Statistical Challenges in fitting stellar orbits around the super-massive black hole at the Galactic center.

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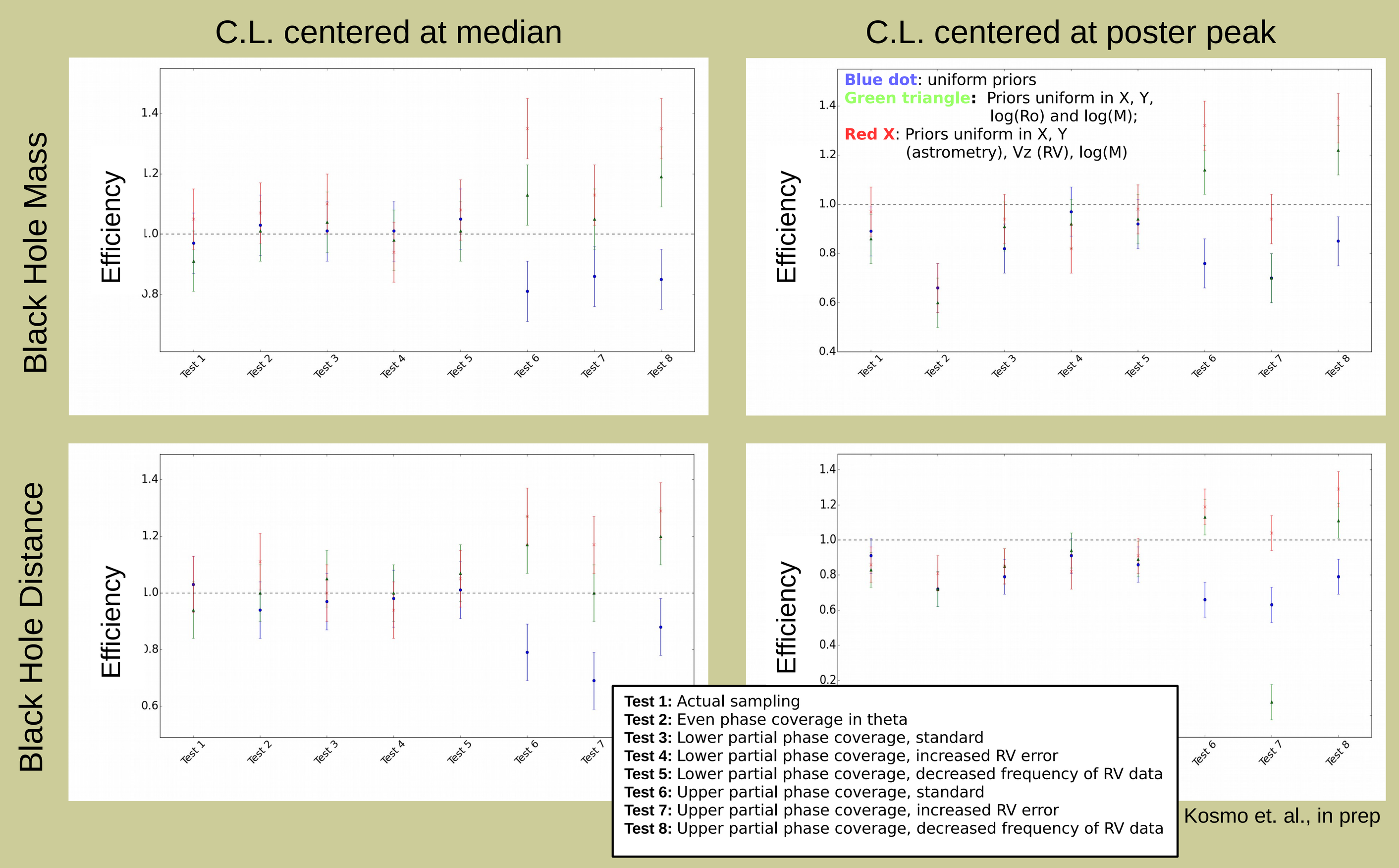
Over two decades of astronomic and radial velocity data of stars at the Galactic center, has the potential to provide unprecedented tests of General Relativity and insight into the astrophysics of the super-massive black hole. Fundamental to this is understanding the underlying statistical issues of fitting stellar orbits. Reference frame effects and unintended prior effects can obscure actual physical effects from General Relativity and underlying extended mass distribution. At the heart of this is dealing with large parameter spaces inherent to multi-star fitting and ensuring acceptable coverage properties of the resulting confidence intervals in the Bayesian framework. This poster will detail some of the UCLA's group analysis and work in addressing these statistical issues.

