Hierarchical Probabilistic Inference of Cosmic Shear

Statistical Challenges in Modern Astronomy VI

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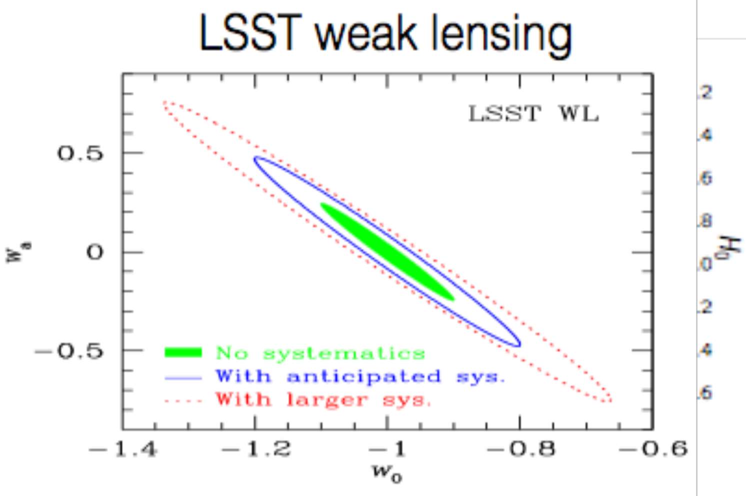
Cosmic Shear

- The lensing by large scale structure
- Looking for very small signal under very large amount of noise
- We **don't know "unsheared" shapes**, but can (roughly) assume they are isotropically distributed
- Cosmic shear distorts statistical isotropy; galaxy ellipticities become correlated
- Exquisite probe of DE, if systematics can be controlled
- LSST: will measure few billion galaxy ellipticities. Excellent sensitivity to both DE and systematics!

Cosmic shear signal is comparable to ellipticity of the Earth, ~0.3%

- D. Wittman

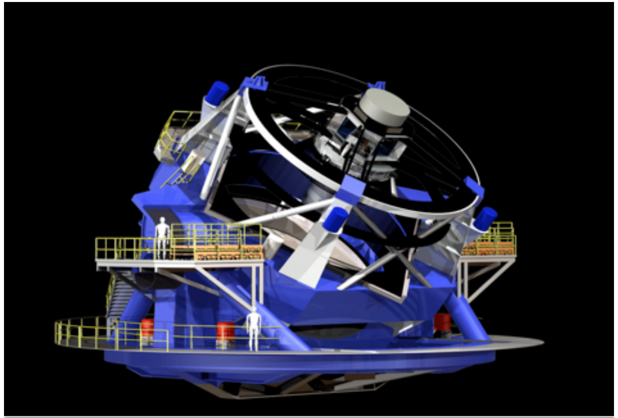


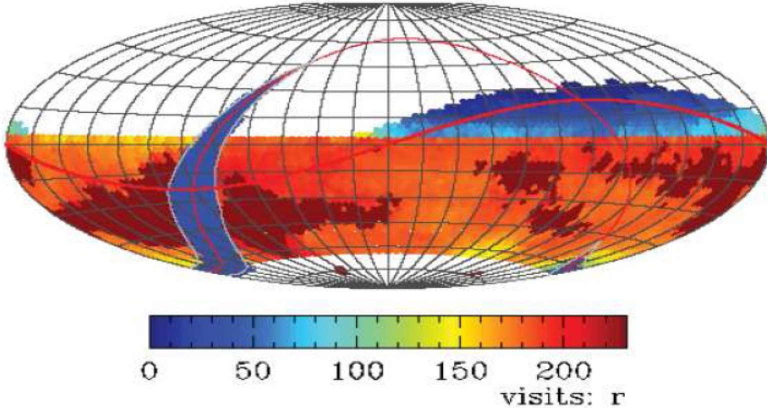
The Large Synoptic Survey Telescope (LSST) is a driver for many statistics and computing innovations in the next decade

Construction start: 2014 First light: 2020 Survey end: 2030

8.4m telescope 18,000+ deg^2 10mas astrometry r<24.5 (<27.5@10yr)

6 broad optical bands (*ugrizy*) 0.5-1% photometry

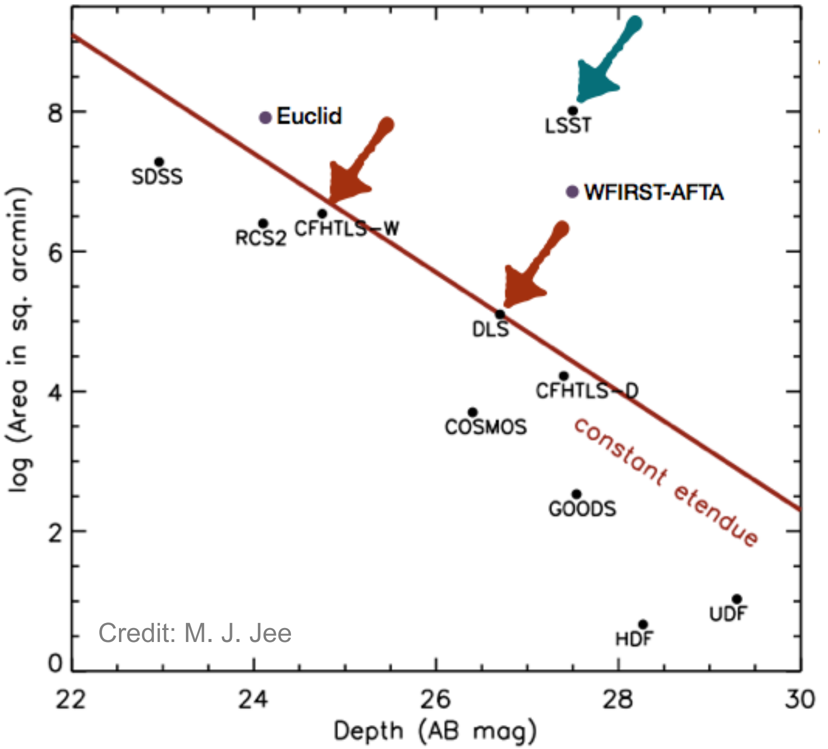




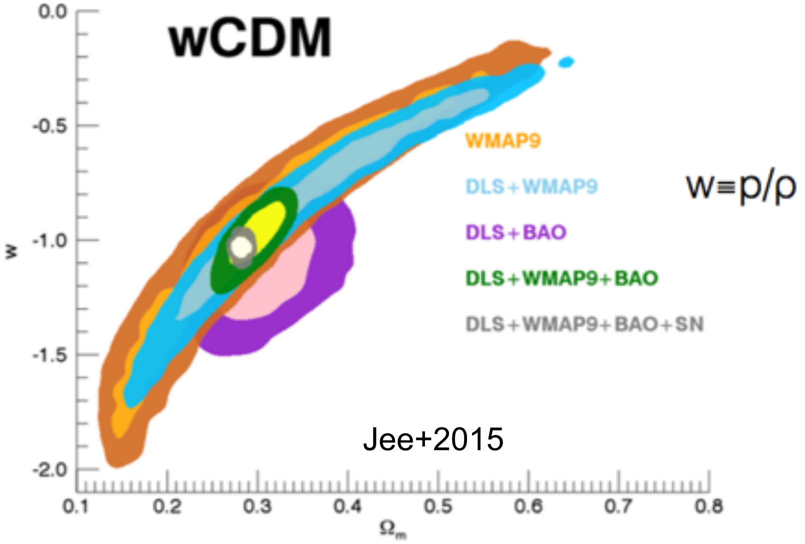
3.2Gpix camera 2x15sec exp/2sec read 15TB/night 20 B objects

Imaging the visible sky, once every 3 days, for 10 years (825 revisits)

Cosmic shear today: Stage II dark energy



DLS shear tomography



We won't just have more data with 'Stage IV' surveys.
- We're in an era with qualitatively new computing capabilities

<u>Dijkstra's Law</u>

A quantitative difference is also a qualitative difference if the quantitative difference is greater than an order of magnitude.

Microprocessor Transistor Counts 1971-2011 & Moore's Law

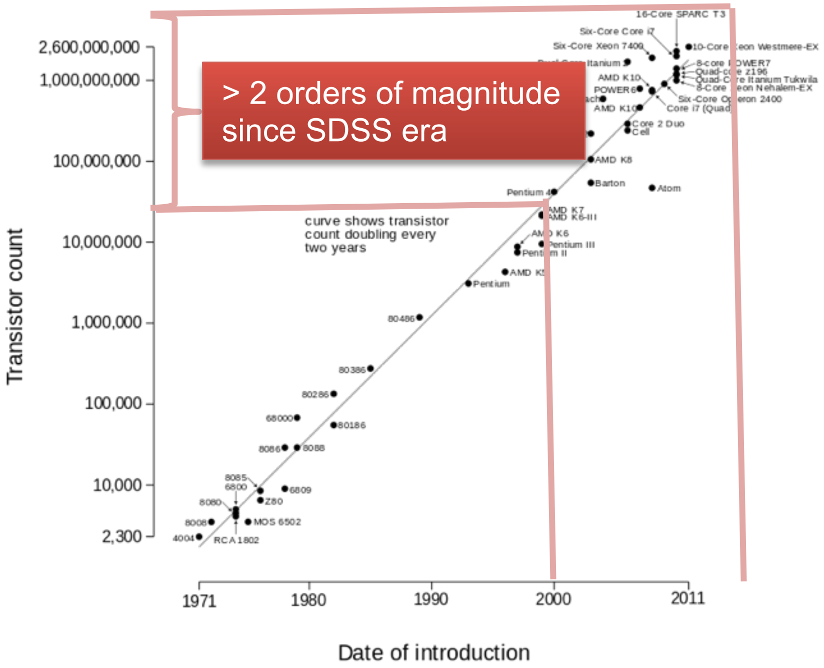


Figure credit: Wikipedia

Qualitative changes in computing enable new scientific methods

"…predictive simulation has brought together theory and experiment in such a compelling way that it's fundamentally extended the scientific method for the first time since Galileo Galilei invented the telescope in 1609…"

