A non-parametric solution to the core-cusp modelling problem using evolutionary algorithms: application to Fornax dSph

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1. BACKGROUND

Dwarf galaxies, some of the most dark matter dominated structures of our universe, are excellent test-beds for dark matter theories. Unfortunately, mass modelling of these systems suffers from the well documented mass-anisotropy degeneracy. This makes discriminating between core and cuspy profiles a very difficult problem. For the case of spherically symmetric systems, we describe a method for non-parametric modelling of the radial and tangential velocity moments. In this way the mass-anisotropy degeneracy is reduced into mass model inference, irrespective of kinematics. Building on previous work, we use computer aided geometric design (CAGD) tools, specifically b-splines, to represent the velocity moments. We use evolutionary algorithms to perform model inference and we extend the notion of Empirical Bayes priors by using mock stellar systems to construct prior information. We test our method using synthetic data. Our algorithm constructs the best kinematic profile and discriminates between competing dark matter models. We apply our method to the Fornax dwarf spheroidal galaxy. Using a King brightness profile and testing various dark matter mass models, our model inference conclusively favours a simple mass-follows-light system. We find that the anisotropy profile of Fornax is tangential and we estimate a total mass of

$$M_{tot} = 16.13^{+0.50}_{-0.75} \times 10^7 M_\odot$$

and a mass-to-light ratio of $\Upsilon = 8.93^{+0.32}_{-0.47}(M_\odot/L_\odot)$

2. DATA

Synthetic Data: a) We constructed a "difficult" set of synthetic data points from a King (1966) brightness profile, a Burkert (1995) DM profile, and an MLT (Tiret & Combes 2007) anisotropy profile.
b) Mock Nbody systems: *Work in progress.*

Fornax: We used published heliocentric velocity values and membership characterization from Walker et al. (2009). For the brightness profile, we constructed the projected number density profile, normalized to the total luminosity of Fornax (Lokas 2009).

3. METHODS

A. Following the Jeans formalism, we expand the radial velocity dispersion in a b-spline basis: $\sigma^2_{rr}(r) = \sum_{i=1}^{N_{coeffs}} a_i B_i(r)$

Hence the line-of-sight velocity dispersion becomes a linear equation of the unknown coefficients $\sigma^2_{los} = \sum_i a_i I_i(R) + C(R)$

and the kinematic (anisotropy) profile is estimated directly from the data, without unnecessary assumptions.

B. We use evolutionary algorithms (EAs) for fitting and model selection. The fitness function is related to the AICc model selection criterion.

$$f(\theta) = \frac{1}{1 + AICc(\theta)}$$

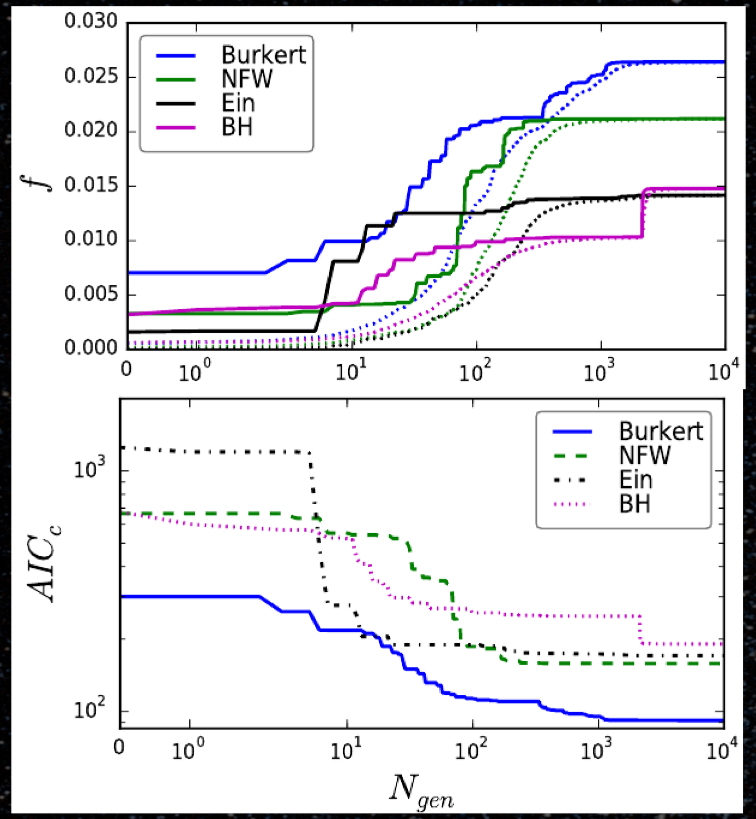
C. Once best model is recovered, we use MCMC to estimate uncertainties in model parameters.