Statistical Coverage Analysis of Stellar Orbital Fitting at the Galactic Center

It is not unreasonable to expect data sets with incomplete orbital phase coverage to not produce accurate confidence intervals. Shown below are the Statistical efficiencies of various Bayesian confidence intervals. The efficiencies are calculated from 100 mock data sets assuming an reasonable set of 'true' parameters. Plotted is the ratio of the fraction of confidence intervals that cover the assumed true value as compared to the defined confidence level. An efficiency of 1 denotes exact coverage whereas an value >1 or <1 overcovers or undercovers respectively. We compare confidence intervals derived using three priors: uniform priors and two different priors uniform in the observable (not model) parameter space.



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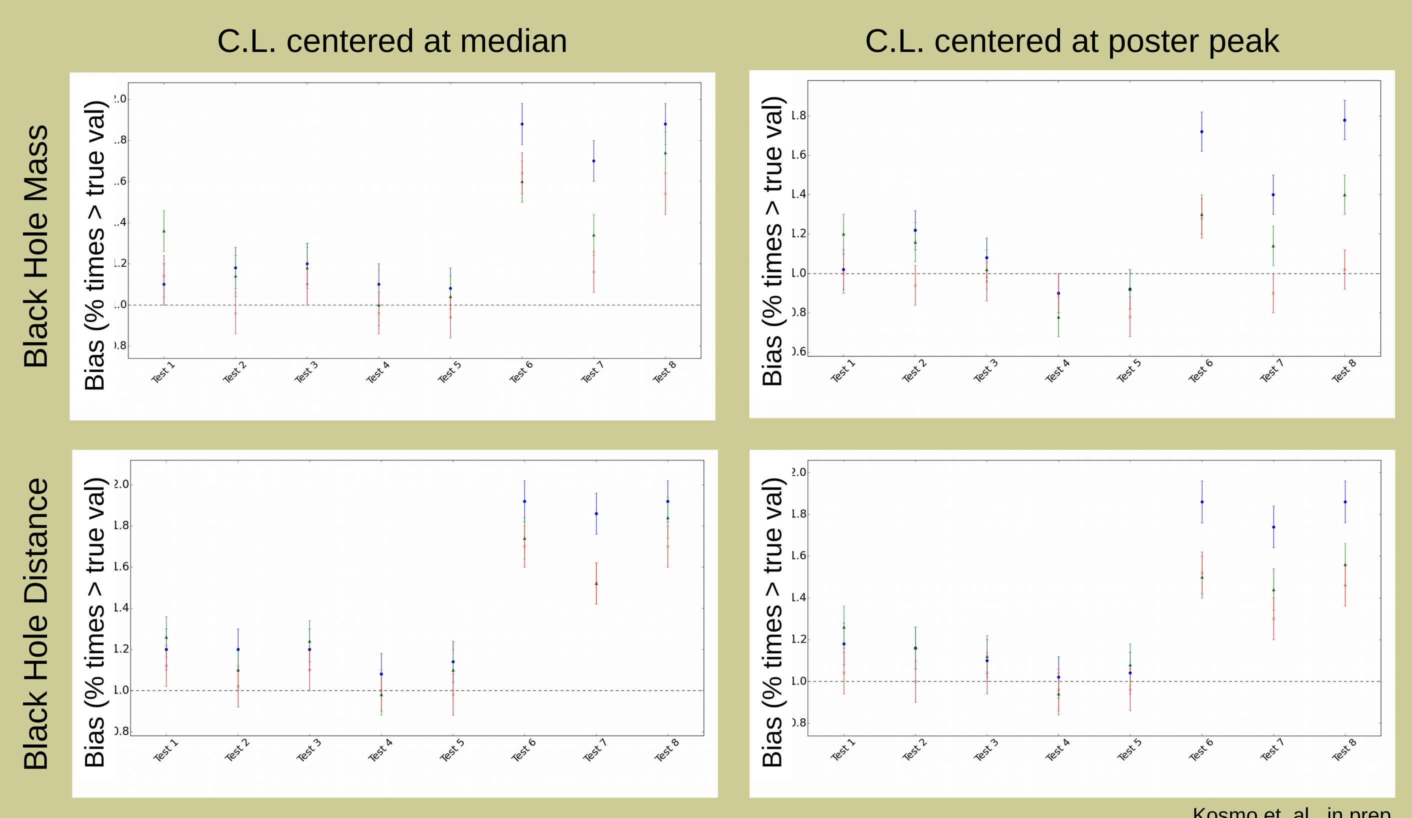
Although the confidence levels of data sets with low phase seem to be helped using our observable-based priors, it is instructive to analyze the bias on the quoted central value. Plot below are the bias defined to be the number of times the 'true' is above the median (or posterior peak). Compared are the assumed true values and the median (or posterior peak) that is inferred from a randomly drawn mock data sets.