- Mark Seager, CTO for the HPC Ecosystem at Intel

(interview in Inside HPC on June 6, 2016)

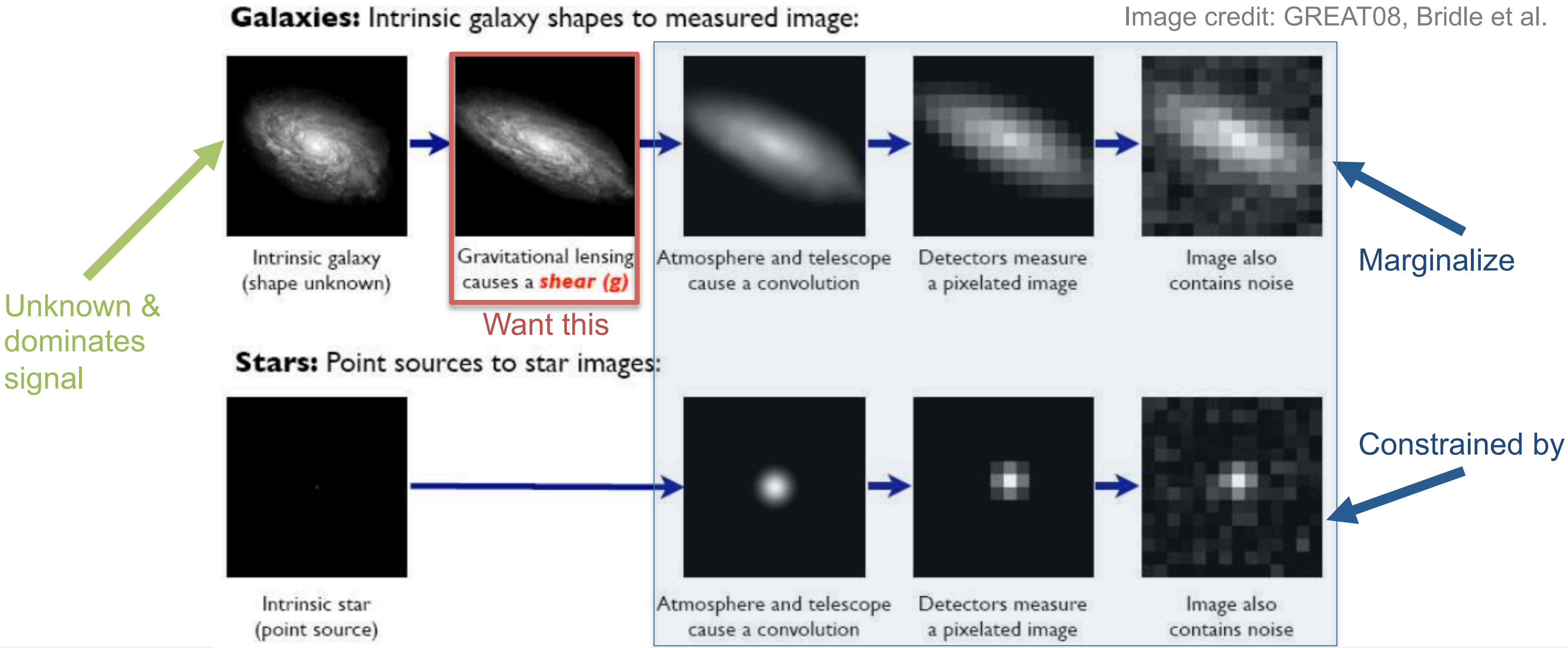
Data + Compute convergence in cosmology – DOE ASCR initiative, April 2016

- We're facing **systematics-limited** measurements
  - End-to-end simulations of the experiment are the best approach to improve accuracy & precision
  - Ties data and simulation more intricately than in past cosmology pipelines
- Image and catalog **summary statistics** are no longer good enough to meet next generation science requirements
  - Probabilistic hierarchical models and related machine-learning approaches show promise but are much more computationally intensive
  - Potential changes to the traditional 'facility' / 'user' separate analysis stages

Removing the line between 'analysis' and 'simulation'.

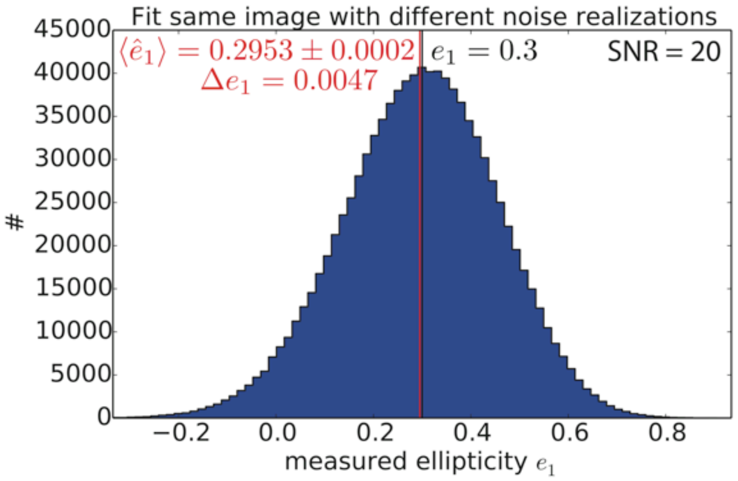
Weak lensing of galaxies: the forward model

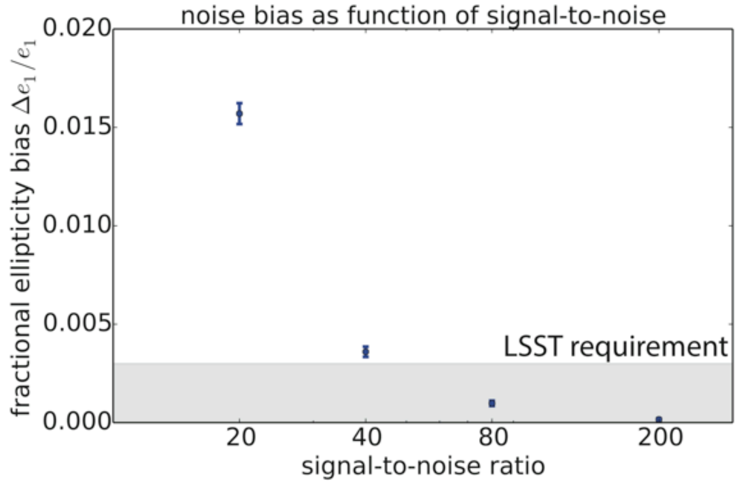
Image credit: GREAT08, Bridle et al.



Shape to Shear: Noise Bias

- Ellipticity: $e = \frac{a-b}{a+b}\exp(2i\theta)$
- Ensemble average ellipticity is an unbiased estimator of shear.
- However, maximum likelihood ellipticity in a model fit is **not** unbiased.
- Ellipticity is a non-linear function of pixel values.





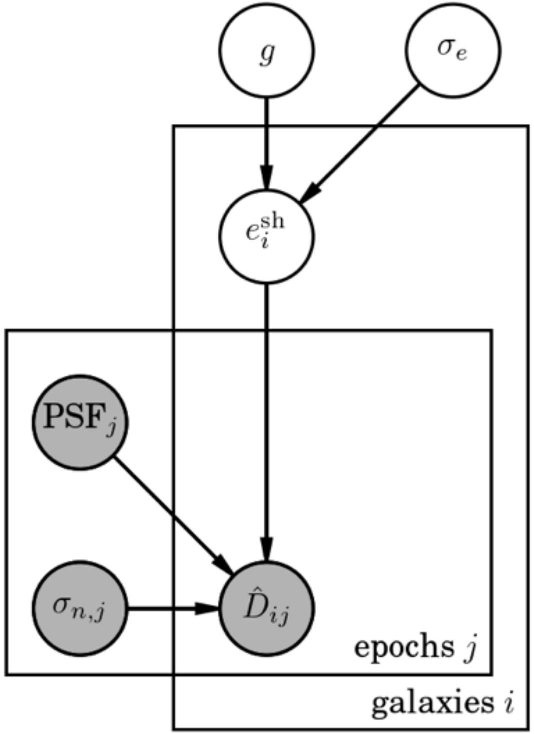
Mitigating Noise Bias – at least 2 strategies

1. Calibrate using simulations. (im3shape, sfit)
   - But corrections are up to 50x larger than expected sensitivity!

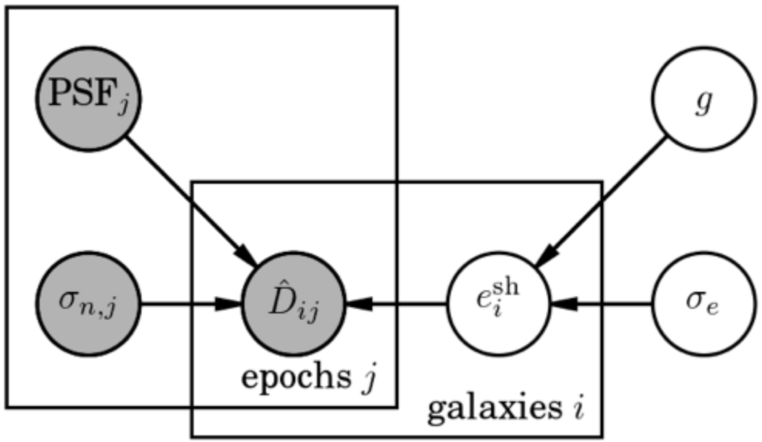
2. Propagate entire ellipticity distribution function P(ellip | data).
   - Use Bayes' theorem: P(ellip | data) $\propto$ P(data | ellip) P(ellip)
   - Measure P(ellip) in deep fields. (lensfit, ngmix, FDNT).
   - Infer simultaneously with shear in a hierarchical model. (MBI).