Advantages:
1. Reduces estimation bias.
2. b-splines convolved with equations from physics results strong constraints on the b-spline geometric shape
3. Model equations are linearized, the problem is simplified.

Difficulties:
1. Optimum smoothing/regularization. *Solution:* We build prior information for the optimum smoothing from ideal theoretical models, thus we generalize the notion of empirical Bayes.
2. Optimum b-spline knots. *Solution:* We use Evolutionary Algorithms (EAs) for the estimation of the optimum knot distribution.

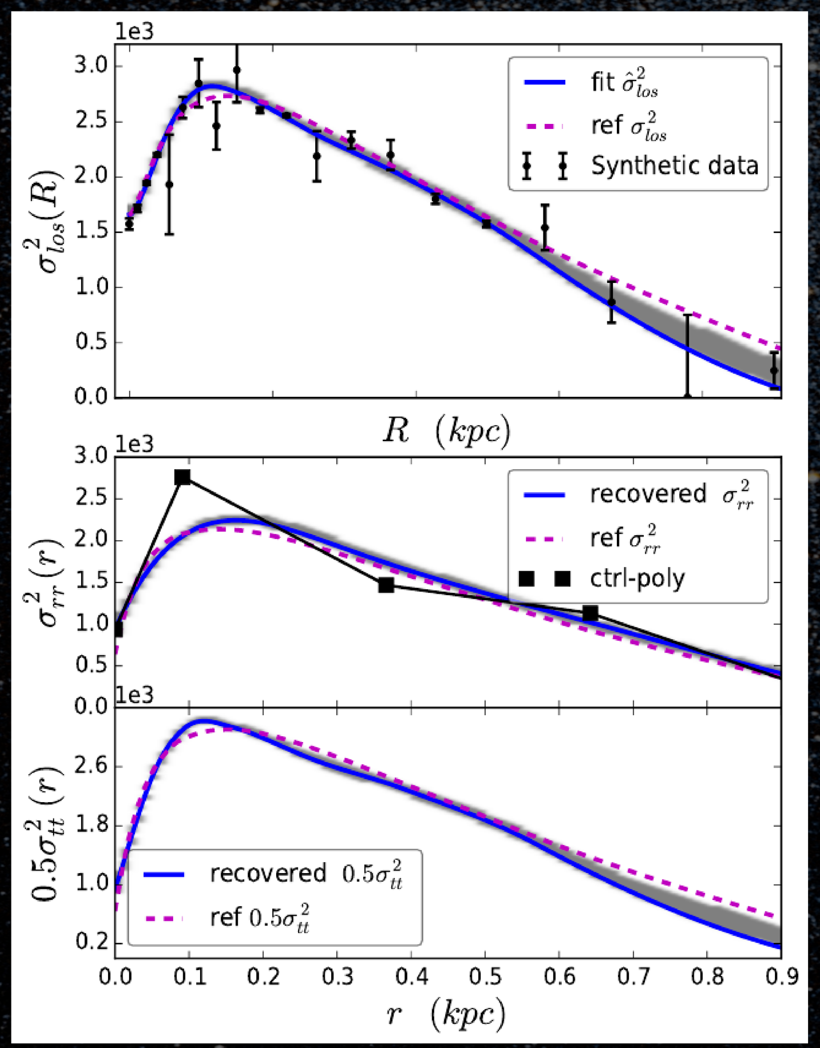
4. RESULTS

Synthetic Data: the evolutionary algorithm identifies the reference DM model from which the synthetic data were created (Burkert). It also reconstructs the radial and velocity moments with excellent accuracy.



