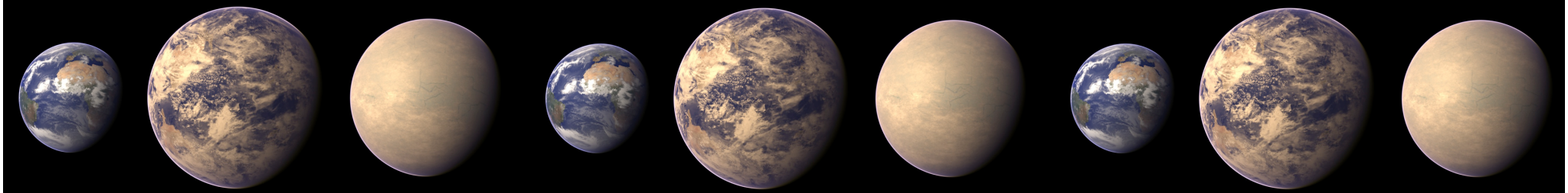
Hierarchical Bayesian Modeling

of Exoplanet Compositions



Angie Wolfgang
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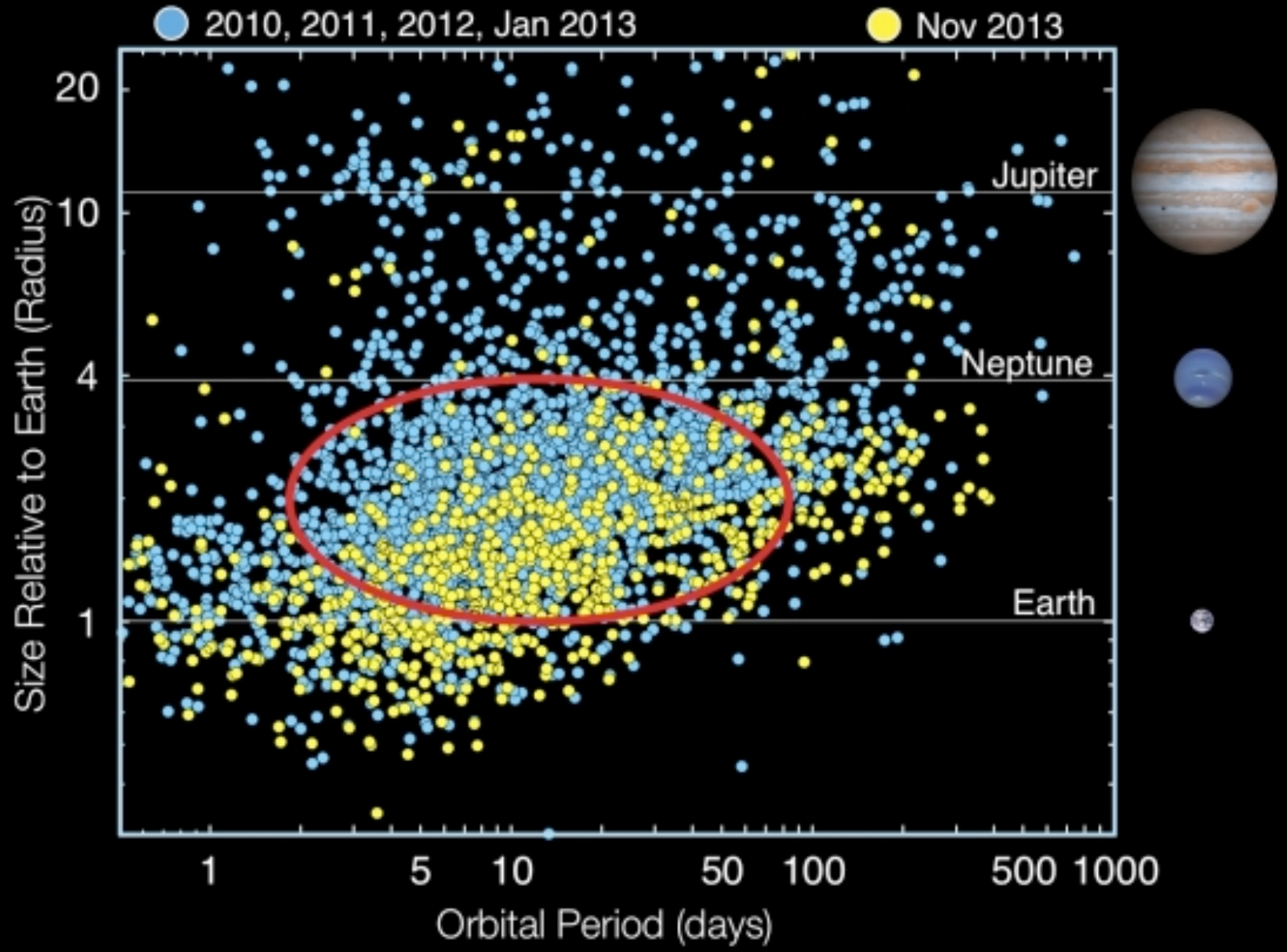
Eric Ford, Leslie Rogers, SAMSI Bayesian Exoplanet Populations Group, Eric Lopez

Kepler: An Unexpected Population



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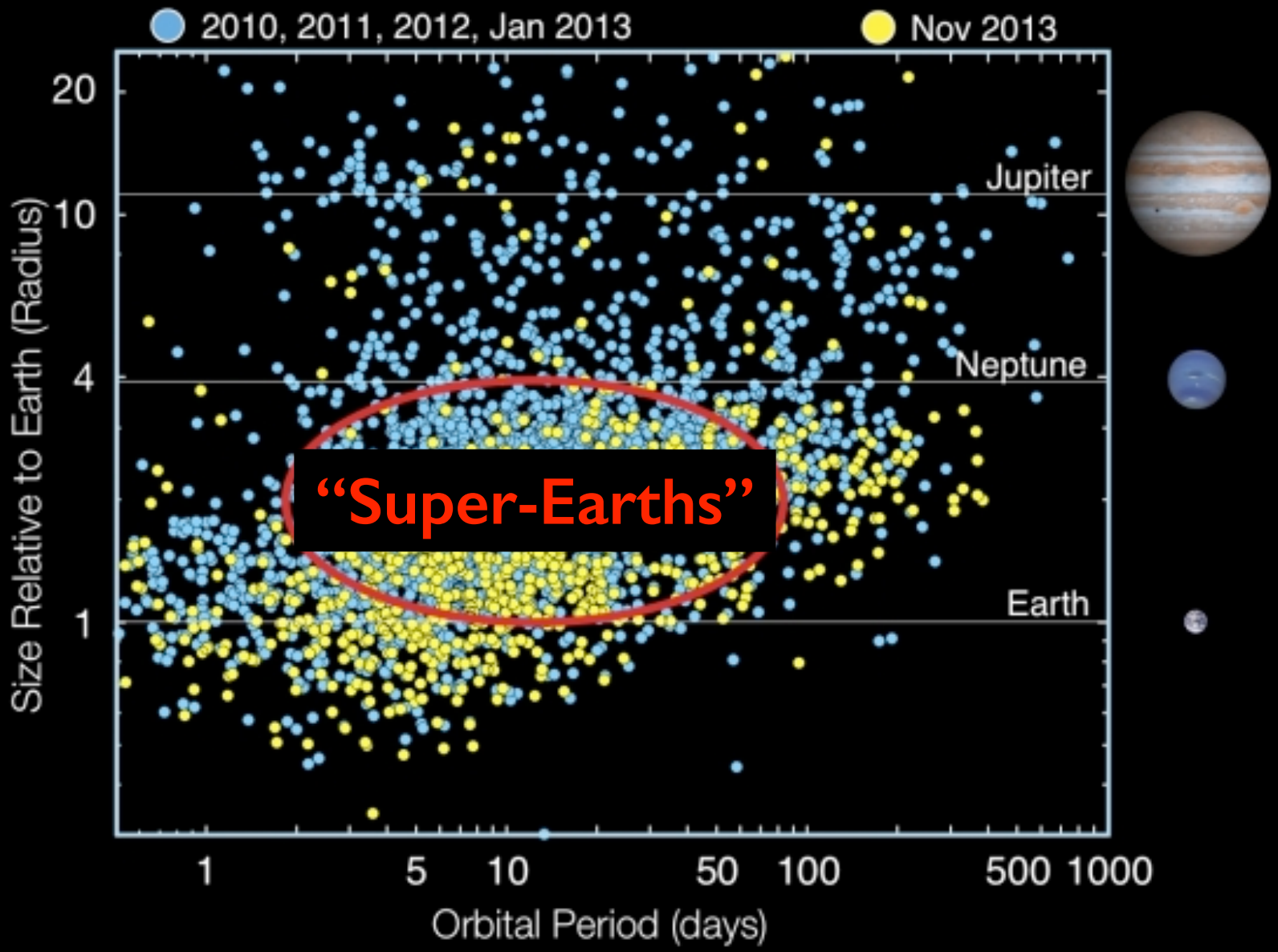


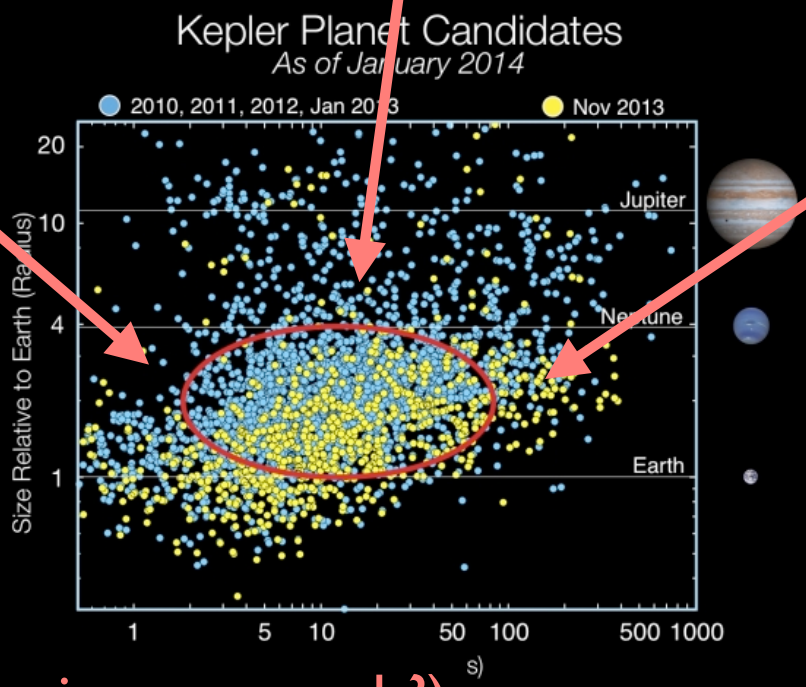
So, what does this mean for planet formation?