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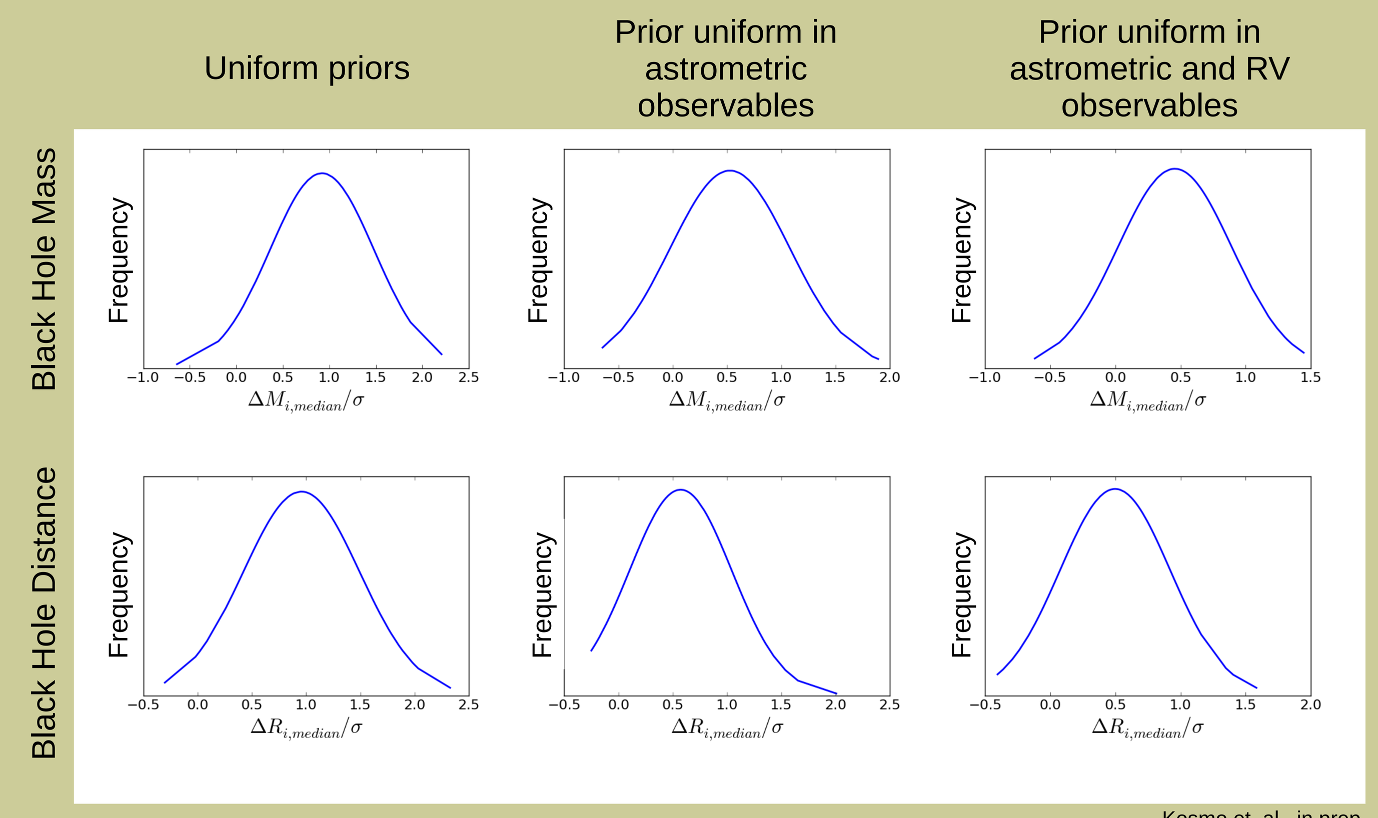
Inferred median or posterior peak values seem to be consistently biased when compared to the true value – an effect not alleviated by our observable-based priors. Although, as shown right, these priors help reduce the magnitude of this effect. To the right is the frequency of the amount the median is biased relative to the assumed 'true' value assuming various priors.