Top panel: evolution of the maximum (solid line) and average (dotted line) fitness value for all generations. As the evolutionary algorithm approaches the best solution the average value of fitness converges to the fitness value of the best solution (individual).

Bottom panel: corrected Akaike Information Criterion (AICc) for each generation. The difference in AICc values between the best and the other candidates is > 10, thus the model selection conclusive.



MCMC highest likelihood fit of second order velocity moments. Grey region corre-sponds to 1σ unce-rtainty region in all model parameters. The dashed lines correspond to the true values of the reference profile from which the synthetic data were created. Blue line is the highest likelihood fit.

Top panel: line-of-sight velocity dispersion. Black dots are the synthetic data with their uncertainty.

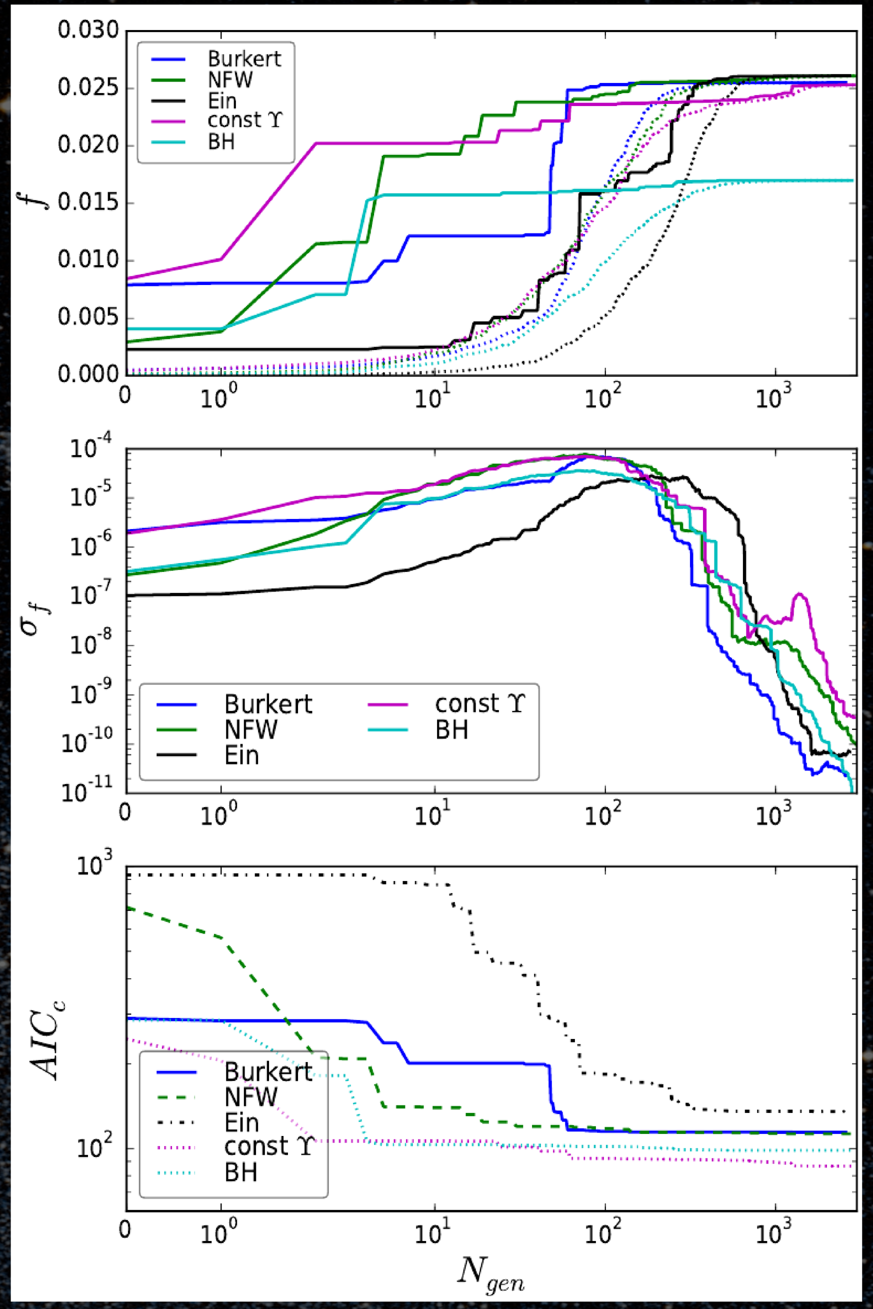
Middle panel: radial velocity dispersion, σ_{rr}^2. Black squares are the vertices of the control polygon of the b-spline representation.

