Assignment 2

36-462, Spring 2008

Due 29 February 2008

You can either do questions 4 and 5, or question 6. If you want to do all three, 6 will be extra credit.

When a problem asks you to do a simulation or write a program, include your code in your answer.

1. BASIC PROPERTIES OF ENTROPY

   (a) Prove that the entropy of a random variable is non-negative.

   (b) If the number of values X can take on is m, prove that $H[X] \leq \log_2 m$, and the maximum is attained just when every possibility is equally likely. (*Hint*: One way to do this is to express the probabilities as $p_i = \frac{1}{m} + \delta_i$, and use the fact that the sum of the p_i must be one.)

2. THE $r = 1$ LOGISTIC MAP AS IID COIN-TOSSING Write a program to simulate the symbolic dynamics of the logistic map with $r = 1$. Tabulate the frequencies of sub-sequences of length $2L$. Test whether X_t^{t+L-1} is independent of X_{t-L}^{t-1}. Explain how you decided how long the simulation should be.

3. TOPOLOGICAL ENTROPY RATE ESTIMATION

   (a) Write a program to calculate the topological entropy rate for the logistic map at any r.

   (b) How would you put a standard error on your estimate of h_0?

4. MARKOV CHAIN ESTIMATION

   (a) Write a program (`markov.mle.1`) to estimate the transition matrix of a binary-valued Markov chain by maximum likelihood.

   (b) Write a program (`markov.mle.k`) to estimate the transition matrix of a Markov chain of order k.

   (c) Write a program (`markov.mle.bic`) which uses BIC to select a k between 1 and $k_{\max}$. Your program should output the estimated k as well as the transition matrix.

5. CHAIN ESTIMATION, CONTINUED

   (a) Write a program to simulate the Foulkes process (lecture 6) for an arbitrary number of time-steps.

   (b) Use the output of this simulation as input to your `markov.mle.bic` program. How many time-steps must you use before the estimate k becomes 4? How many before all the transition probabilities are within 0.05 of the true probabilities?

6. JANET (extra credit; worth as much as previous two problems combined)

   (a) Read Foulkes's paper (on syllabus page, under lecture 6).

   (b) Write an implementation of JANET.

   (c) Show that your program reproduces Foulkes's results. If you cannot reproduce his results, how do you explain the differences?