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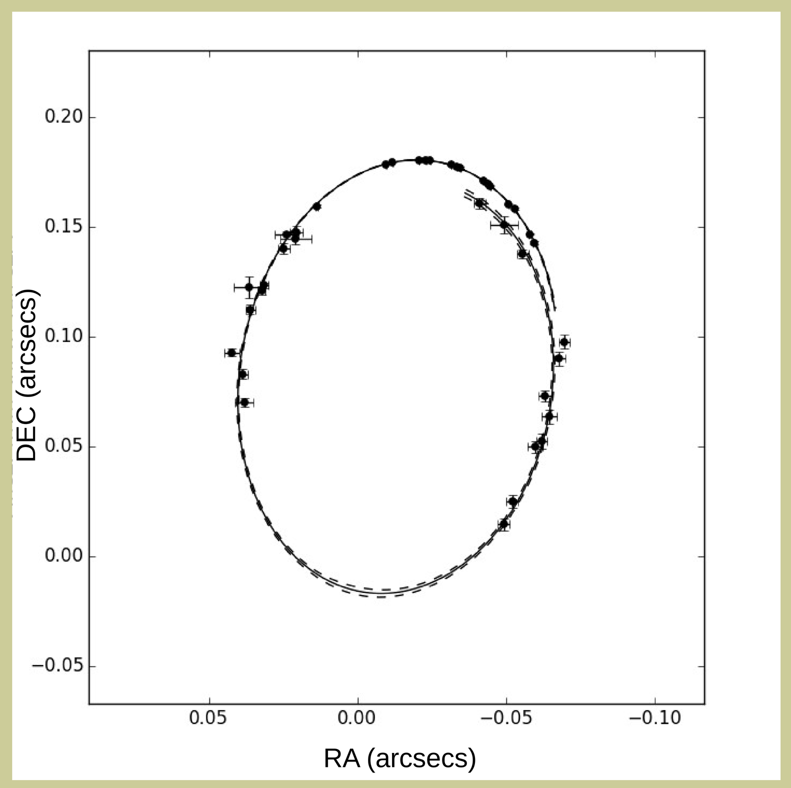