From Detection to Characterization

What are they made of? Probability of each composition?

How did they get so close in? What features explained by which migration mechanisms?

What processes dominate the formation and evolution of a "typical" super-Earth?

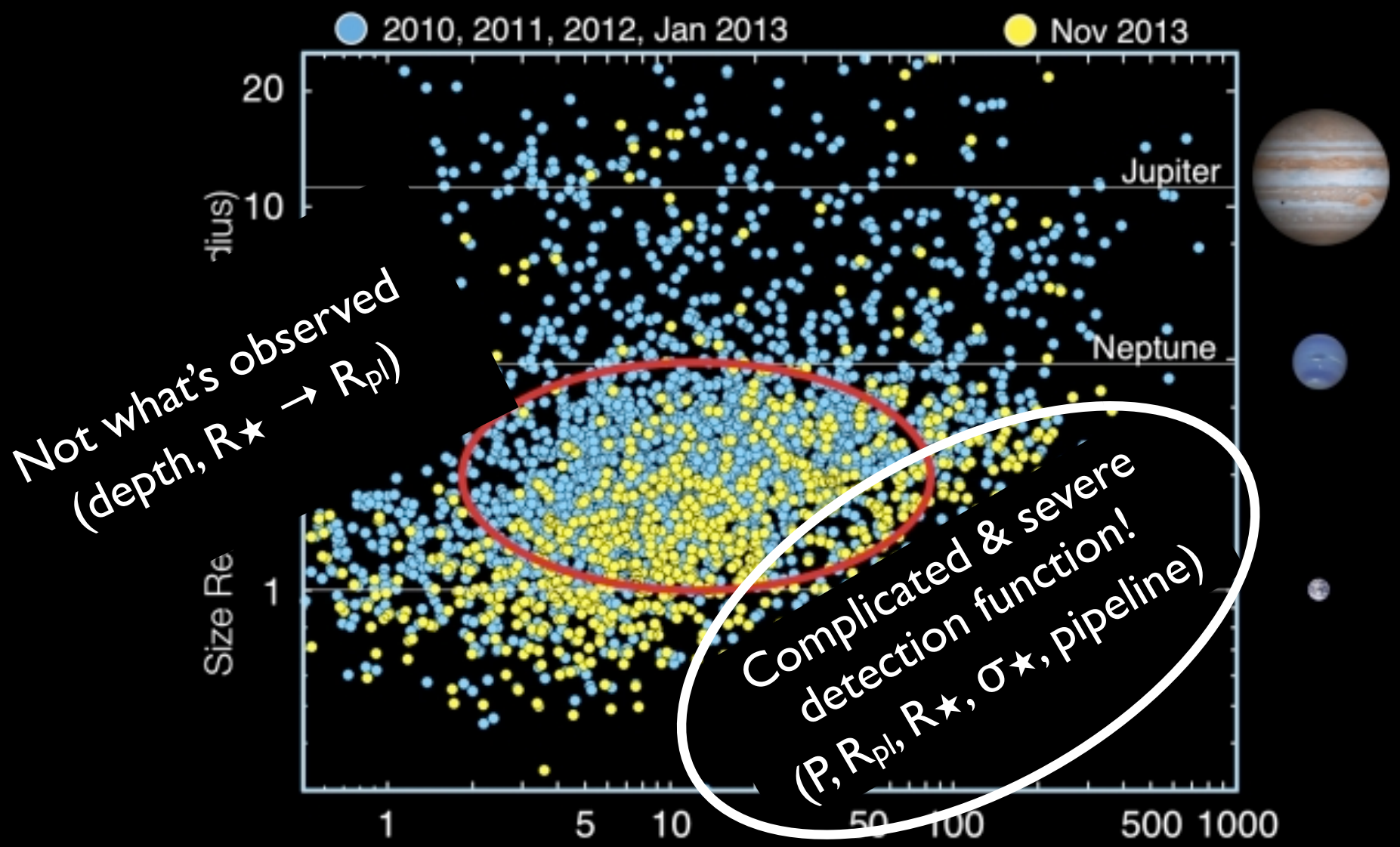


On the population level, planetary physics will be a ***probabilistic*** endeavor!

Complicated processes . . . more than one . . . tease apart which are ***most likely.***

(But aren't by-eye comparisons enough?)

No!!



1) Detection probability must be accounted for.

2) Latent variables and large uncertainties unavoidable

Small-Planet Compositions

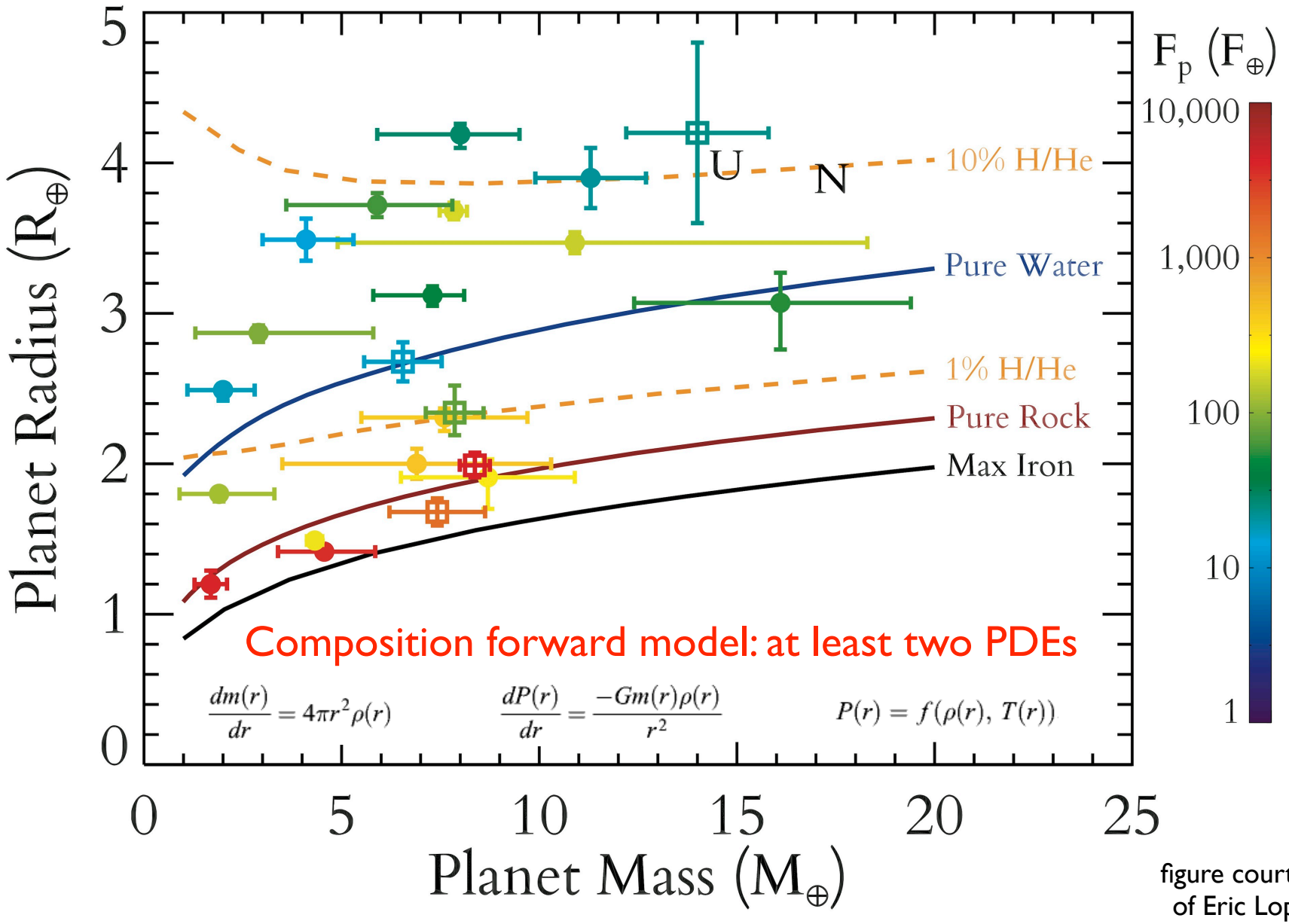
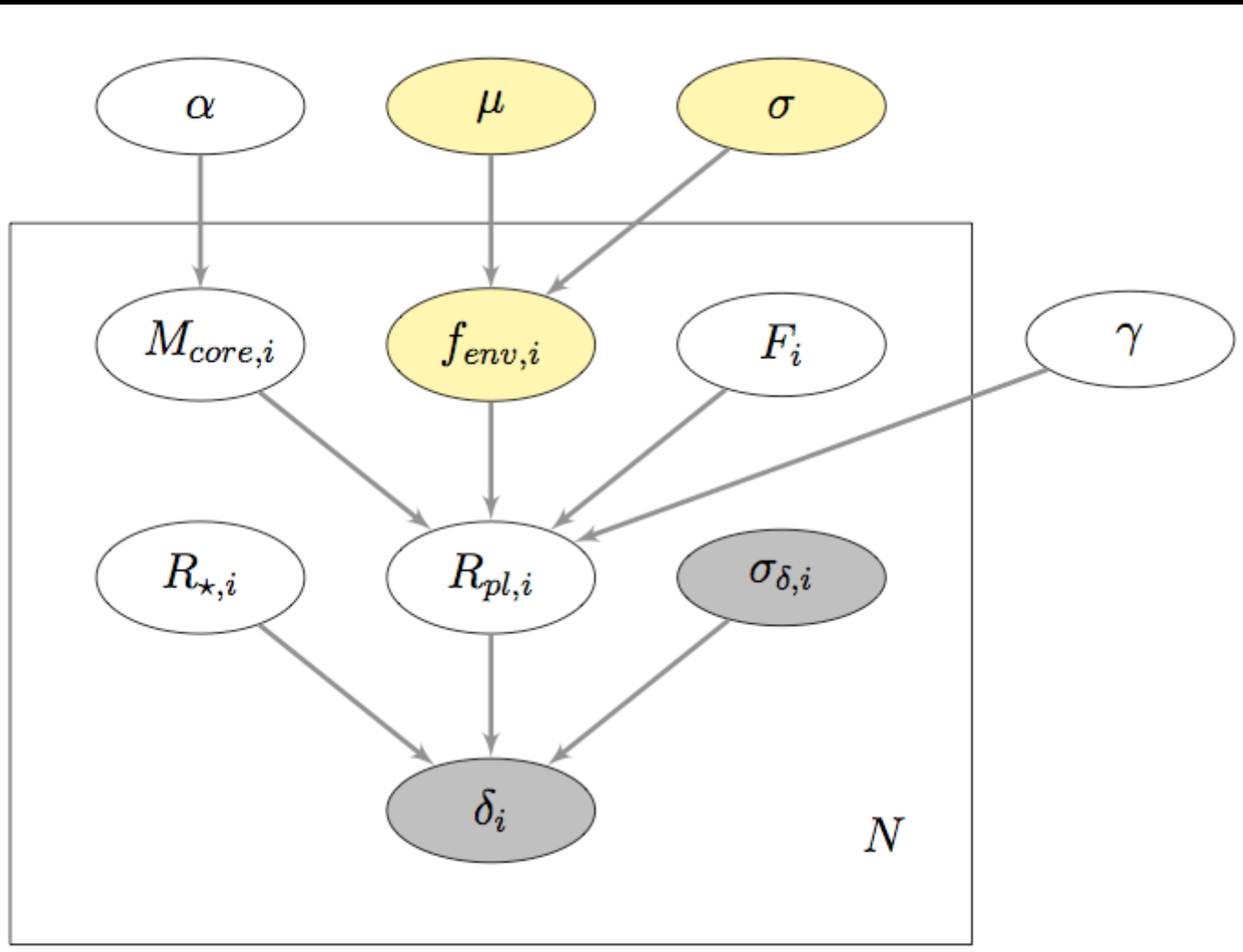