A hierarchical model for the galaxy distribution

- σ_e = intrinsic ellipticity dispersion
- e^{int} = galaxy intrinsic ellipticity
- g = shear
- e^{sh} = galaxy sheared ellipticity
- PSF = point spread function
- D = model image
- σ_n = pixel noise
- D = data: observed image



Our graphical model tells us how to factor the joint likelihood



- Use a probabilistic graphical model to encode the factorization of the joint probability distribution of variables in the model.
- We don't care about e^{sh} for cosmology, so integrate it out.

$$\Pr\left(g, \sigma_e | \left\{\mathrm{PSF}\right\}_j, \left\{\sigma_{n,j}, \left\{D_{ij}\right\}\right\}\right)$$

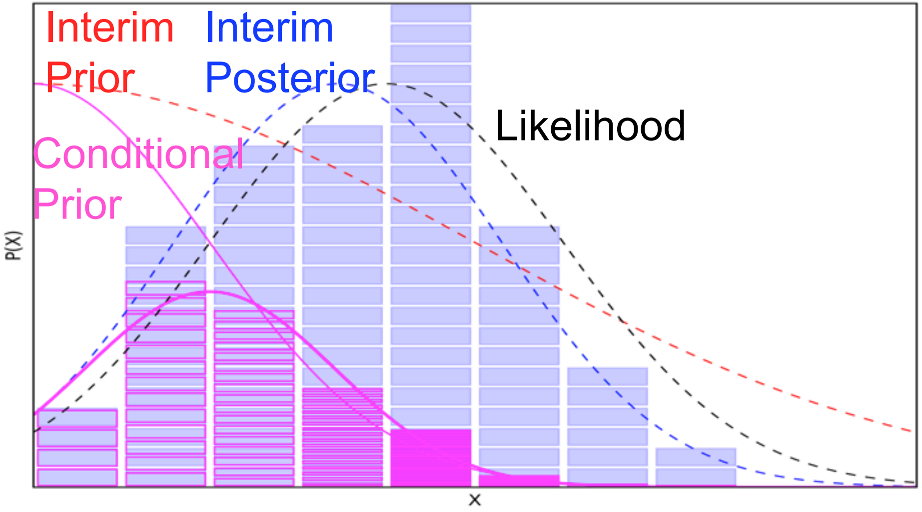
$$\propto \int d^{n_{\mathrm{gal}}}\left\{e_i^{\mathrm{sh}}\right\} \left[\prod_{ij} \Pr\left(D_{ij}|\mathrm{PSF}_j, \sigma_{n,j}, e_i^{\mathrm{sh}}\right)\right] \left[\prod_i \Pr\left(e_i^{\mathrm{sh}}|g, \sigma_e\right) \Pr(g) \Pr(\sigma_e)\right]$$

Huge complicated integral to compute for every posterior evaluation.

Importance Sampling: the pseudo-marginal likelihood

- Don't go back to pixels for every time we sample a new g or σ_e.
- For each galaxy, draw image model parameter samples under a fixed "interim" prior. This is embarrassingly parallelizable.
- Use reweighted samples to approximate the integral via Monte Carlo.

How many interim samples are needed?



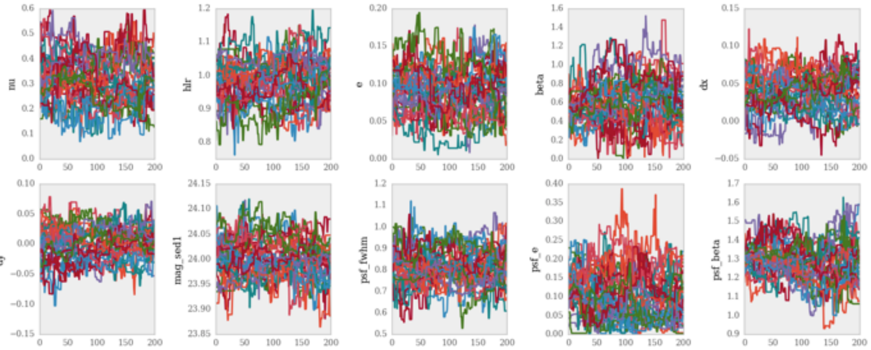
$$\text{Draw } K \text{ samples } e_{ik}^{\rm sh} \sim \mathrm{P}\left(e_i^{\rm sh} \middle| \hat{D}_i, I_0\right) \propto \mathrm{P}\left(\hat{D}_i \middle| e_i^{\rm sh}\right) \mathrm{P}\left(e_i^{\rm sh} \middle| I_0\right)$$

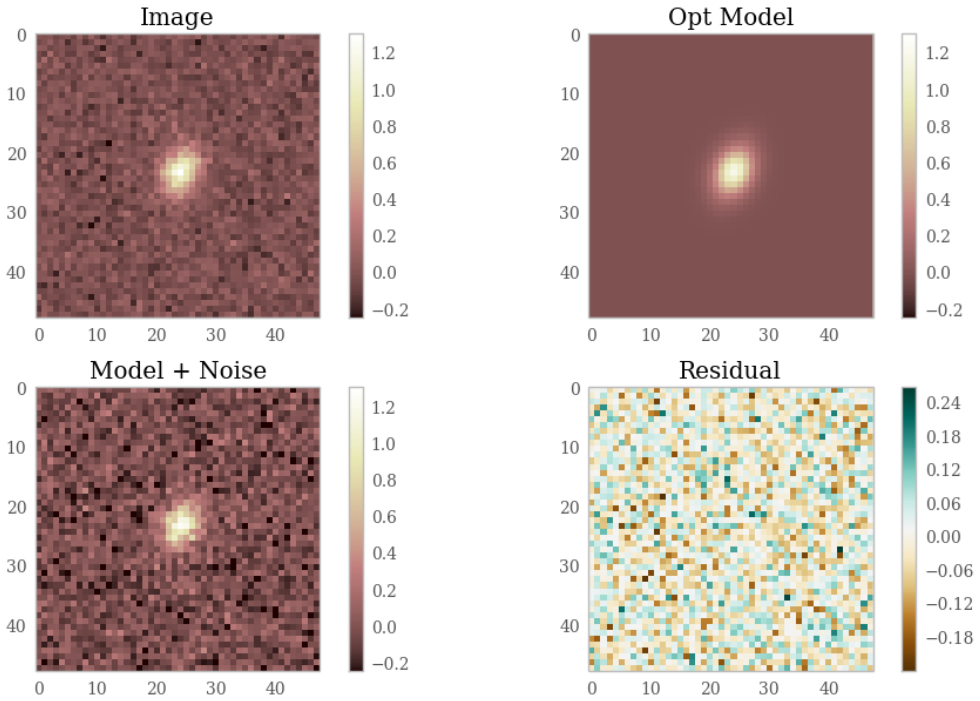
1

Source characterization via probabilistic image modeling

Infer image model parameters via MCMC under an interim prior distribution for the galaxy and PSF parameters.

MBI GREAT3 analysis with:
The Tractor (Lang & Hogg)

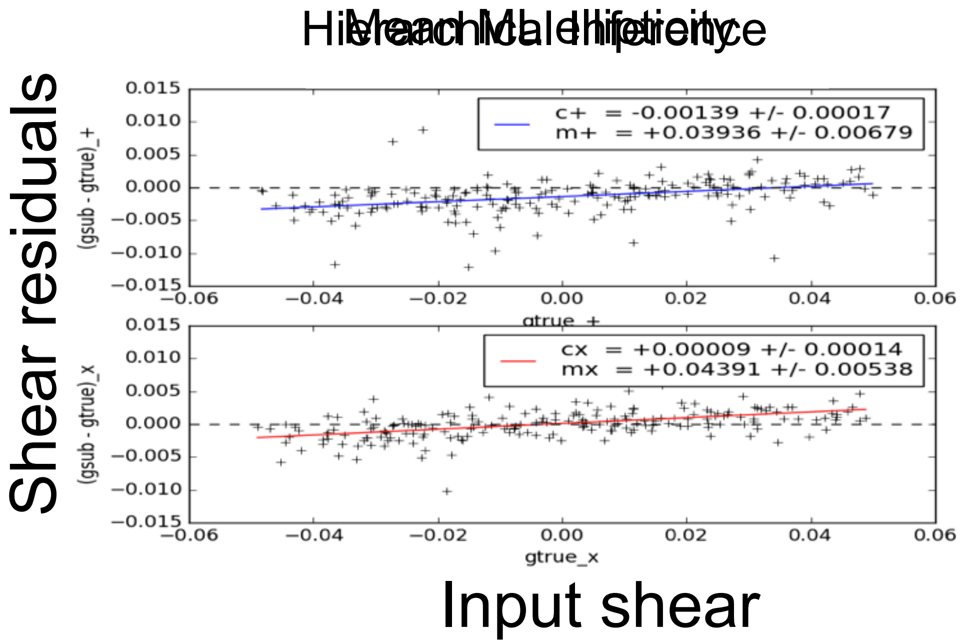




GalSim models inside an MCMC chain – **Can it be made fast enough?**

GREAT3 results

- Tested hierarchical approach using simulations from the third GRavitational lEnsing Accuracy Test (GREAT3).
- Hierarchical inference performs significantly better than ensemble average maximum likelihood ellipticity.