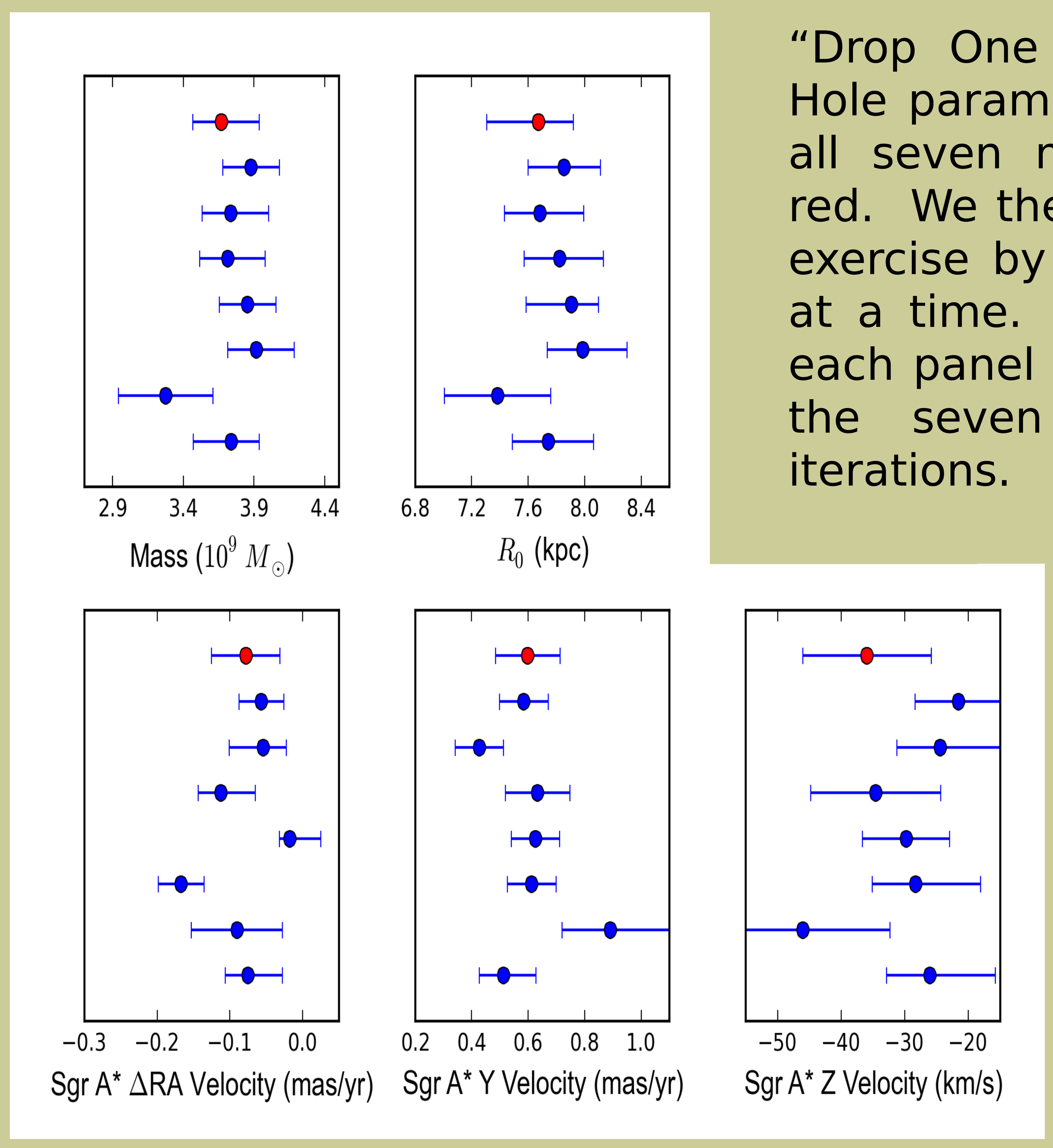
Estimating systemic uncertainties in the reference frame