Bottom panel: tangential velocity dispersion.

Fornax: We model Fornax by assuming a King brightness profile, a constant mass-to-light ratio, Υ, and in addition the following separate DM components: NFW, Burkert, Einasto and a black hole in the center of the galaxy. In all of the models there is a constant mass-to-light ratio, Υ, but only in one we do not use a separate DM mass profile, thus in total we have 5 different mass models. We refer to the model with no separate DM component as const Υ.

Based on the available brightness and kinematic data sets, our algorithm predicts conclusively (ΔAICc > 20), that there is no need for a separate dark matter component in the dwarf galaxy. That is, from a variety of cored and cuspy DM profiles and modelling independent of the MAD our best candidate is a simple mass-follows-light model. This does not imply that there is no dark matter in the dSph, however it does suggest that in a well mixed system, like Fornax, there is no need for a separate DM component that does not follows the stellar profile. The second best candidate, which is also strongly disfavoured (ΔAICc ~ 12), is a simple mass-follows-light model with a black hole in the center. We emphasize that these results should be verified with the use of proper Bayesian inference and the use of different tracer profiles; (work in progress).

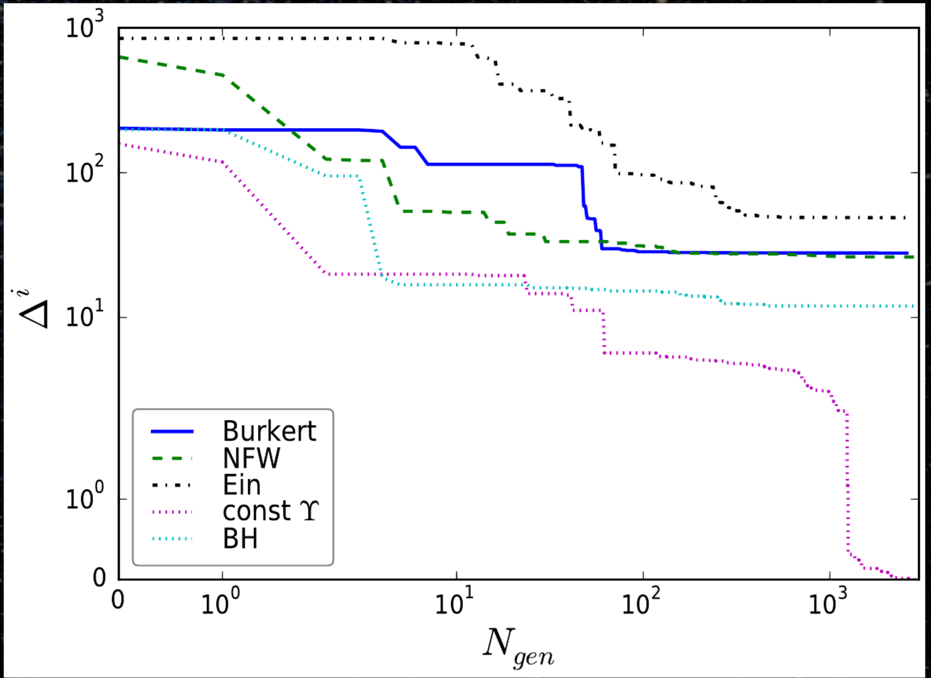
FORNAX (cont...)



Top panel: Evolution of the fitness value (max: solid line, average: dotted) for all geenerations.

Middle panel: evolution of the standard deviation (std) of the fitness values of the population for each generation. As the algorithm converges, the std goes to lower values, indicating convergence.