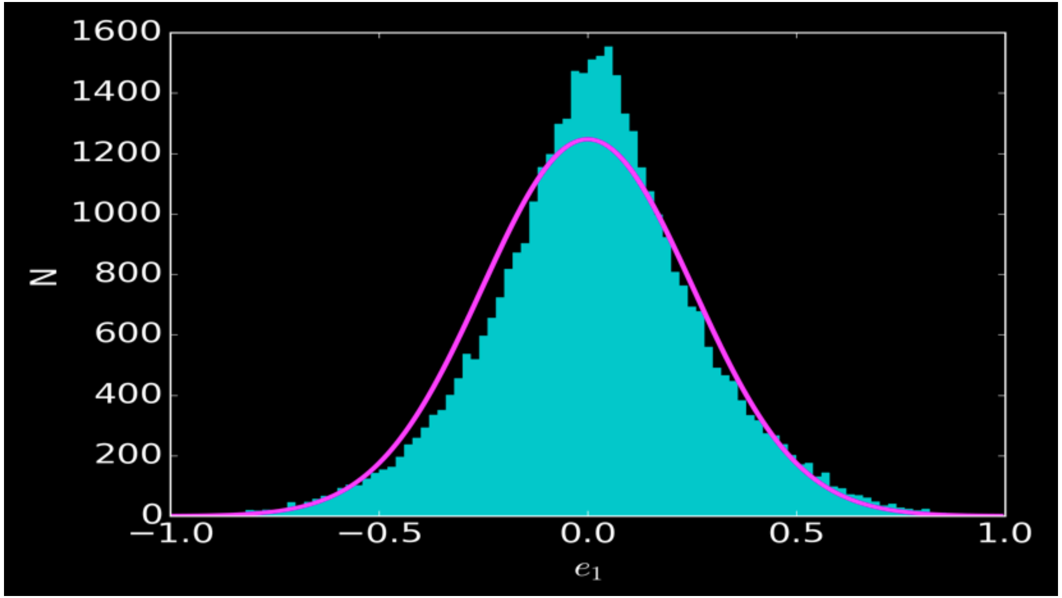
<ML> : 13% shear calibration errors

H.I. : 4% shear calibration errors

Pr(e^{int}) is not Gaussian!

- Would rather not assert a particular parametric form for P(e^{int}).
- Use a "non-parametric" distribution: a Dirichlet Process Mixture Model

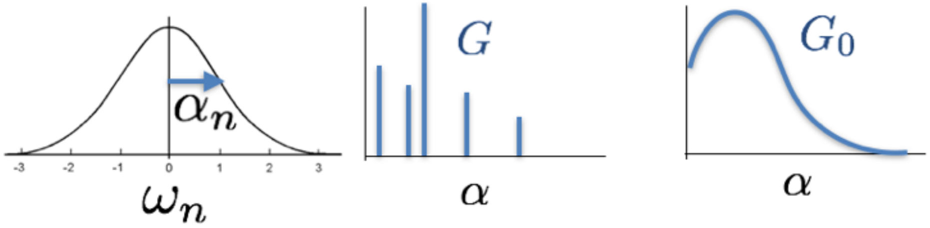
Ellipticities from COSMOS



Hierarchical inference of intrinsic galaxy properties

Specify a Dirichlet Process (DP) for the distribution of intrinsic galaxy property hyper-parameters

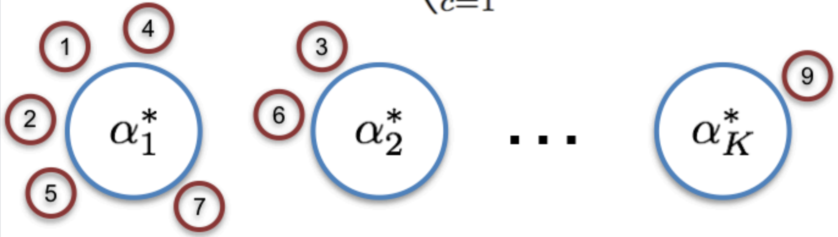
$$\omega_n \sim \mathcal{N}(0, \alpha_n), \quad \alpha_n \sim G(\alpha_n|\mathcal{A}), \quad G \sim \mathrm{DP}(\mathcal{A}, G_0)$$



The DP is a ‘non-parametric’ distribution with discrete support

The DP distribution allows clustering of data points (e.g., galaxies) to infer *latent structure* in the data.

$$\alpha_n|\alpha_1, \ldots, \alpha_{n-1} \sim \frac{1}{n-1+\mathcal{A}} \left(\sum_{c=1}^{K} N_c \delta_{\mathrm{D}}\left(\alpha_c^*\right) + \mathcal{A} G_0\left(\cdot\right) \right)$$



Gibbs updates in the Dirichlet Process model

Latent class assignments are updated with different conditional distributions depending on whether any other observations are assigned to the current class.

$$\Pr(c_n = c_\ell | c_{-n}, \omega_n, \alpha, \mathcal{X}) = b\, N_{-n,c}\, \Pr(\mathbf{d}_n | \alpha_{c_\ell}, \mathcal{X}), \qquad \forall \ell \neq n$$

$$\Pr(c_n \neq c_\ell \forall \ell \neq n | c_{-n}, \omega_n, \mathcal{X}) = b\, \kappa \int \Pr(\mathbf{d}_n | \alpha, \mathcal{X})\, G_0(\alpha)\, d\alpha,$$

The DP mixture parameters are simply updated with the posterior given all observations currently associated with the given latent class.

$$\alpha_{c_n} \sim G_0\left(\alpha_{c_n}\right) \prod_{\ell=1}^{N_{c_n}} \Pr(\mathbf{d}_\ell | \alpha_{c_n}, \mathcal{X}))$$

Neal (2000)

Highlighted integral is expensive to compute in general.

$$\Pr(c_n \neq c_\ell \forall \ell \neq n | c_{-n}, \omega_n, \mathcal{X}) = b\, \kappa \int \Pr(\mathbf{d}_n | \alpha, \mathcal{X})\, G_0(\alpha)\, d\alpha$$

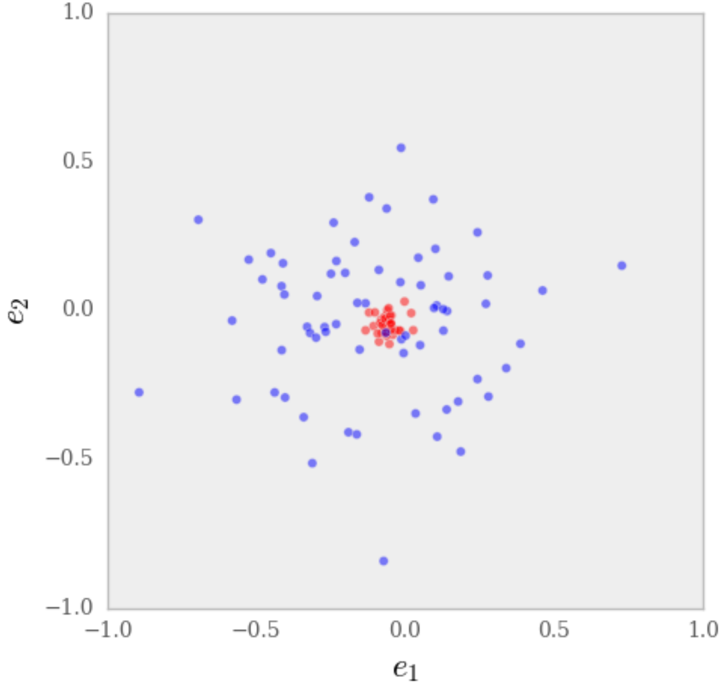
With importance sampling we only require the DP base distribution to be conjugate to the distribution of galaxy properties – *NOT* the likelihood.