figure courtesy of Eric Lopez

Composition Distribution

Wolfgang & Lopez, 2015

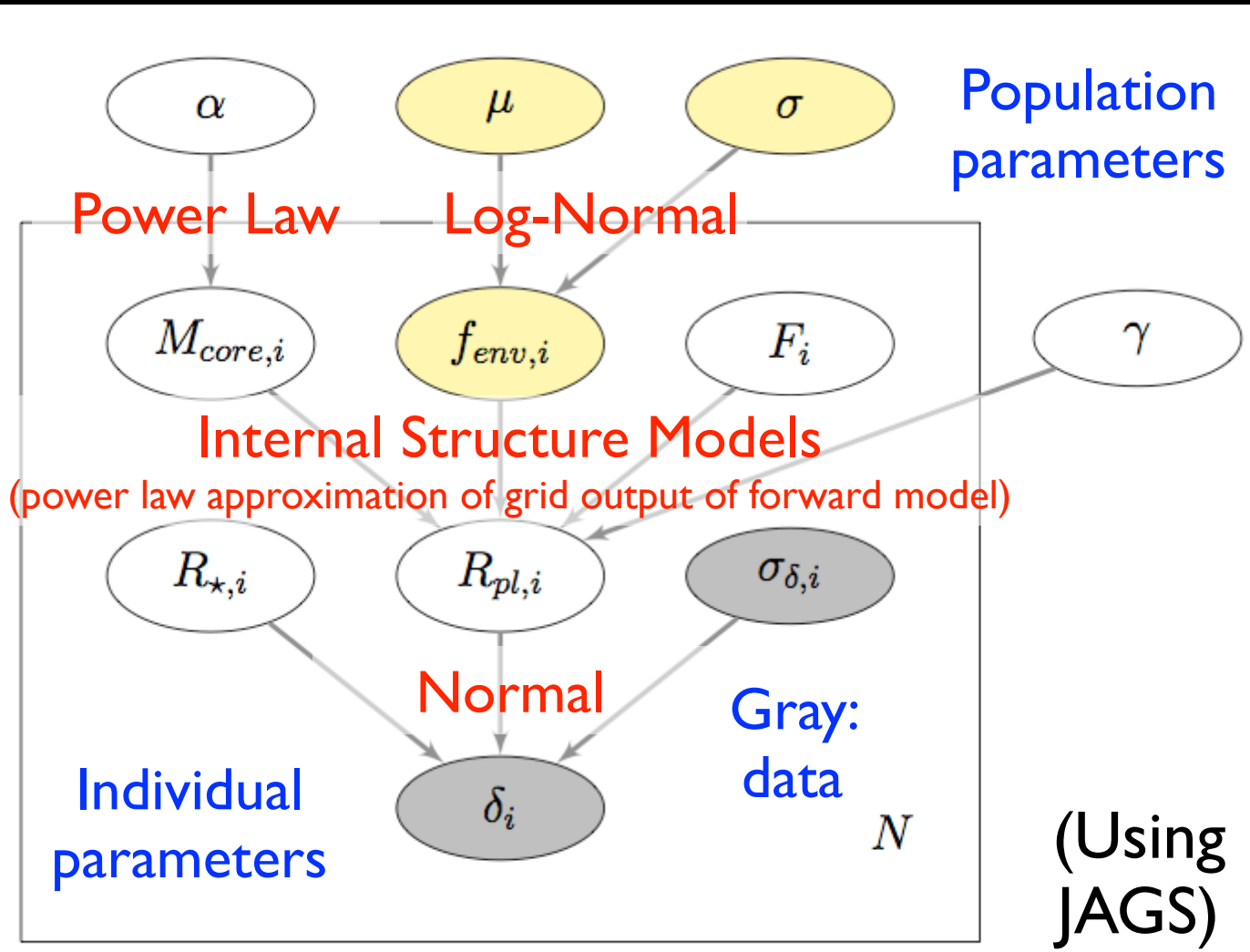


Wanted to understand both:

- compositions of individual super-Earths (fraction of mass in a gaseous envelope: f_{env})
- the distribution of this composition parameter over the Kepler population (μ, σ).

Composition Distribution

Wolfgang & Lopez, 2015



(Using JAGS)

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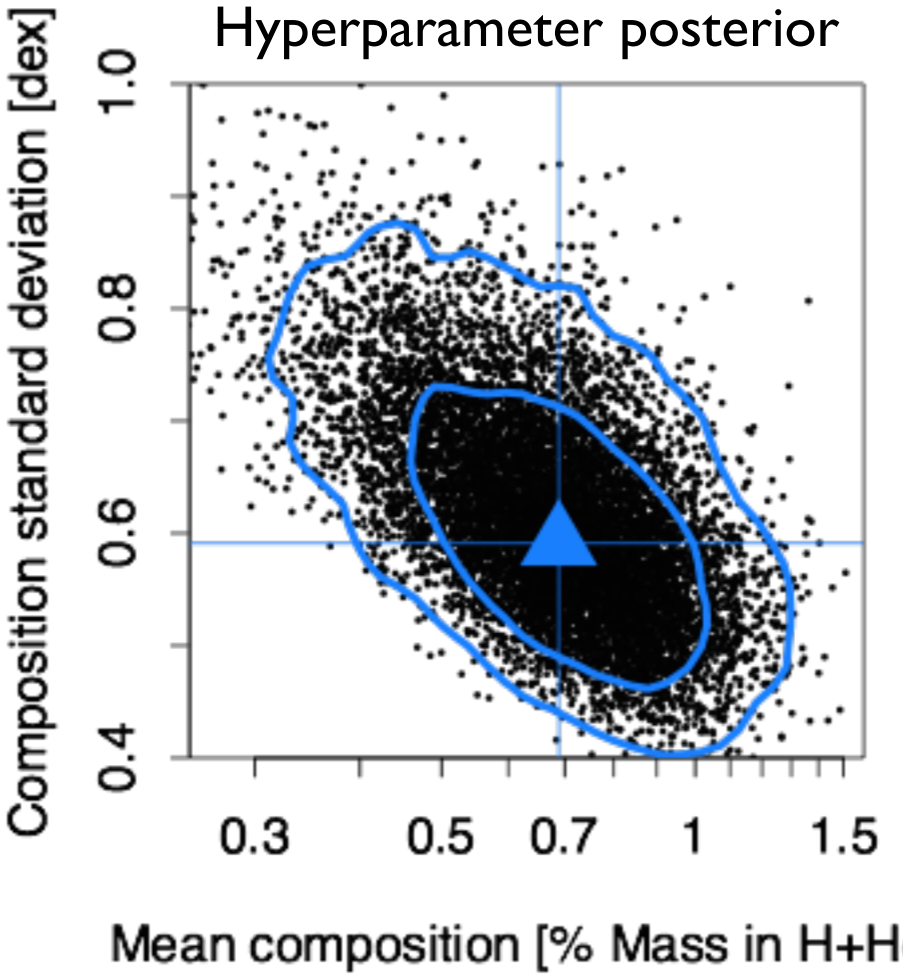
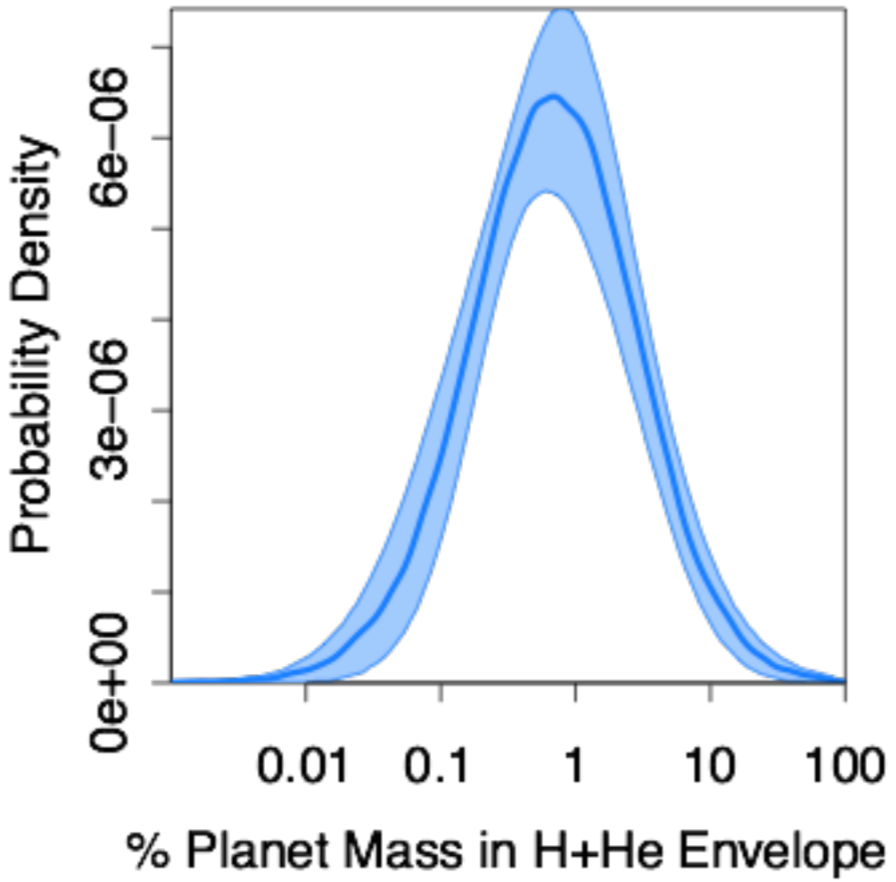
Results

