for LMC FO mode Cepheids.

Band	FU FT	FU RW	FU TM	FU DT	FO FT	FO RW	FO TM	FO DT
V	Y	N	Y	Y	Y	Y	Y	Y
I	Y	N	Y	Y	Y	Y	Y	Y
J	Y	Y	Y	Y	Y	N	Y	N
H	Y	Y	Y	Y	N	N	Y	N
K_s	Y	Y	Y	Y	N	N	Y	N

Table 2 : For each band P-L relations, Y/N represents the break/no break under each test statistics.

Conclusions

- For fundamental-mode Cepheids, we find that the optical P-L, P-W and P-C relations are non-linear at 10 days.
- The near-infrared P-L and the SH0ES Wesenheit relations are non-linear around 18 days. This break is attributed to a distinct variation in mean Fourier amplitude parameters near 20 days for longer wavelengths as compared to optical bands [6].
- For first-overtone mode Cepheids, a significant change in the slope of P-L, P-W and P-C relations is found around 2.5 days only at optical wavelengths.
- We determine a global slope of -3.212±0.013 for the SH0ES Wesenheit relation by combining our LMC data with observations of Cepheids in Supernovae host galaxies [7].
- We find this slope to be consistent with the corresponding LMC relation at short periods, and significantly different to the long-period value.
- We do not find any significant difference in the slope derived from the global fit using linear or non-linear P-L relations, due to the large observational scatter in the distant Cepheid data.

References

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