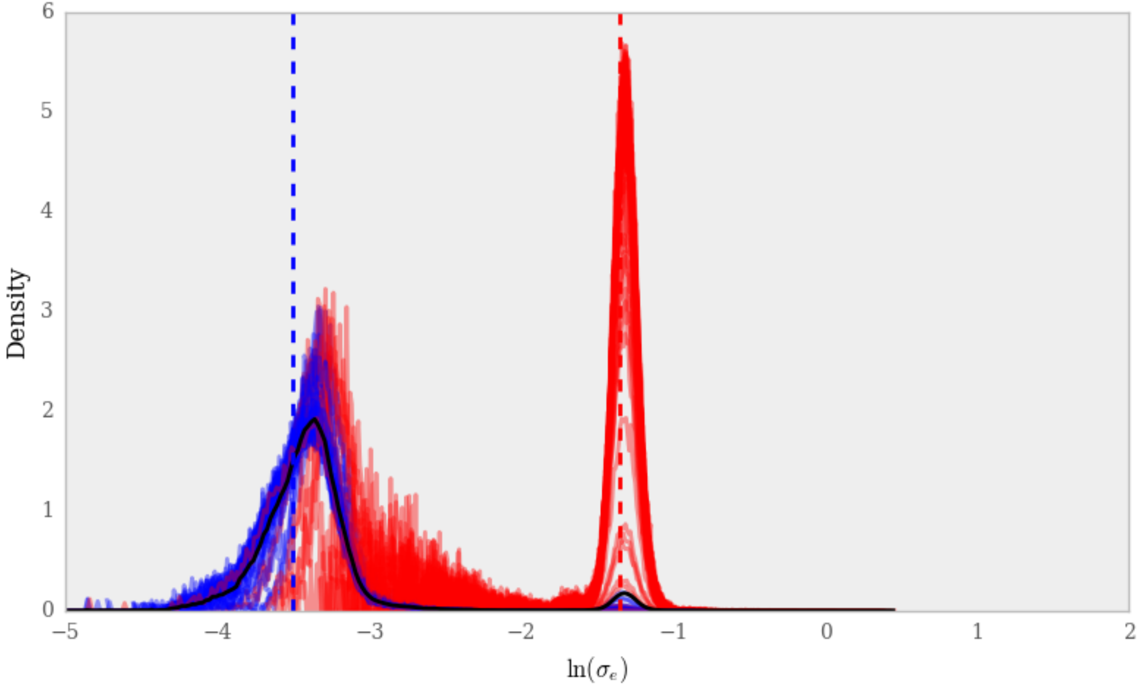
$$\int \Pr(\mathbf{d}_n | \alpha, \mathcal{X})\, G_0(\alpha)\, d\alpha = \frac{Z_n}{N} \sum_{k=1}^{N} \frac{\Pr_{\mathrm{marg}}(\omega_{nk} | a)}{\Pr(\omega_{nk} | I_0)}$$

$$\Pr_{\mathrm{marg}}(\omega_{nk} | a) \equiv \int d\alpha_{c_n}\, G_0(\alpha_{c_n} | a) \Pr(\omega_{nk} | \alpha_{c_n})$$

A simulation study with 100 galaxies validates the DP model

100 galaxies drawn from 1 of 2 Gaussian ellipticity distributions

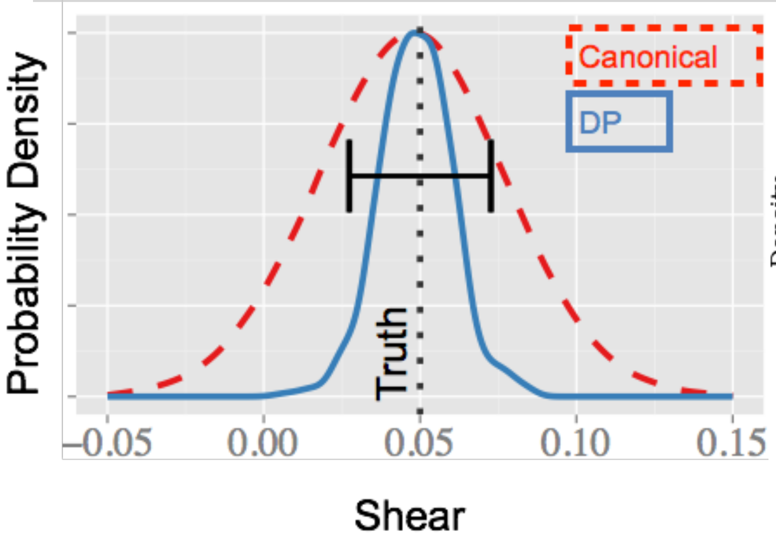


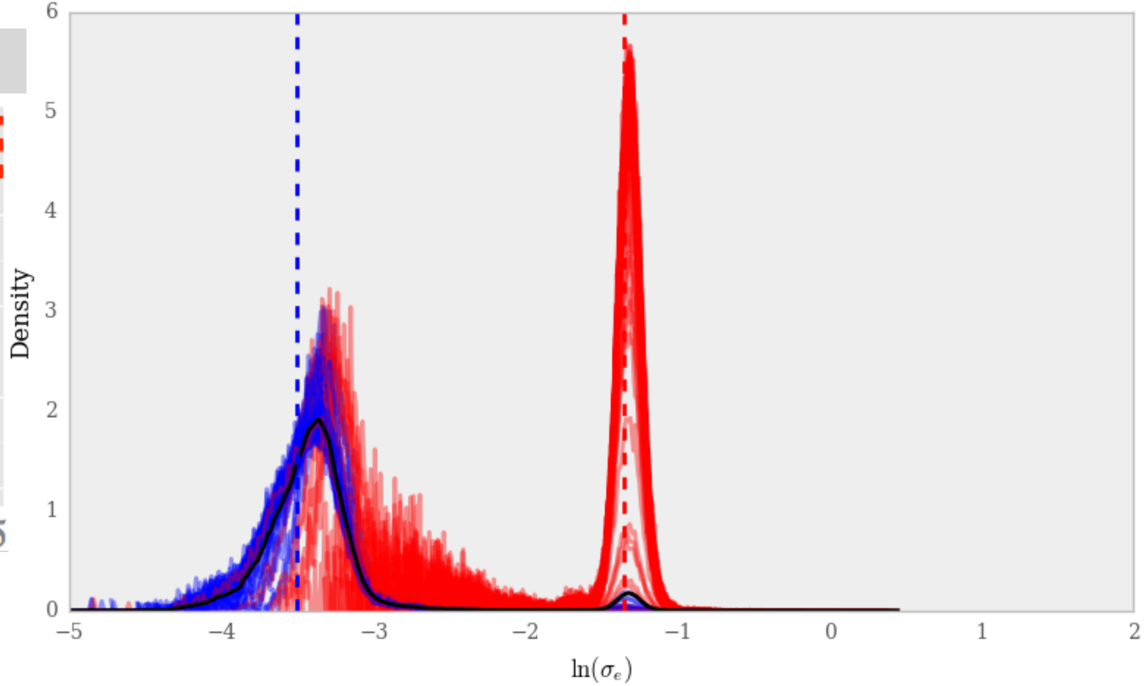


Simulation study: We can beat the traditional 'shape noise' statistical error bound by inferring latent structure in the data

100 galaxies drawn from 1 of 2 Gaussian ellipticity distributions

3x improvement in cosmic shear precision

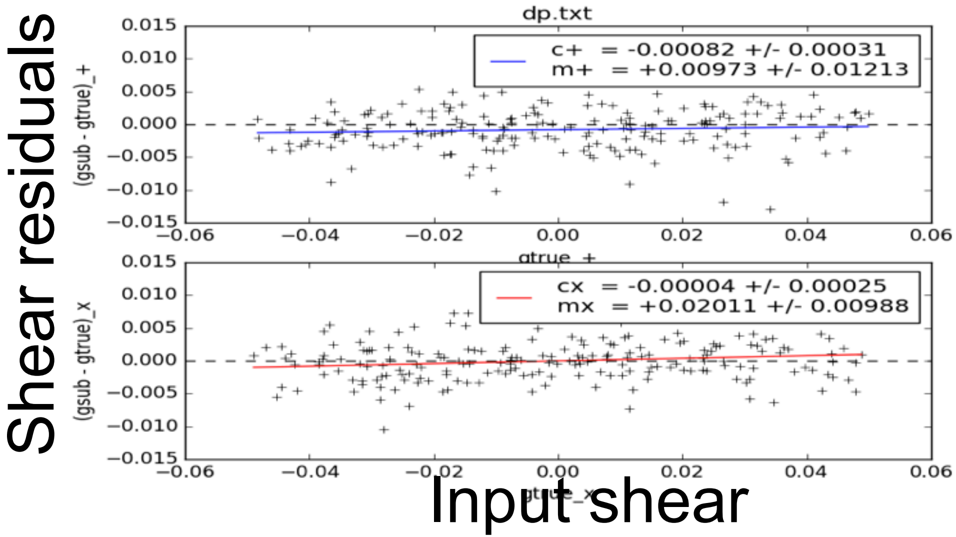




GREAT3 results

- Tested hierarchical approach using simulations from the third GRavitational lEnsing Accuracy Test (GREAT3).
- Hierarchical inference performs significantly better than ensemble average maximum likelihood ellipticity.
- The DPMM ellipticity prior performs better than the single Gaussian ellipticity prior.

Dirichlet Process Inference

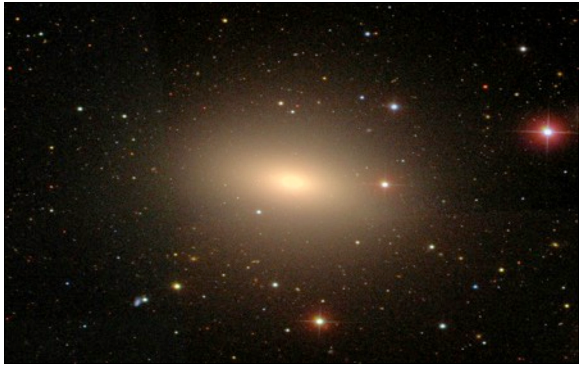


<ML> : 13% shear calibration errors

H.I. : 4% shear calibration errors

DP : 1-2% shear calibration errors

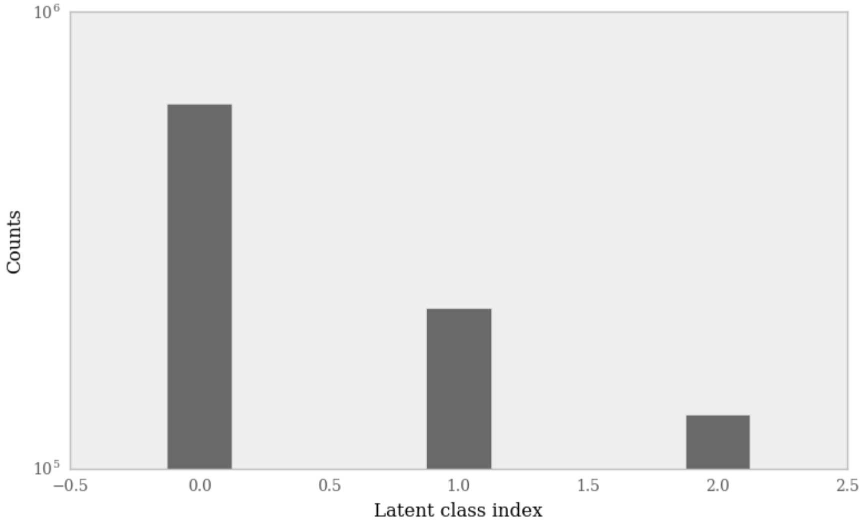
Multi-variate DP mixture model (in progress): "standardizable" ellipticities.



