36-220 Lab#4
Continuous Random Variables

Week of September 26, 2005

Please write your name below, tear off this front page and give it to a teaching assistant as you leave the lab. It will be a record of your participation in the lab. Please remember to include your section (A, B or C). Keep the rest of your lab write-up as a reference for doing homework and studying for exams.

Name:

Section:

- The symbol ♣ at the beginning of a question means that, after you answer that question, you should raise your hand and have either the TA or lab assistant review your answer. Once they have reviewed your work they will place a check in the appropriate space in the table below. The purpose of this check is to be sure you have answered the question correctly.
- Try to complete as much of the lab exercise as possible. We understand that students work at different paces and have tried to structure the exercise so that it can be completed in the allotted time. If you work systematically through the handout and still don't complete every question don't worry. The important thing is that you understand what you are doing. Nonetheless, you are encouraged to complete the lab on your own.

Check-Problem ♣	Instructor's Initials
Question 5	
Question 10	
Question 13	

36-220 Lab #4
Continuous Probability Distributions

1. We are going to generate random data from the exponential distribution. To do this select **Calc** > **Random Data** > **Exponential**. In the box that appears, indicate that you want to generate 200 rows of data and store the results in column C1. You now have 200 observations in column 1. Each observation is an independent random draw from a exponential distribution with mean $\frac{1}{\lambda} = 1$.

   Label the first column by typing *Exp1* in the cell at the top of the column.

   Repeat this process to generate 200 data from an exponential distribution with mean $\frac{1}{\lambda} = 10$, and store these values in column C2. Label this column *Exp10*.

   Compare the distributions of the two variables by creating histograms. Select **Graph** > **Histogram**. In the box that appears, double-click on *Exp1*, then on *Exp10*. You should see the variable names appear in the grid on the right to indicate they will be graphed.

   Question #1: In the space below, sketch the distribution of each of the two variables.

   Question #2: Describe the distribution of *Exp1*. Compare it to the distribution of *Exp10*. Comment on the location and the spread of both of the two variables.

   Be sure to keep these graphs open. We will refer back to them later.

   Question #3: Using the formulae provided in class, calculate (by hand) the population mean and standard deviation for each of the two variables.

In Question #3, you calculated the population mean and standard deviation based on your knowledge of λ. Now let's compare the population values to the means and standard deviations of your two samples. Select **Stat** > **Basic Statistics** > **Display Descriptive Statistics**. Select *Exp1* and *Exp10*. Click **OK**.

Question #4: How do the sample means and sample standard deviations compare to their population counterparts?

♣ Question #5: Repeat the above steps with a sample size of 1000. How do the sample means compare to the population means for the sample size of 1000 versus that for 200?

2. A system with an exponential life time has a constant failure rate — the system deteriorates at a constant rate. This is common for systems that undergo regular preventative maintenance. However, a more general life time distribution, that allows for a varying failure rate is called the Weibull distribution. It has two positive parameters: a shape parameter γ and a scale parameter θ. The Weibull density and cumulative distributions are given by:

$$f(y) = \begin{cases} \frac{\gamma}{\theta} y^{\gamma-1} e^{-\frac{y^\gamma}{\theta}} & y > 0, \gamma > 0, \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$F(y) = \begin{cases} 1 - e^{-\frac{y^\gamma}{\theta}} & y > 0, \gamma > 0, \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

To see what kind of density shapes this distribution can obtain, you will generate a few samples from a Weibull distribution.

Generate 3 columns of Weibull data, each including 200 rows. Label them *Weibull_1,1* (shape parameter 1 and scale parameter 1), *Weibull_1,10* (shape parameter 1 and scale parameter 10) and *Weibull_10,10* (shape parameter 10 and scale parameter 10).

Let's compare the distributions of the two variables by creating histograms. Select **Graph** > **Histogram**. In the box that appears, double-click on

Weibull1_1, then on *Weibull1_10*, and then *Weibull10_10*. You should see the variable names appear in the grid on the right to indicate they will be graphed.

Question #6: Compare the first two histograms. What does the scale parameter seem to influence? (hint: look at the horizontal scale of the graph)

Question #7: Compare the last two histograms. What does the shape parameter seem to influence?

Question #8: Compare the first two histograms with the ones you had for the exponential data. Now look at the formulas for the exponential and Weibull densities. How is the exponential distribution a special case of the Weibull distribution?

Question #9: To learn about the failure rate of a Weibull variable, compute the Weibull *hazard function*, which is $h(t) = \frac{f(t)}{1-F(t)}$, where $F(t)$ is the cdf. The hazard function can be interpreted as follows: Given that the system has not failed before time t, the probability it fails during the period $(t, t+c)$ is approximately $c \times h(t)$, when c is small.

♣ **Question #10: Draw three possible shapes that the Weibull hazard function can assume (Hint: start by considering $\gamma = 1$). In what situations might these functions be appropriate?**

3. In some systems, a customer is allocated to one of two service facilities. If the service time for a customer served by facility i has an exponential distribution with parameter $\lambda_i (i = 1, 2)$ and p is the proportion of all customers served by facility 1, then the pdf of X=the service time of a randomly selected customer is

$$f(x; \lambda_1, \lambda_2, p) = \begin{cases} p\lambda_1 e^{-\lambda_1 x} + (1-p)\lambda_2 e^{-\lambda_2 x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

This often called the mixed exponential distribution.

Question #11: Verify that $f(x; \lambda_1, \lambda_2, p)$ **is a pdf.**

Question #12: Find the cdf, $F(x; \lambda_1, \lambda_2, p)$**.**

♣ **Question #13: Compute** $\mathbf{E}[X]$**.**