The reference frame is built by comparing the infrared and radio positions of seven SiO masers found in the GC and minimizing the difference between the two sets of measurements. This approach should put Sgr A* at rest. When fitting stellar orbits, we allow Sgr A*'s velocity as a free parameter. Any non-zero velocity indicates systematic errors in the reference frame. This often leads measured orbits not closing which can be confused as actual physics!





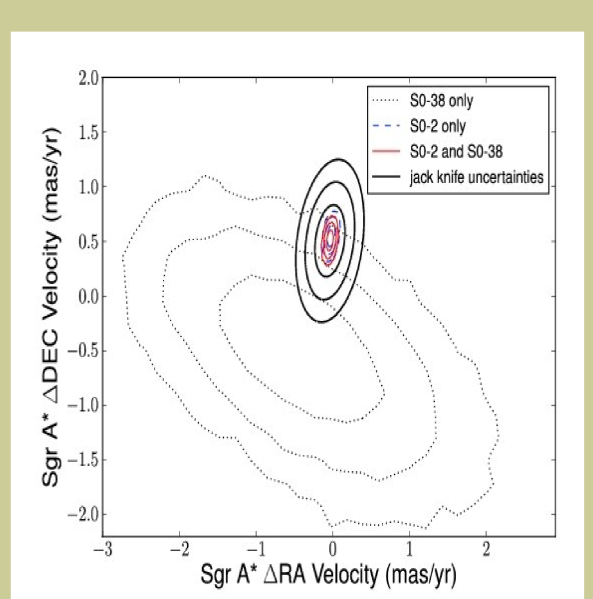
"Drop One Maser" Test: Black Hole parameters estimated using all seven masers are shown in red. We then carry out the same exercise by excluding one maser at a time. Seven blue circles in each panel show the results from the seven 'drop one maser' iterations.

Given an hypothetical dataset of infinite masers such that it produces the "true" reference frame, our 7 maser dataset represents a mere subset that produces a statically biases reference frame. Using our "Drop One Maser" result we can estimate this bias through jack knifing.

Jack Knife Uncertainties

$$x_{\text{bias}} \approx (n-1)(\bar{x}_{n-1} - x_n)$$

$$\sigma^2(x_n) \leq \frac{n-1}{n} \sum_{i=1}^{n} \left(x_{n-1,i} - \bar{x}_{n-1}\right)^2$$



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Given Posteriors showing the dynamical velocity of Sgr A inferred from stellar orbits. Color lines represent the posteriors inferred from orbits of S0-2 and S0-38. The solid represents the uncertainties from the jack knife.

Much of our discrepancies are alleviated with the inclusion of our jack knife results indicating a large amount of our systematics come from unaccounted for reference frame statistical errors.

Black Hole Properties:	S0-2	S0-38	S0-2+S0-38
Distance (kpc)	$8.02 \pm 0.36 \pm 0.04$	$[6.5, 9.5]^b$	$7.86 \pm 0.14 \pm 0.04$
Mass ($10^6 M_\odot$)	$4.12 \pm 0.31 \pm 0.04$	$[2.5, 5.5]^b$	$4.02 \pm 0.16 \pm 0.04$
X Position of Sgr A* (mas)	$2.52 \pm 0.56 \pm 1.90$	$-5.25 \pm 9.41 \pm 1.90$	$2.74 \pm 0.50 \pm 1.90$
Y Position of Sgr A* (mas)	$-4.37 \pm 1.34 \pm 1.23$	$-6.85 \pm 5.00 \pm 1.23$	$-5.06 \pm 0.60 \pm 1.23$
X Velocity (mas/yr)	$-0.02 \pm 0.03 \pm 0.13$	$-0.40 \pm 0.70 \pm 0.13$	$-0.04 \pm 0.03 \pm 0.13$
Y Velocity (mas/yr)	$0.55 \pm 0.07 \pm 0.22$	$-0.48 \pm 0.43 \pm 0.22$	$0.51 \pm 0.06 \pm 0.22$
Z Velocity (km/sec)	$-15 \pm 10 \pm 4$	$[-80, 40]^b$	$-15.48 \pm 8.36 \pm 4.28$

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Next generation reference frame construction techniques will directly incorporate these errors through a global likelihood analysis..