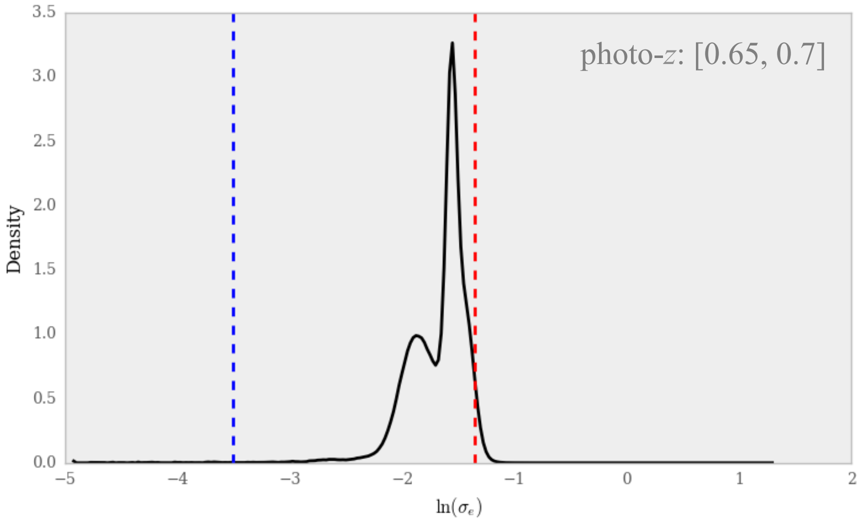
- Elliptical galaxies have a narrower intrinsic ellipticity distribution than late-type. Higher sensitivity to shear!
- Ellipticals/spirals also distinguishable by color and morphology (e.g., Sersic index, Gini coefficient, asymmetry), potentially providing additional variables with which to cluster.
- Other correlations to exploit?

Application to the Deep Lens Survey: real galaxies require at least 2 latent classes (ignoring lensing)

We infer 2 latent classes given only an ellipticity catalog

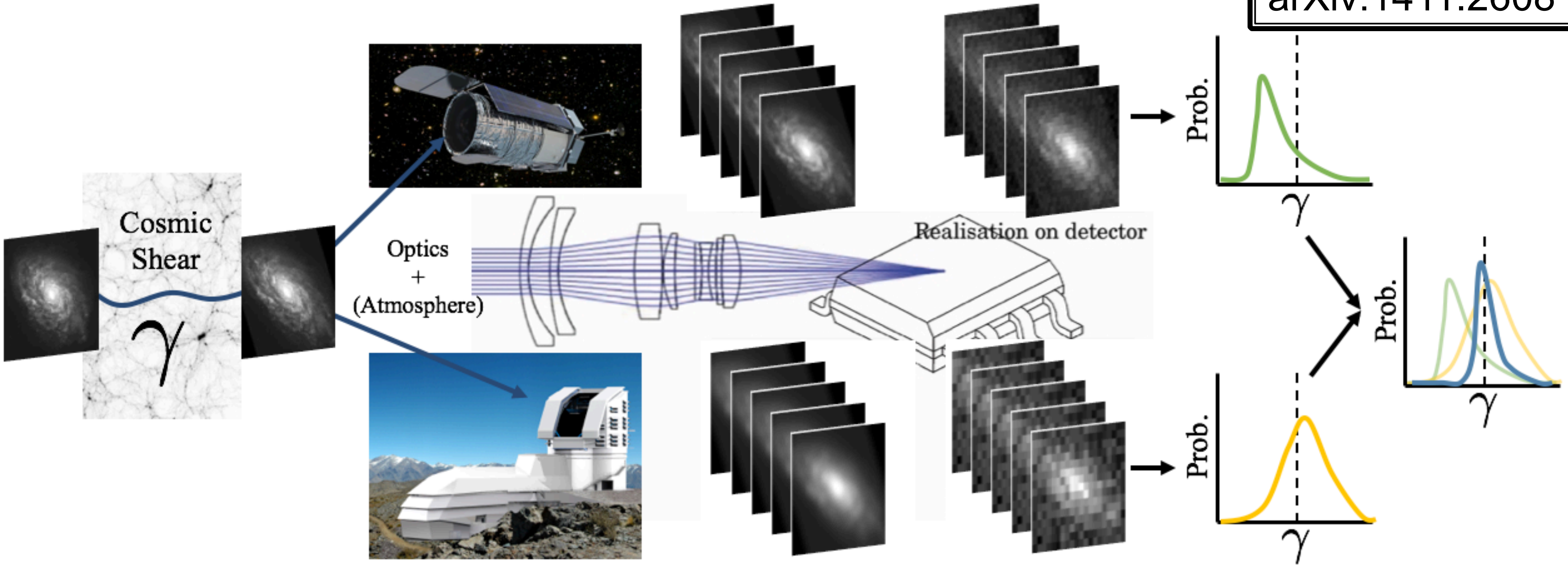


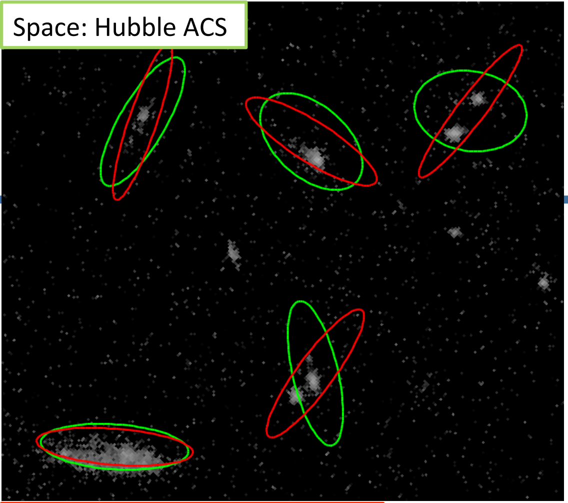
Preliminary: The marginal posterior distribution of ellipticity variance from the Deep Lens Survey

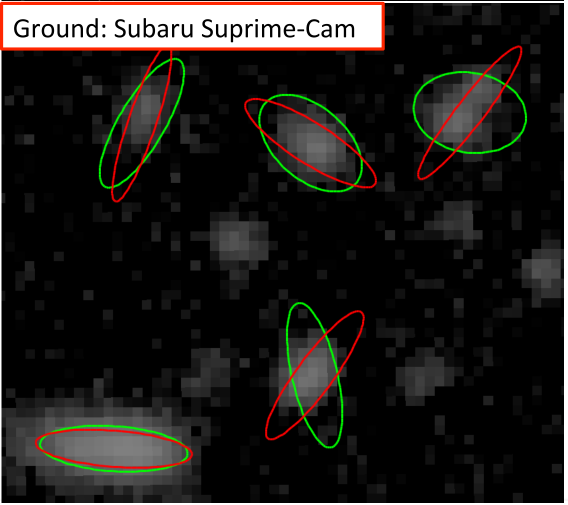


Unlike in the past, we will have many observations of the same sources that must be combined, while marginalizing distinct systematic errors
– A new processing paradigm

arXiv:1411.2608

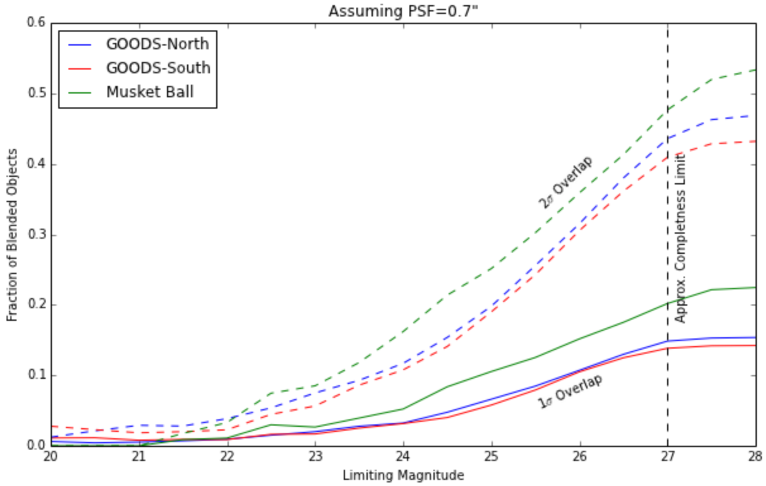






Aside: catalog cross-matching between space and ground is confused by significant object blending as seen by LSST

LSST blend fractions estimated from Subaru & HST overlapping imaging



Dawson+2015

How do we combine multiple observations of the same galaxy?
Naïvely we must joint fit all epochs simultaneously