Hyperparameter posterior



Marginal composition distribution

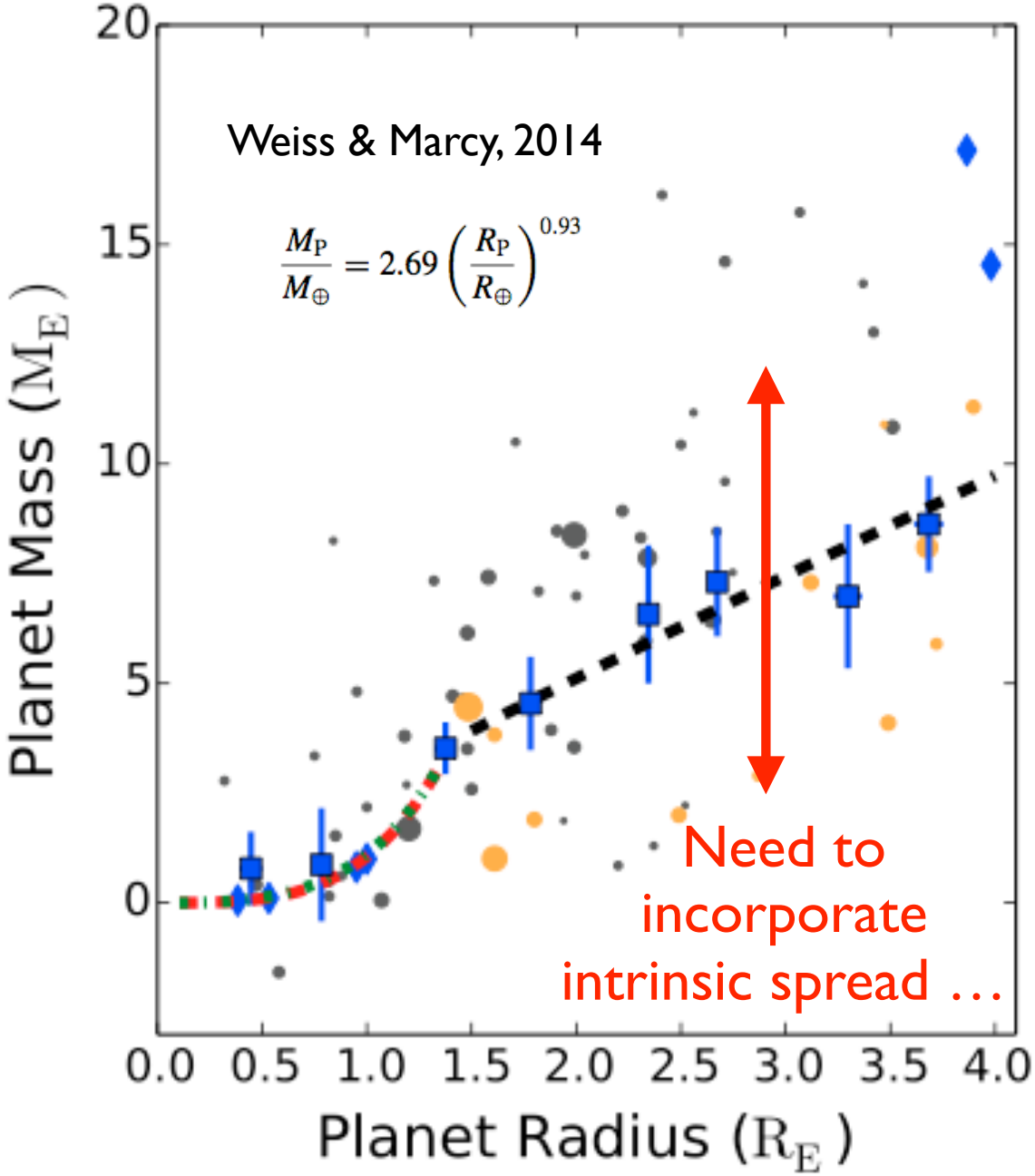


First composition distribution:

~ 1% envelope mass fractions are the most likely

Width had not been previously characterized

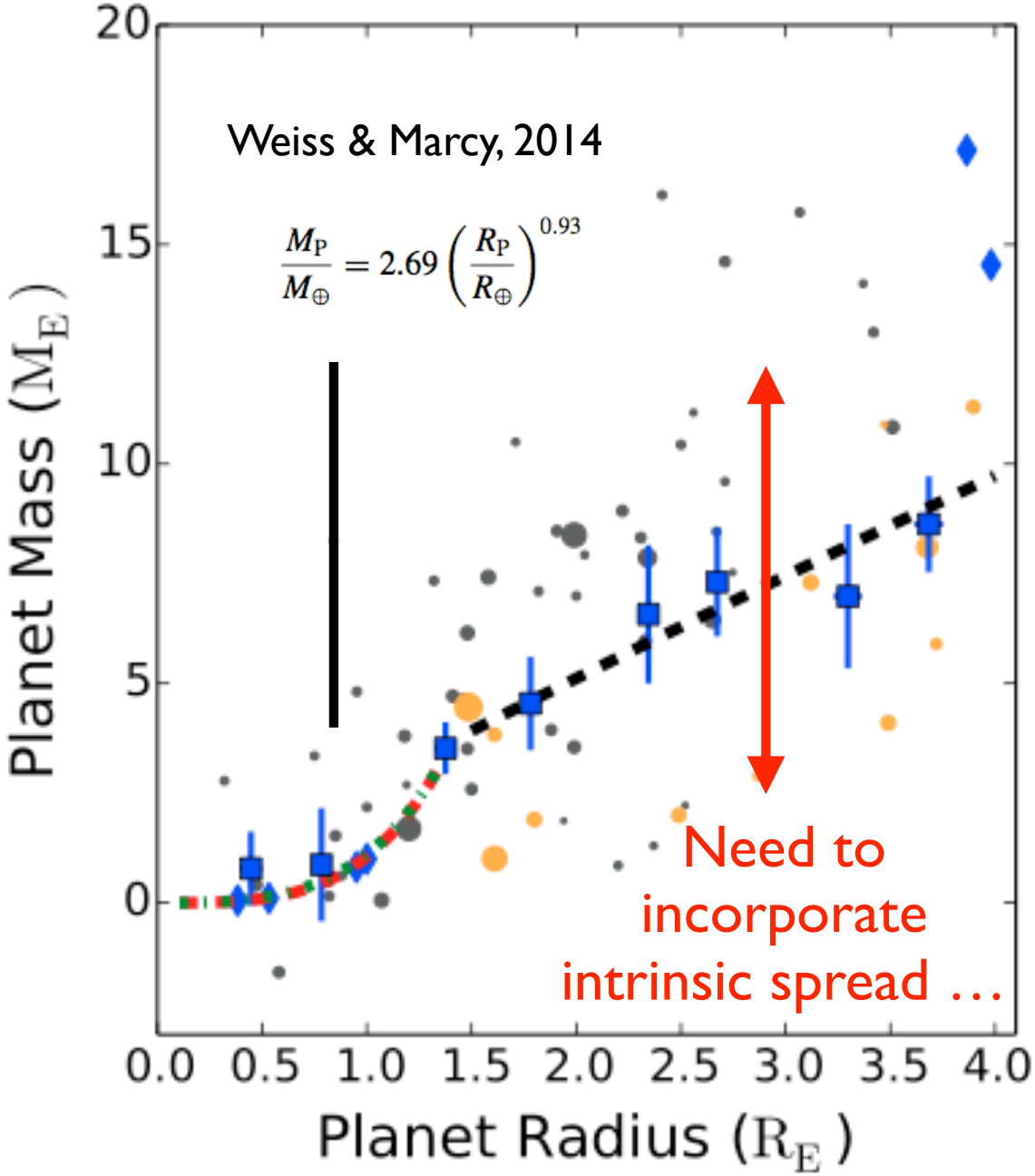
Mass-Radius Relationship?



