Midterm Examination, Statistics 220, Fall 2005

19 October 2005

Name:

- You have 50 minutes.
- You can use your own cheat-sheet and your own calculator. Please don't share anything.
- Show your work.
- Ask us if you need extra scratch-paper.

Exercise 1	/10
Exercise 2	/5
Exercise 3	/5
Exercise 4	/10
Problem 5	/20
Problem 6	/20
Problem 7	/30
TOTAL	/100

1 Binomial vs. Poisson

X is a binomial random variable, $n = 10$, $p = 0.1$. Y is a Poisson random variable, $\lambda = 1$.
(a) What is $\mathbf{E}[X]$? (1 pt) What is $\mathrm{Var}(X)$? (1 pt) What is the probability that $X = 2$? (3 pts)
(b) What is $\mathbf{E}[Y]$? (1 pt) What is $\mathrm{Var}(Y)$? (1 pt) What is the probability that $Y = 2$? (5 pts)

2 Camels

Bactrian camels (*Camelus bactrianus*) are used as beasts of burden in Central Asia. A survey of the camels available in a certain town in Afghanistan shows that their mean weight is 1600 pounds, with a standard deviation of 150 pounds.
(a) What are the mean and standard deviation of the camels' weights in kilograms? (2 pts) *Hint*: 1 kg = 2.2 lb.
(b) The camels are now loaded with water casks weighing exactly 120 kilograms each. What is the mean and standard deviation of their total weight? (3 pts)

3 Coins

(a) What are the probabilities of getting the following sequences of heads and tails from 10 consecutive tosses of a fair coin? (2 pts)
(i) HHHHHHHHHH
(ii) HTHTHTHTHT
(iii) HHTTTTTTHH
(b) What is the probability of getting 0 heads in 10 consecutive tosses? 5 headsout of 10? 6 out of 10? (3 pts)

4 Dice

A fair die is one where all six faces are equally likely.
(a) Let X = the number that comes up on a fair die. What is $\mathbf{E}[X]$? (5 pts)
(b) Let X_1, X_2 and X_3 be three independent fair die, and $S = X_1 + X_2 + X_3$. What is $\mathbf{E}[S]$? (2 pts)
(c) What is $\Pr(S = 18)$? (3 pts)

5 A Continuous Random Variable

Consider a continuous random variable X which lies between -1 and $+1$. The density of X is proportional to $1 - x^2$.
(a) What is the density of X? (10 pts)
(b) What is the cumulative distribution function of X? (5 pts)
(c) Find the expectation of X. (5 pts) *Hint*: you do not need to integrate to do part (c).

6 Quality Control Charts

Your job is to monitor the concentration of xylene in the output of a municipal water-treatment plant. When the process is in control, the mean concentration of xylene is 6 parts per million, and the standard deviation per measurement is also 6 parts per million. Because measurements are small and cheap, you take $n = 36$ independent measurements every day.

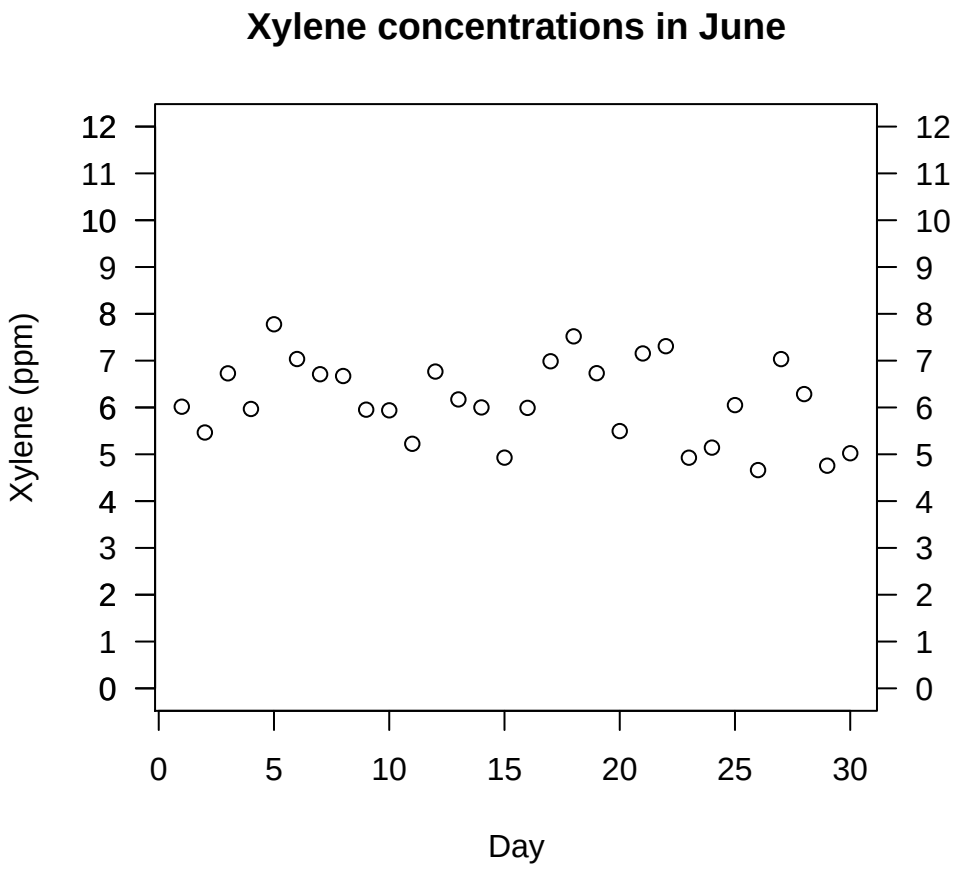
(a) What is the standard error (standard deviation of the sample mean)? (5 pts.)

(b) What are the upper and lower control limits (UCL and LCL), at the usual "three sigma" level? (5 pts)

(c) The figure (next page) shows the daily sample mean concentrations of xylene over a month. Is the process in control? (5 pts)

(d) Is the distribution of the sample mean Gaussian? Explain. (5 pts)

problem 6 (continued)



7 Quality Control and False Alarms

The output of an fully-functioning power plant has a Gaussian distribution, with a mean of 10 MW and a standard deviation of 0.5 MW. Upper and lower control limits are established as 11.0 MW and 9.0 MW respectively. If a certain component fails, the mean output drops to 9.0 MW, without changing the standard deviation.
(a) What is the probability that the output lies between the control limits, when the power plant is working properly? (10 pts)
(b) What is the probability that the output lies between the control limits during one of the partial failures? (10 pts)
(c) If the power plant works properly 99% of the time, what is the probability that going outside the control limits means a component has failed? (10 pts) *Hint 1*: Use Bayes's rule. *Hint 2*: What fraction of the time is the output outside the control limits?

SELECTED VALUES OF THE STANDARD GAUSSIAN CDF

$\Phi(z) = \Pr(Z \leq z)$ when $Z \sim \mathcal{N}(0,1)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998

NOTE: $\Phi(-z) = 1 - \Phi(z)$.