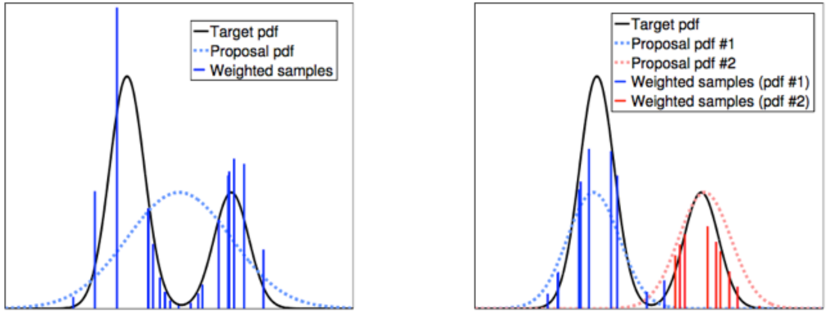
Problem: Imagine we have fit pixel data from LSST year 1.
How do we incorporate year 2 observations without redoing (expensive) calculations?

$$\Pr(\mathbf{d}_n|\alpha, \{\Pi_i\}) = \int d\omega_n \Pr(\omega_n|\alpha) \prod_{i=1}^{n_{\rm epochs}} \Pr(\mathbf{d}_{n,i}|\omega_n, \Pi_i)$$

Solution: Consider single-epoch samples as draws from a multi-modal importance sampling distribution:

$$q(\omega_n) = \frac{1}{n_{\rm epochs}} \sum_{i=1}^{n_{\rm epochs}} \Pr(\omega_n|\mathbf{d}_{n,i}, \Pi_i, I_0)$$

arXiv:1511.03095
Generalized Multiple Importance Sampling
Elvira, Martino, Luengo, & Bugallo



(a) Single proposal pdf (standard IS). (b) Two proposal pdfs (MIS).

Fig. 1: Approximation of the target pdf, $\pi(\mathbf{x})$, by the random measure χ.

Generalized multiple importance sampling (MIS) weights

MIS sampling distribution: sample from the conditional posterior for each epoch individually

$$q(\omega_n) = \frac{1}{n_{\text{epochs}}} \sum_{i=1}^{n_{\text{epochs}}} \Pr(\omega_n | \mathbf{d}_{n,i}, \Pi_i, I_0)$$

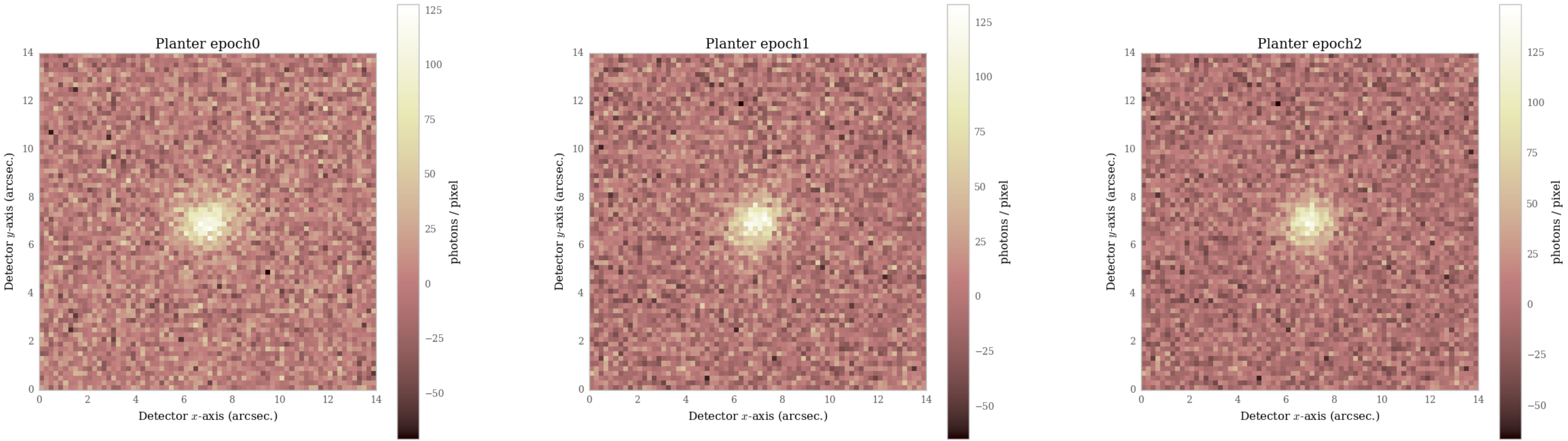
MIS weights: Evaluate the ratio of the conditional posterior for each epoch i to that of the MIS sampling distribution

$$w_i = \frac{\Pr(\mathbf{d}_{n,i}|\omega_n, \Pi_i)\Pr(\omega_n|\alpha)}{\sum_{i=1}^{n_{\text{epochs}}} \Pr(\mathbf{d}_{n,i}|\omega_n, \Pi_i)\Pr(\omega_n|I_0)}$$

'*cross-pollination*' needed:
Evaluate the likelihood of epoch i given model parameter samples from epoch j, for all combinations of i, j.
A standard scatter / gather operation

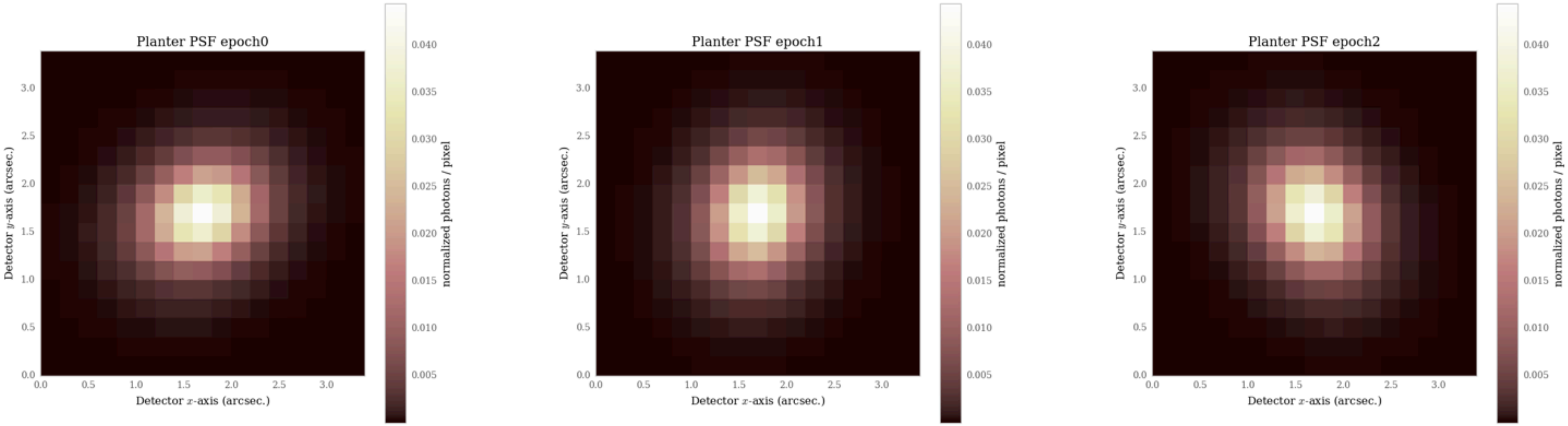
$$\Pr(\mathbf{d}_{n,i}|\omega_n^{(j)}\Pi_i)$$

Example: 1 galaxy, 3 epochs – fit the galaxy model parameters



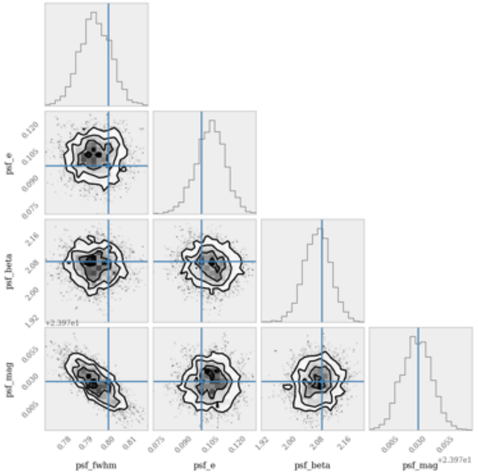
Each epoch has highly elliptical PSFs (|e| = 0.1) of same size, but different orientations

The PSF FWHM also matches the galaxy HLR making the single-epoch inferences noticeably different from each other. There is therefore a large gain of information in combining epochs.