Kepler gives radius, but need mass to:

- perform dynamical studies
- compare planet surveys
- more directly constrain planet formation theory

Mass-Radius PDF (probability density function)



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- perform dynamical studies
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- more directly constrain planet formation theory

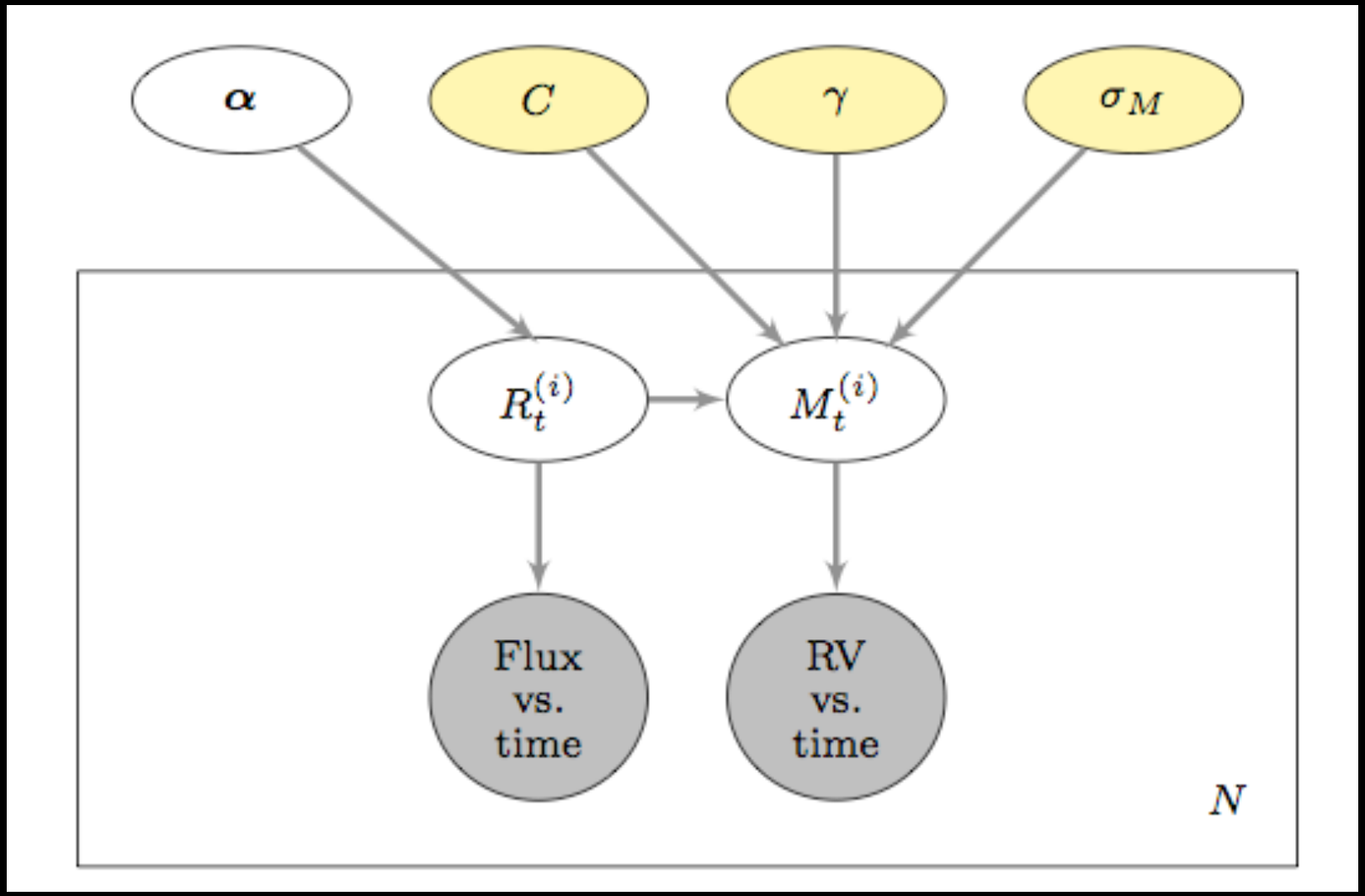
$$\frac{M}{M_\oplus} = C\left(\frac{R}{R_\oplus}\right)^{\gamma}$$

$$\frac{M}{M_\oplus} \sim \text{Normal}\left(\mu = C\left(\frac{R}{R_\oplus}\right)^{\gamma}, \sigma = \sigma_M\right)$$

. . . while accounting for measurement uncertainty

HBM for Mass-Radius Relation

Wolfgang, Rogers, & Ford, 2016



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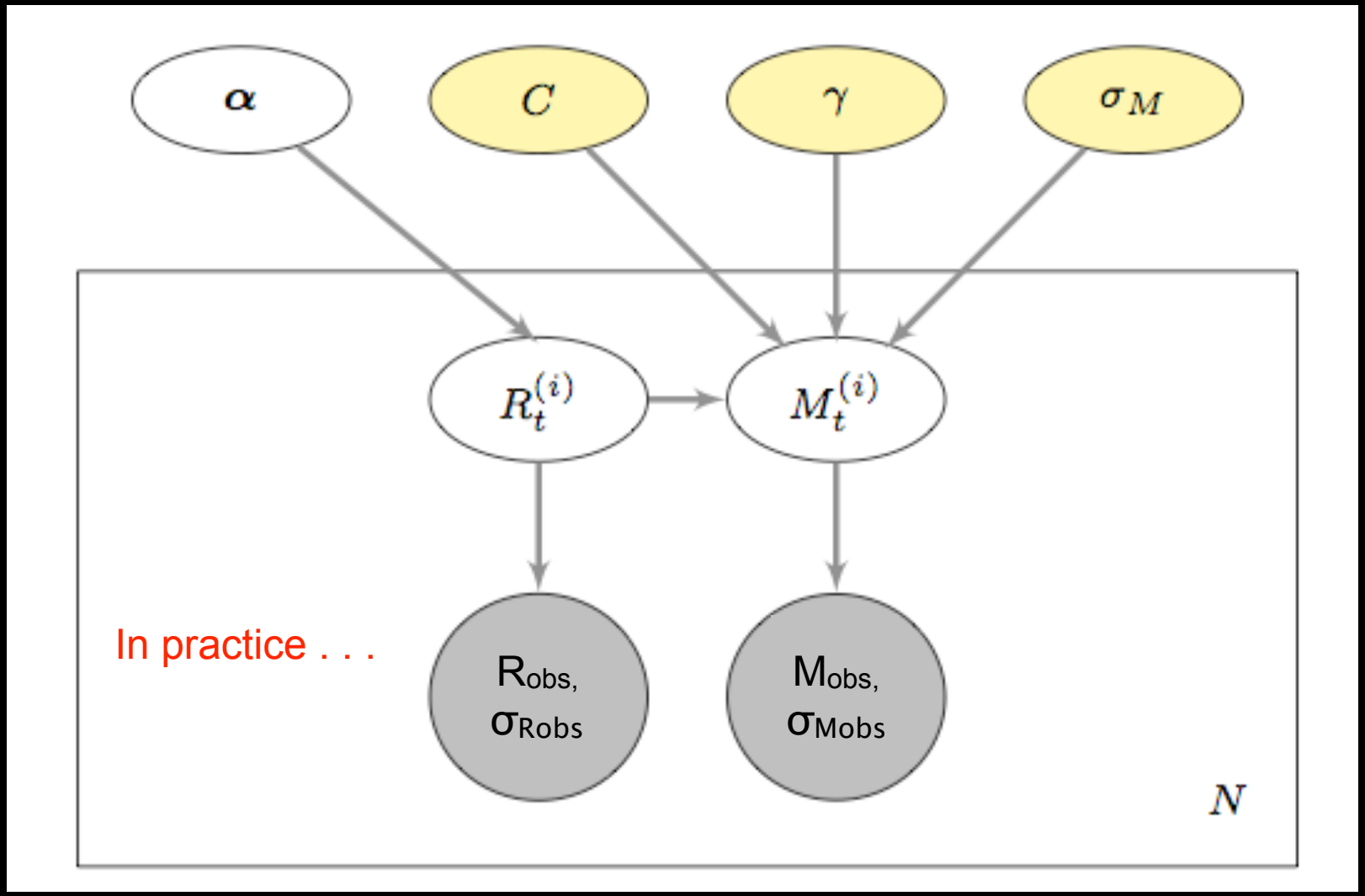