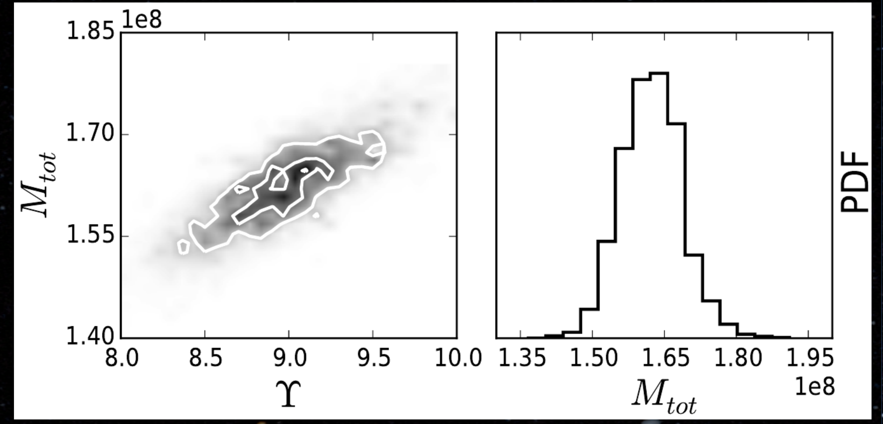
Bottom panel: Evolution of the AICc model selection criterion for each generation.



Evolution of the differences ΔAICc between the best and the other competing models. A difference ΔAICc > 20 indicates conclusive model selection. The horizontal axis is in log scale. The simple constΥ model (no separate DM component) is conclusively the most favored candidate

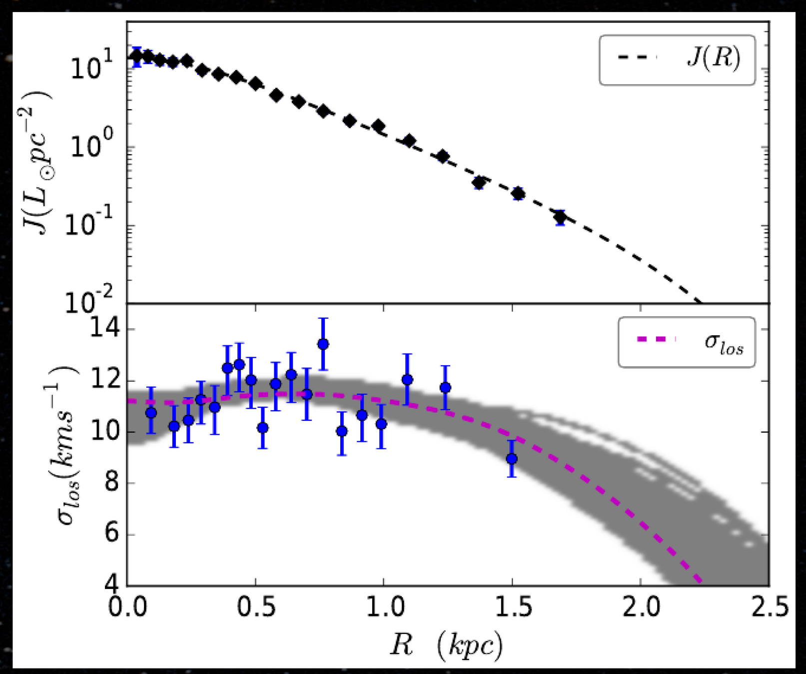
Model	*AICc*	Δ
King, Υ	86.50	0.0
King, Υ, BH	98.49	11.99
King, Υ, NFW	112.66	26.15
King, Υ, Burkert	114.36	27.85
King, Υ, Einasto	135.38	48.88

Numerical values of the differences ΔAICc between the various competing mass models.



Marginalized distributions of total Mass and mass-to-lighte ratio, Υ, for the best fitted model for the Fornax dSph.