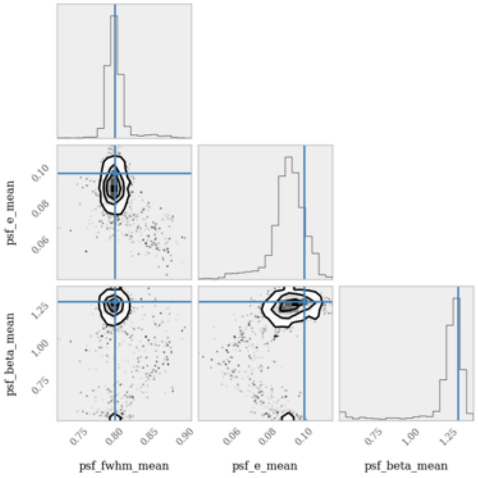


Interim posterior samples at each stage of the PSF hierarchical model

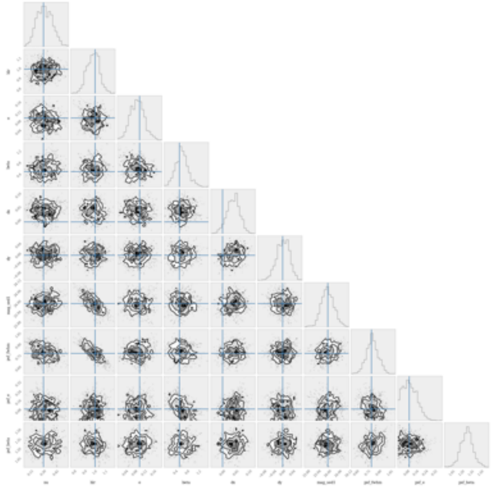
1) Fit stars



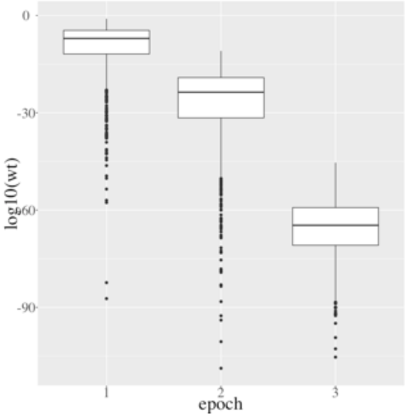
2) Constrain PSF model



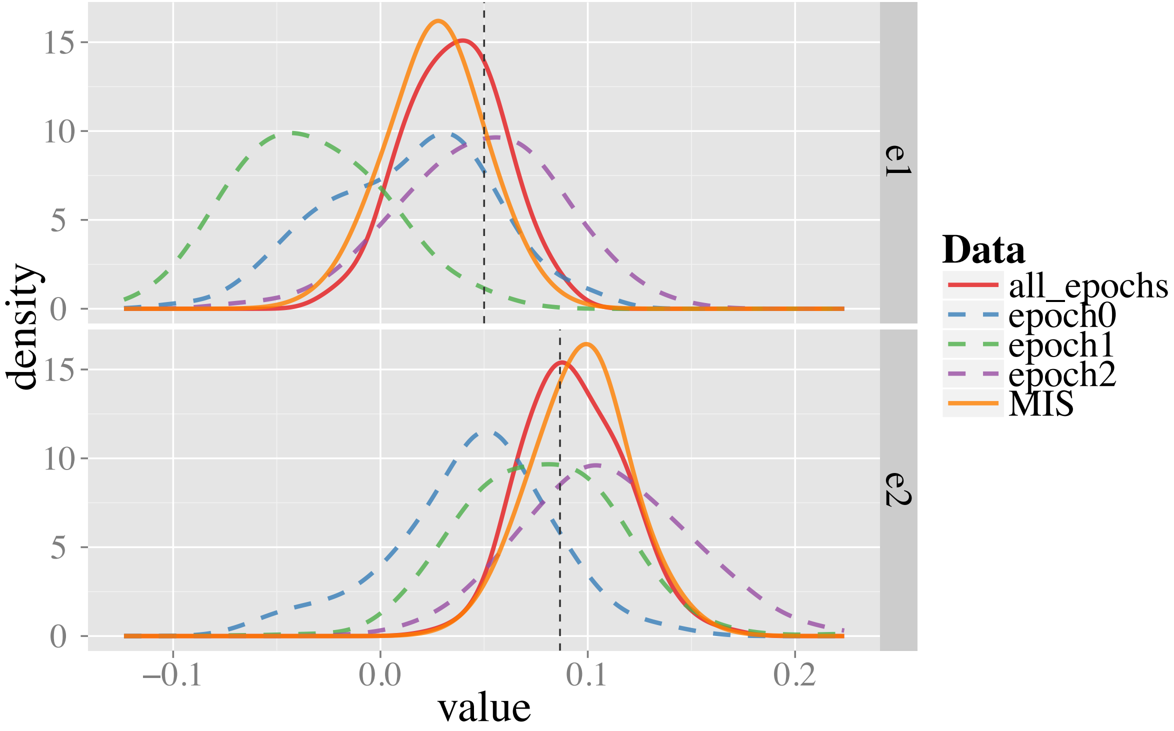
3) Fit galaxies & PSFs



4) Calculate MIS weights to combine epochs

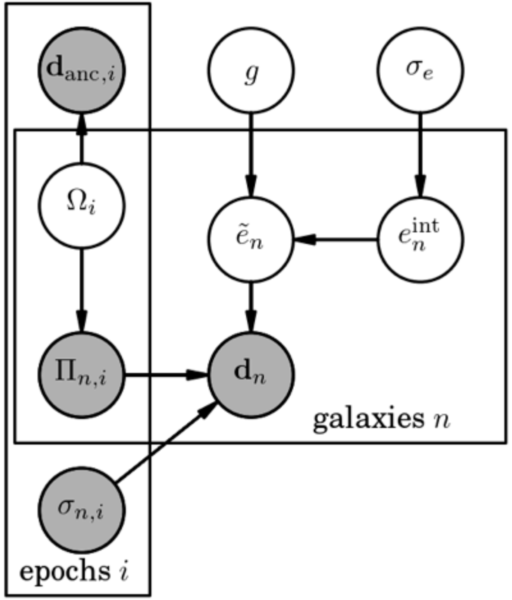


Comparison of single-epoch and combined epochs marginal posteriors

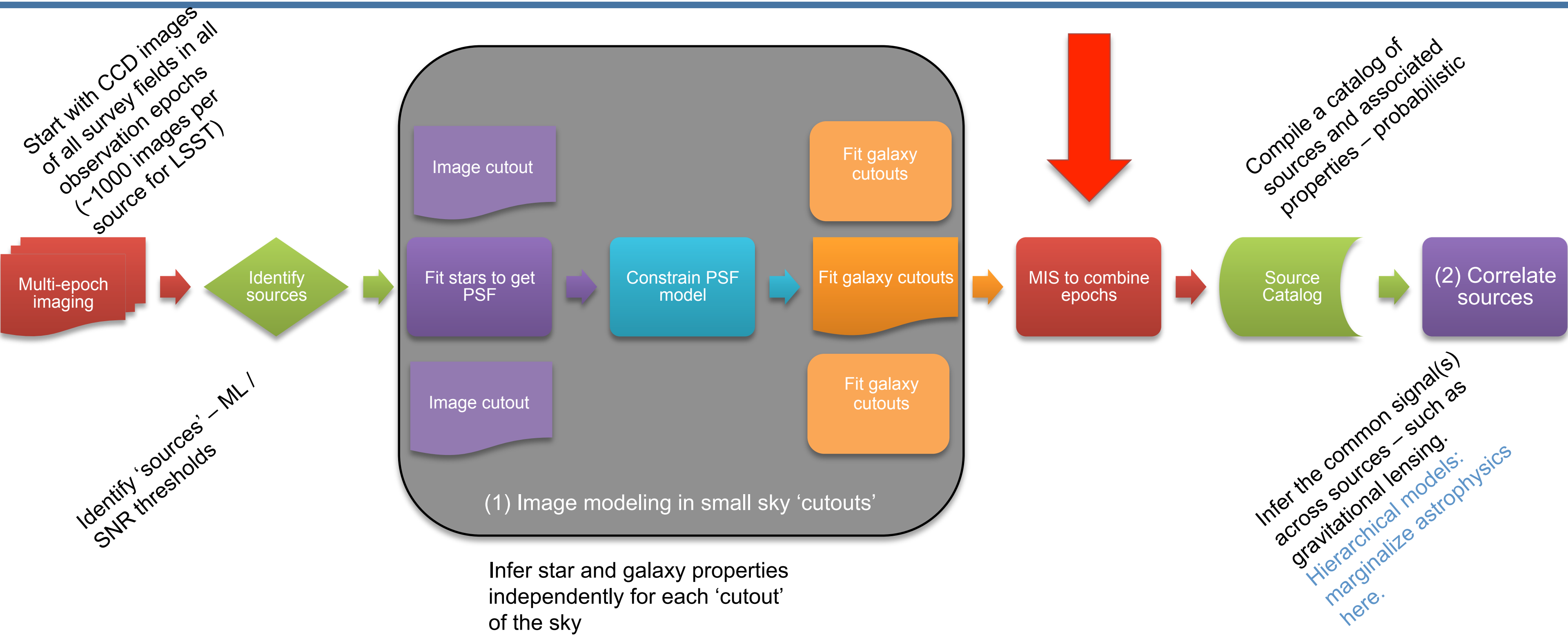


Marginalizing PSFs: MIS makes this tractable

- LSST will have ~200 epochs per object per filter
  - We aim to marginalize the PSF $\prod_{n,i}$ in every epoch
  - The marginalization is constrained by:
    - Consistency of PSF realizations over the focal plane for each epoch
    - Consistency of the underlying source model across epochs
- Simplest approach (statistically, not computationally): Infer galaxy models given all epoch imaging simultaneously
  - "Interim" samples are of size: ~10 galaxy params + 200 * ~4 PSF params = ~1k parameters!



Simulation and analysis pipeline: MIS-enabled



Summary

- Cosmic shear is systematics limited & signal is dominated by PSF and astrophysics
  - A probabilistic approach is warranted to infer a small signal and mitigate biases
- A hierarchical probabilistic model for cosmic shear can trade bias for variance, but also can increase precision by learning latent structure in the galaxy distribution.
- Importance sampling methods allow tractable approaches to a probabilistic forward model of LSST imaging
  - With billions of galaxies and hundreds of epochs per galaxy modeling LSST imaging requires an approach to separating analyses of data subsets, even though statistically correlated
- We are able to sample from a probabilistic model with multiple hierarchies to marginalize both correlated image systematics and astrophysical properties of galaxies.