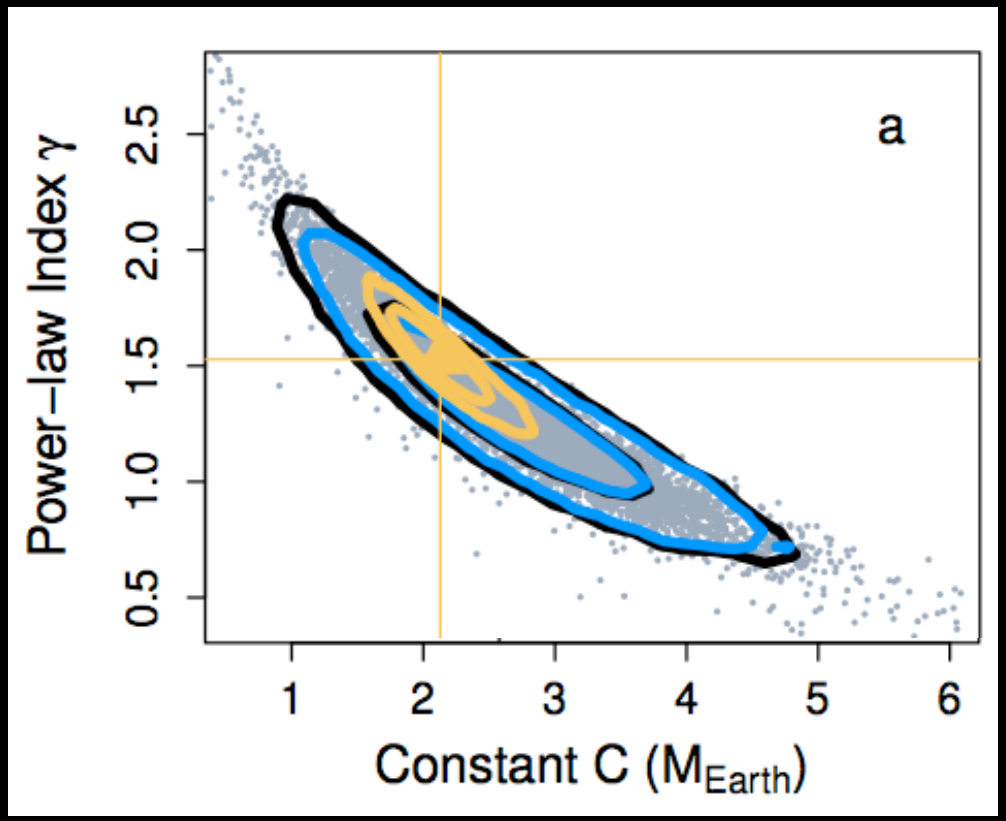
Results

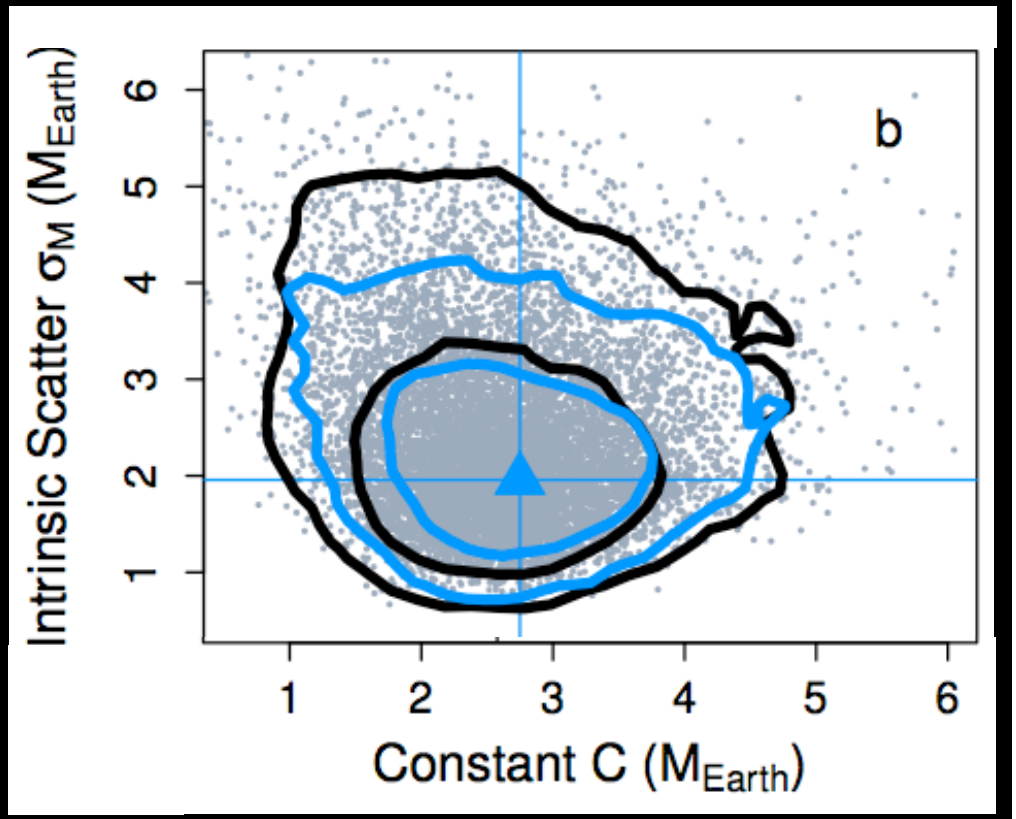
deterministic M-R relation:

$$\frac{M}{M_\oplus} = C\left(\frac{R}{R_\oplus}\right)^\gamma$$

probabilistic M-R relation:

$$\frac{M}{M_\oplus} \sim \text{Normal}\left(\mu = C\left(\frac{R}{R_\oplus}\right)^\gamma, \sigma = \sigma_M\right)$$

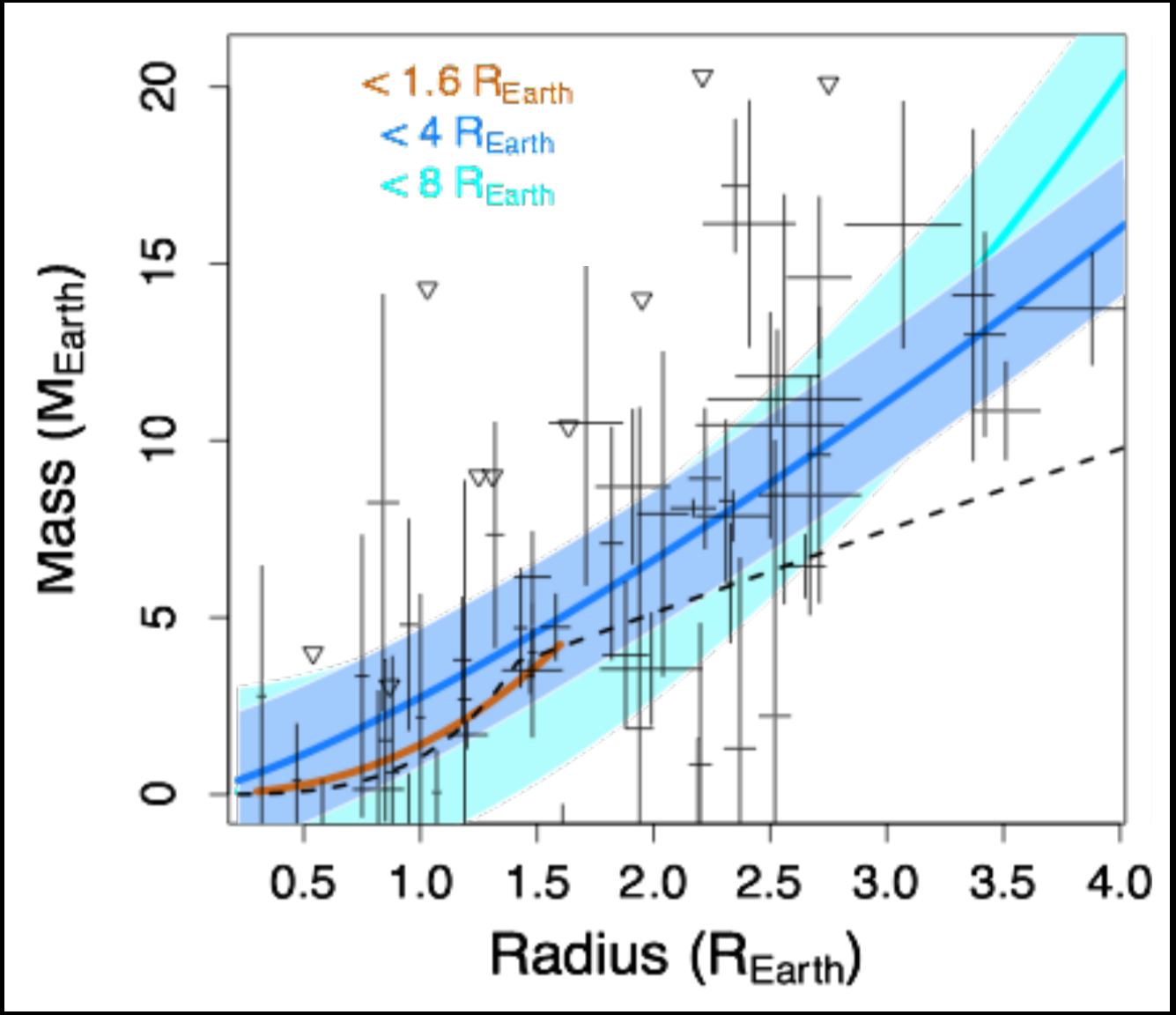




There is intrinsic scatter in the current set of R,M measurements.

Probabilistic M-R Relation:

$$\frac{M}{M_\oplus} \sim \text{Normal}\left(\mu = 2.7\left(\frac{R}{R_\oplus}\right)^{1.3}, \sigma = 1.9\right)$$



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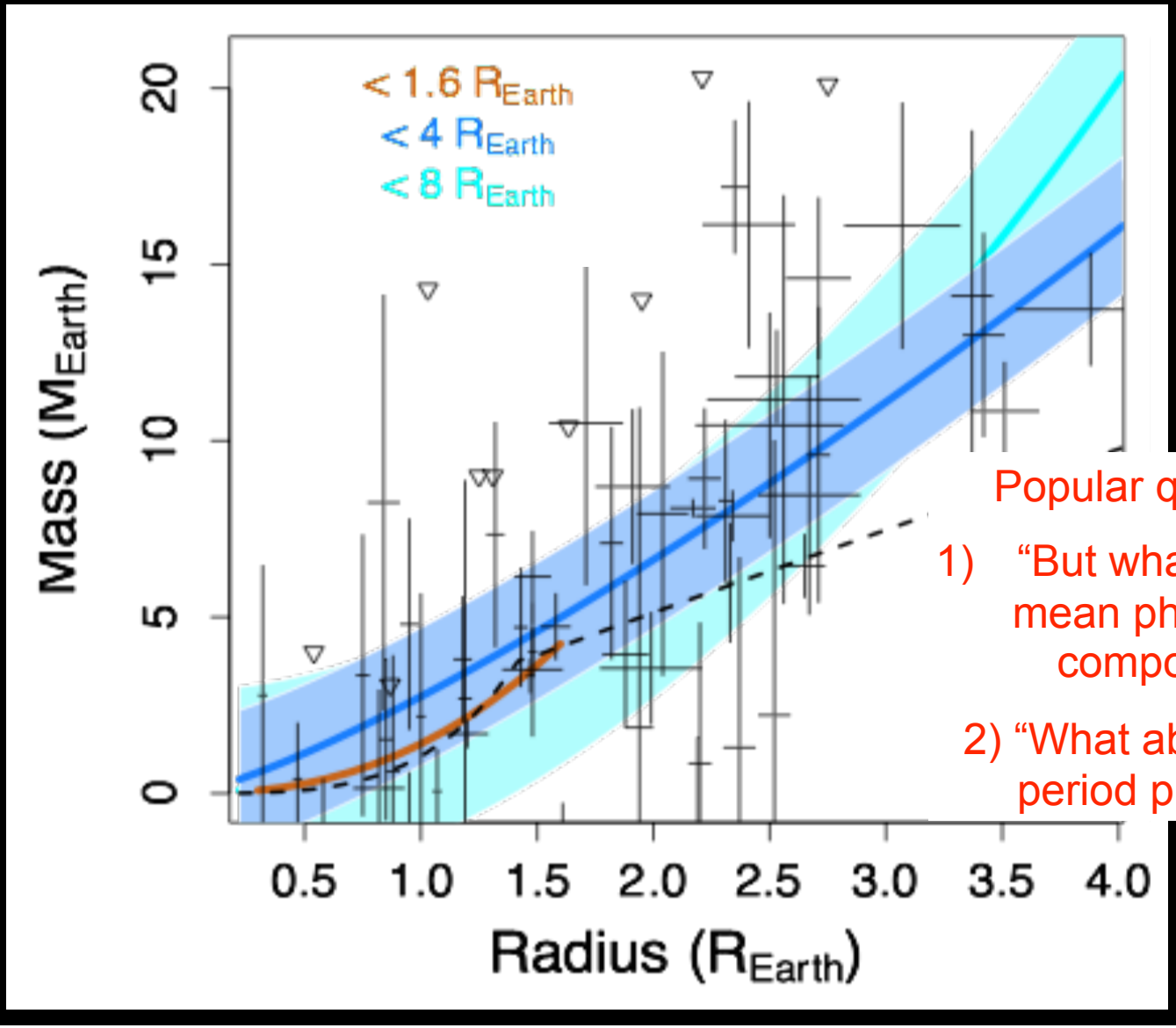

Popular questions:

1) "But what does this mean physically, for compositions?"