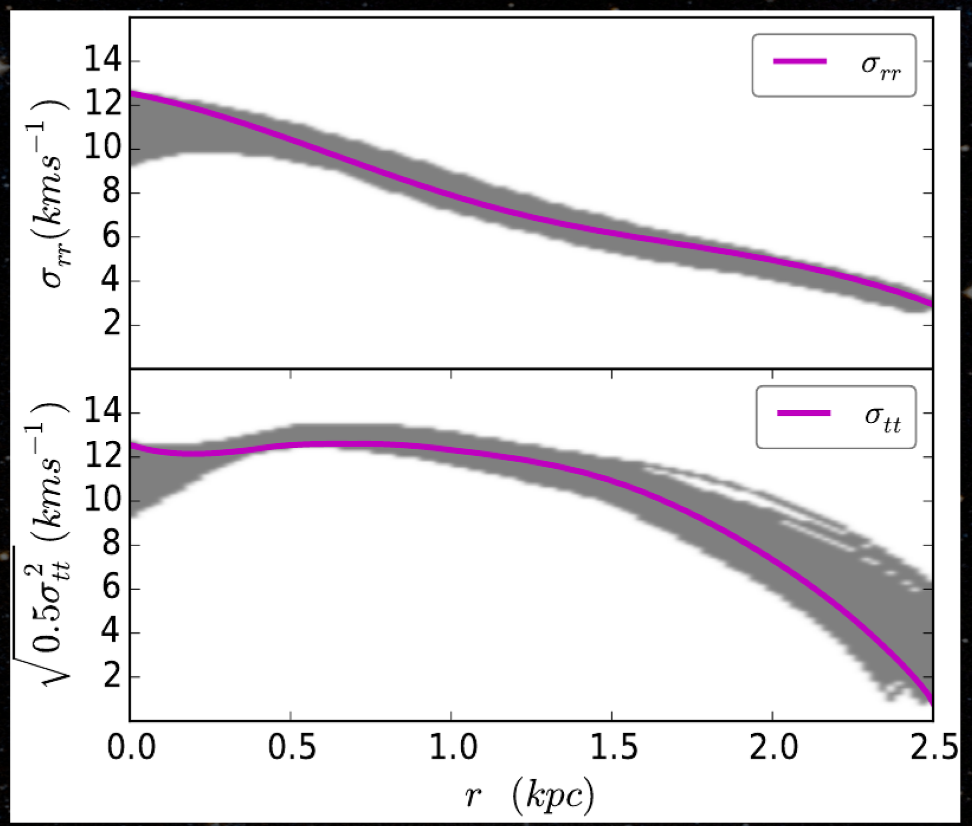
FORNAX (cont...)



Fornax maximum likelihood fit.

Top panel: Brightness fit.

Bottom panel: line-of-sight velocity dispersion. Blue dots are binned σlos values. Purple dashed line is the highst likelihood fit. Grey region corresponds to 1σ uncertainy in all model parameters.



Reconstructed radial (top) and tangential (bottom) velocity moments. The solid purple line is the maximum likelihood fit. Grey region is the 1σ uncertainty region in all model parameters. These velocity moments produce a tangential anisotropy profile, β(r) <0.

5. CONCLUSIONS

J.E.AN.S solver: Building on previous work (Diakogiannis et al. 2014a,b) we further develop our method by introducing an evolutionary algorithm that evaluates the optimum knot distribution for the b-spline representation of the radial velocity dispersion, σ_{rr}^2. That is the best kinematic profile independent of any anisotropy, β(r), assumptions.

Our algorithm uses a fitness function that includes the corrected Akaike information criterion for second order bias, AICc. This model inference criterion has the advantage that is fast to evaluate and it can be applied to small number of available data points, thus it allows fast model selection between competing DM candidates. It has the disadvantage that it does not include information from the whole range of Markov chains in the MCMC. As a result, it is not as robust as Bayesian inference methods, e.g. model inference using Bayesian evidence (Feroz et al. 2009).

We extend the notion of empirical Bayes (Casella 1985), by presenting a new algorithm for the evaluation of prior information from ideal theoretical models for the optimum smoothing for the line-of-sight velocity dispersion, σ_{los}^2 fit. This new version results tighter constrains on the smoothing hyperprior parameters and is computationally much faster.

FORNAX modelling: By using a King brghtness profile and a variety of cored and cuspy DM profiles and modelling independent of the MAD our best candidate is a simple mass-follows-light model. This does not imply that there is no dark matter in the dSph, however it does suggest that in a well mixed system, like Fornax, there is no need for a separate DM component that does not follows the stellar profile.

We estimate an anisotropy profile that is tangentially biased. The tangential anisotropy seems to favour the scenario that Fornax is the remnant of a recent merger.