2) "What about longer period planets?"

Small-Planet Compositions

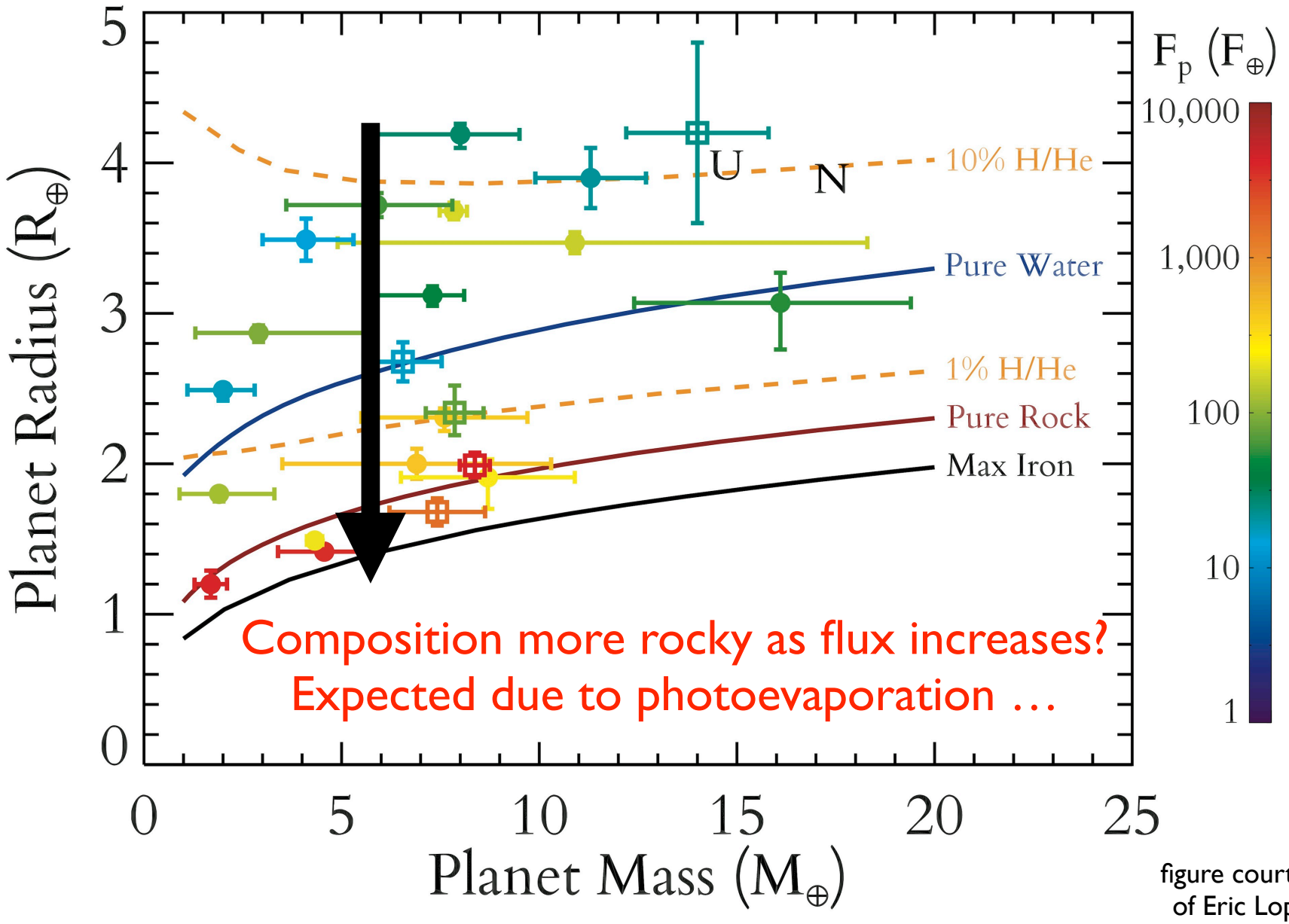

figure courtesy of Eric Lopez

But two methods for mass …

… with major selection effects in both datasets!!

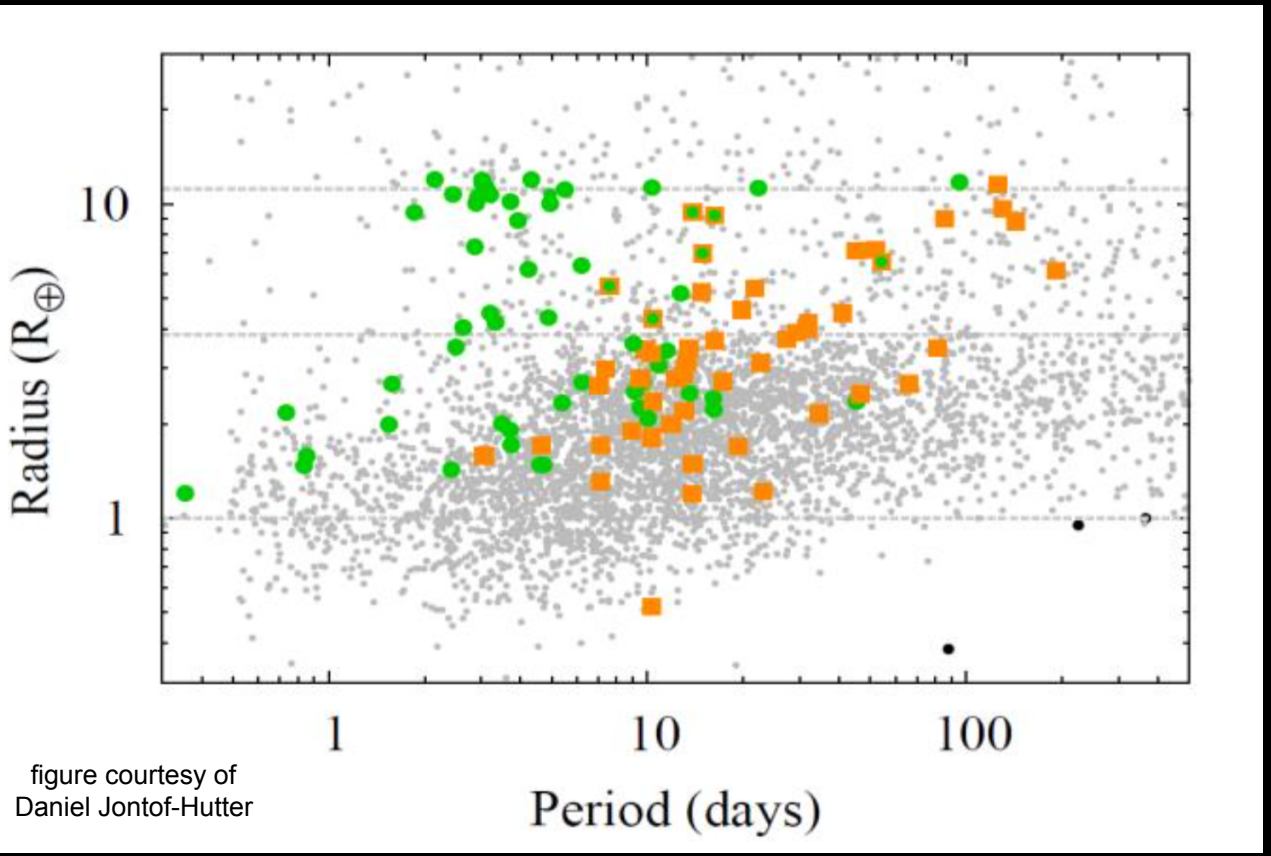

figure courtesy of
Daniel Jontof-Hutter

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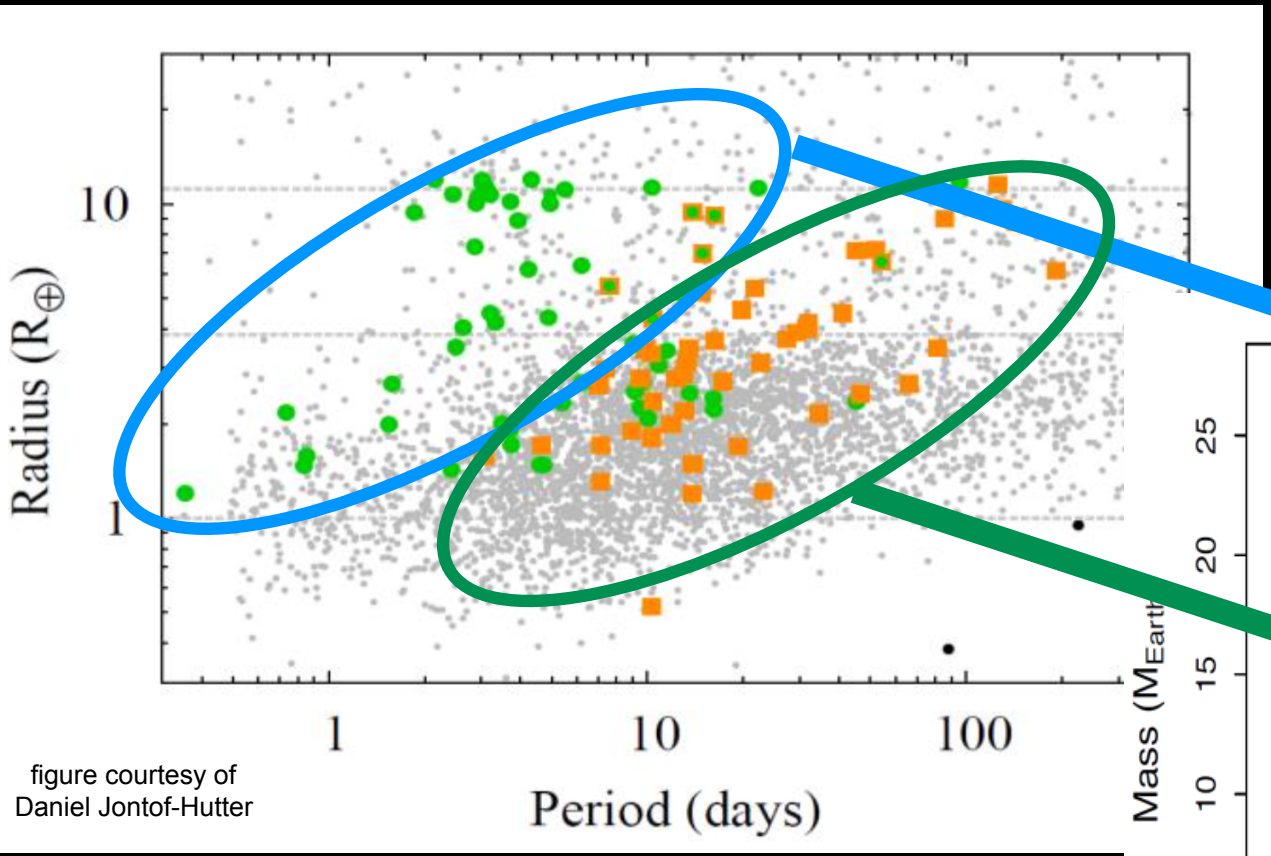

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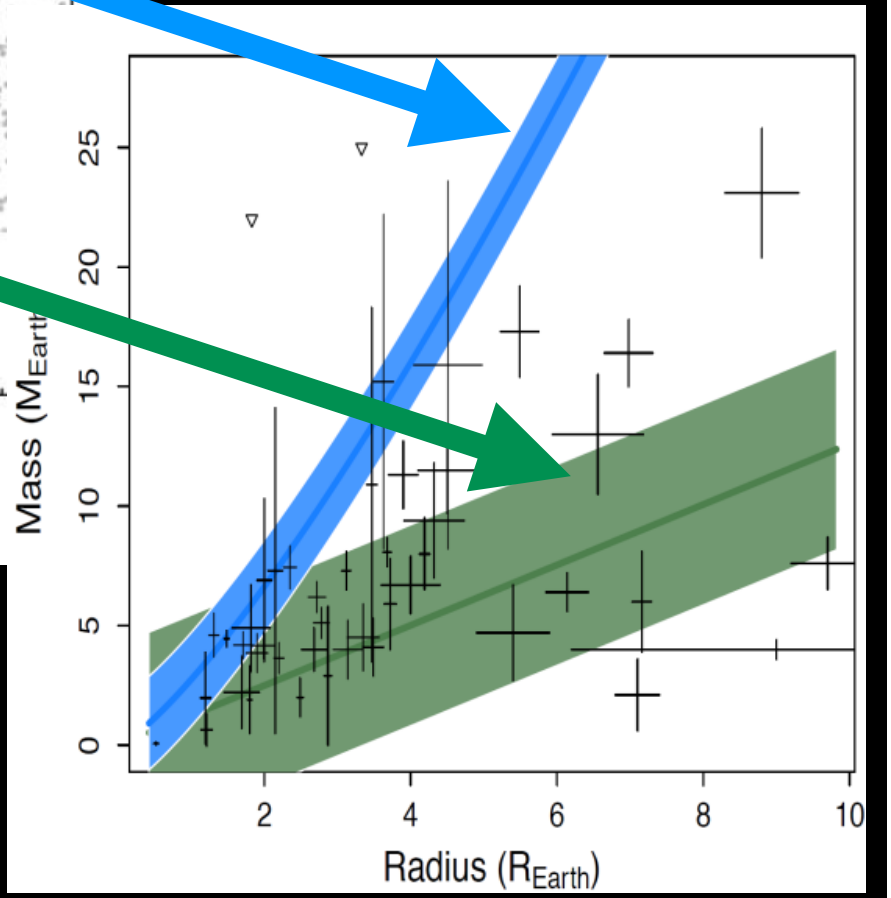
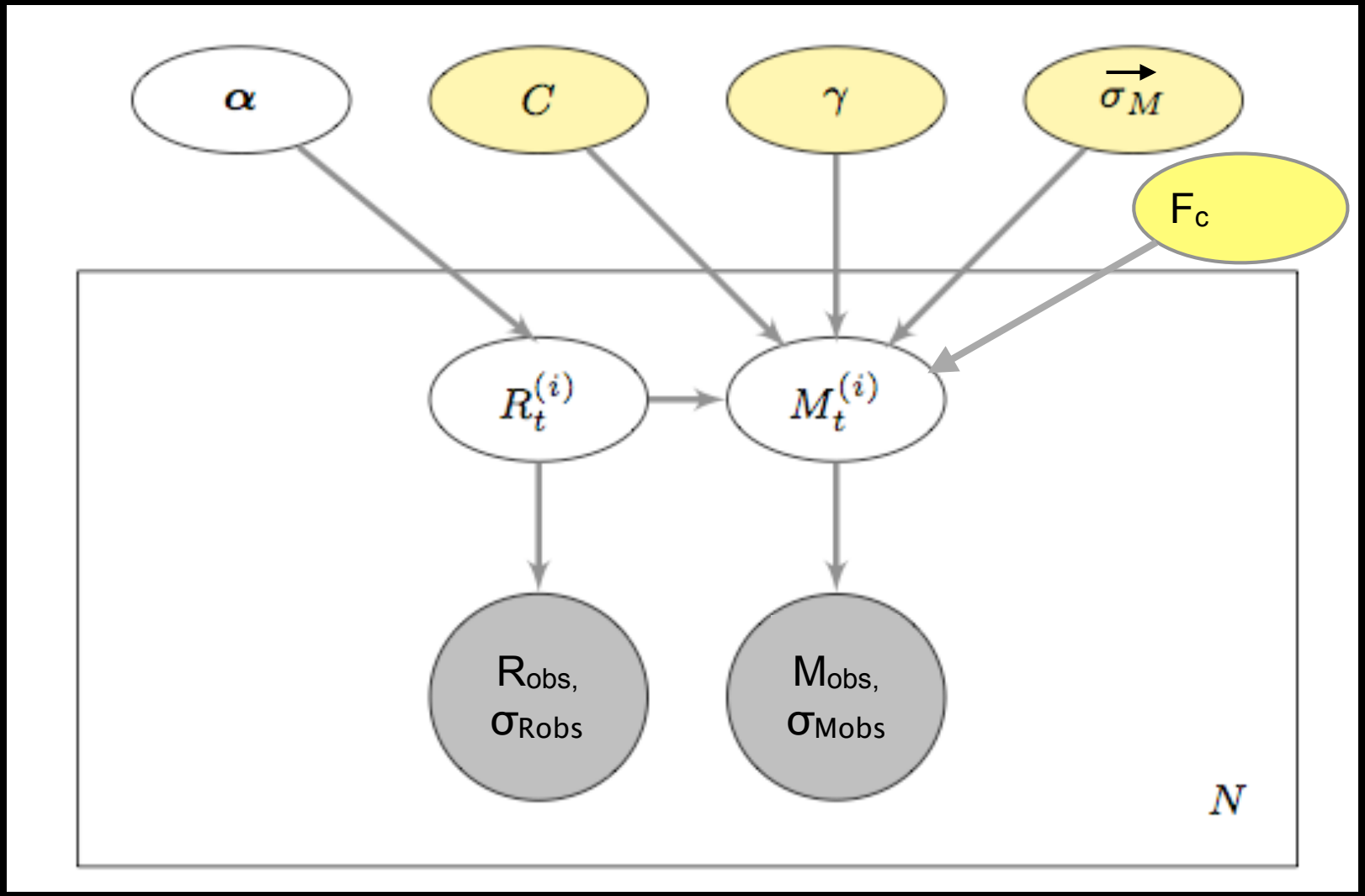


Eventually will want to incorporate this into hierarchical model . . .

M-R Relation by Flux

Start with two flux-dependent populations



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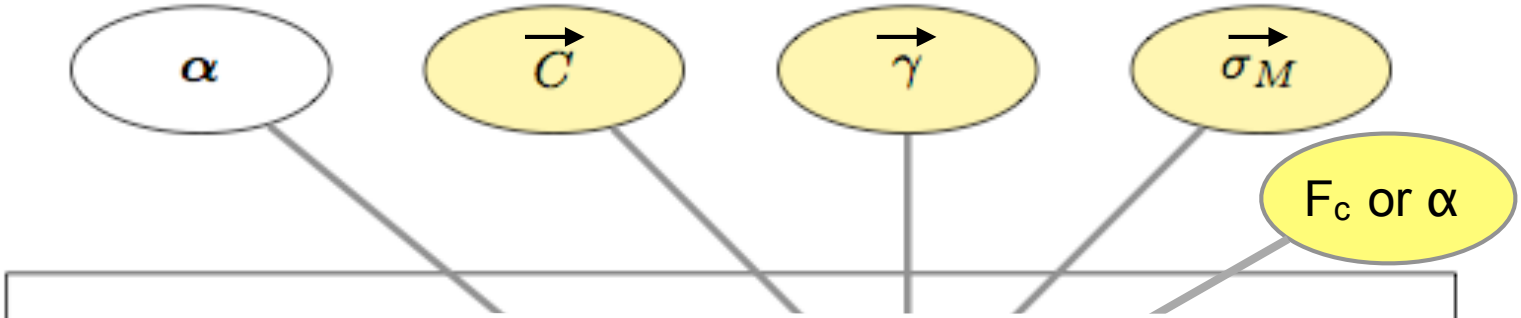

Hierarchical Model Comparison?!?!?!

R_{obs}, σ_{Robs}

M_{obs}, σ_{Mobs}

Mixture model with two components . . .

Mixture model with N (unknown) components …

Gaussian process instead of power law . . .

Summary

Hierarchical modeling is **necessary for exoplanet demographics**; opportunity for quantitative constraints on planet formation theory.

Much work to be done to **understand biases** in M-R relation and how we can **characterize** its **multi-dimensional nature**.

Hierarchical model comparison is needed to guide physical understanding of the population.