# Chaos, Complexity, and Inference (36-462)

## Lecture 10

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Some things you can read:

[1] is what got me interested in the subject;

[2] is the best introduction to CA modeling

code examples are for obsolete mid-1980s hardware

*principles* apply

[3] has nice chapter on CAs

[4, 5]: easier to read the more physics you know

[6, 7, 8]: important paper collections

## Cellular Automata

Completely discrete, spatially-extended dynamical systems
Time: discrete
Space: divided into discrete **cells**
Each cell is in one of a finite number of **states**, a.k.a. colors
Global **configuration**: the states of all cells at one time
Every cell has a **neighborhood**

includes cell itself

**von Neumann neighborhood**: cardinal directions (NSEW)

**Moore neighborhood**: diagonals too

possibly of range $> 1$

# The dynamics

**Local rule**: Given the states of the neighborhood at time $t$, what is the state of the cell at $t+1$?

simultaneous vs. random-order updating

Local rule implies a *global* rule for the configuration

Rules can be stochastic!

Globally, always have a Markov chain

Locally, have a **dynamic Markov random field**

# Majority Vote

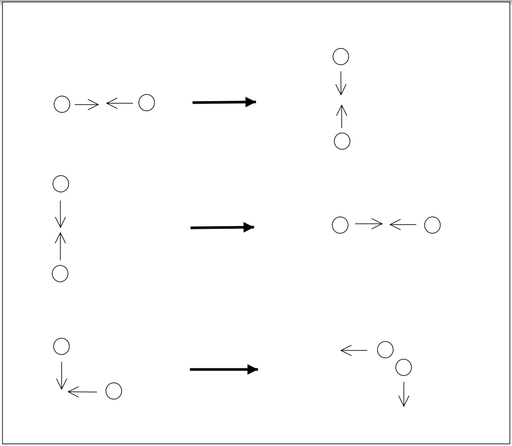
"Majority vote": change to match the majority of the neighborhood (including self)
Magnetism, strains of organisms, conventions, ...
Start from near-50-50 initial conditions
Quickly: solid-color regions form
More slowly: they unbend; "motion by mean curvature"
Variants:

- higher or lower thresholds
- probability of flipping depending on proportion of neighbors
- copy a random neighbor ("voter model")

Difference between finite-size behavior and infinite-size limit

## Lattice Gases

[9, 10, 11]
Models of fluid flow
Fluid consists of discrete particles moving with discrete velocities
State of each cell is number of particles with different velocities there (possibly none)
"atoms and void"
Collision rules implement basic physics: conservation of momentum, conservation of energy, conservation of mass
Hydrodynamics are emergent consequences of local interactions, looking at a large scale and averaging



Collision rules for “HPP” lattice gas [12], gets some hydrodynamics right but isn’t isotropic (rotationally symmetric) — need a non-rectangular lattice [13]

## Game of Life

2D, binary, Moore neighborhood
$<$ 2 neighbors on: turn off
$=$ 2 neighbors on: stay the same
$=$ 3 neighbors on: turn on
$>$ 3 neighbors on: turn off
some common life patterns: gliders, blinkers, beehives, ...
build a computer: use glider streams to represent bits, then build logic gates
build a universal computer by attaching memory

Can build machines which construct new configurations according to plans
These can reproduce themselves and copy their plans: mechanical (if mathematical) self-reproduction
This is actually the historical origin of cellular automata [14, 6]
[1] is a wonderful account

## Diffusion-Limited Aggregation

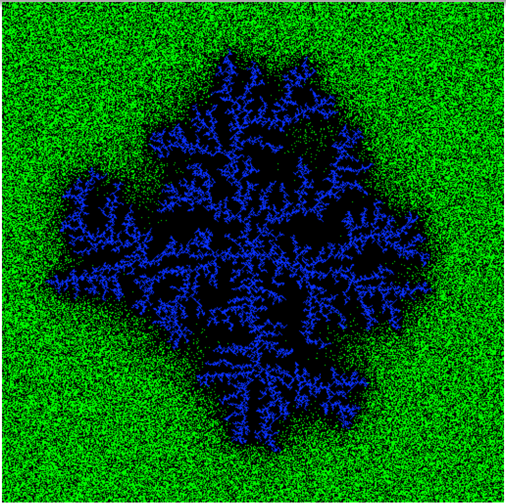
Particles arrive at boundary, move in random walks

some trickery needed to get a random walk in a CA

Particles which hit crystal freeze and become crystal

Models aggregation, freezing

from Wikipedia



me, me, me!

Andy Goldsworthy

# Learning CA rules

Known neighborhood, deterministic rule: wait and see.
Known neighborhood, stochastic rule: counting (as with Markov chain)
Unknown neighborhood: search over neighborhoods to maximize mutual information between neighbors' states and new state [15]
Huge number of neighborhoods, use fancy optimization techniques
Could also try: minimize conditional MI between new state and rest of configuration given neighborhood

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[3] Nino Boccara. *Modeling Complex Systems*. Springer-Verlag, Berlin, 2004.

[4] Bastien Chopard and Michel Droz. *Cellular Automata Modeling of Physical Systems*. Cambridge University Press, Cambridge, England, 1998.

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[6] Arthur W. Burks, editor. *Essays on Cellular Automata*, Urbana, 1970. University of Illinois Press.

[7] Howard Gutowitz, editor. *Cellular Automata: Theory and Experiment*, Cambridge, Massachusetts, 1991. MIT Press. Also published as *Physica D* **45** (1990), nos. 1–3.

[8] David Griffeath and Cristopher Moore, editors. *New Constructions in Cellular Automata*, Oxford, 2003. Oxford University Press.

[9] Gary D. Doolen, editor. *Lattice Gas Methods for Partial Differential Equations: A Volume of Lattice Gas Reprints and Articles*, Reading, Massachusetts, 1989. Adidson-Wesley.

[10] Gary D. Doolen, editor. *Lattice Gas Methods: Theory, Applications, and Hardware*, Cambridge, Massachusetts, 1991. MIT Press. Also published as *Physica D* **47 (1–2)**, 1991.

[11] Daniel H. Rothman and Stéphane Zaleski. *Lattice-Gas Cellular Automata: Simple Models of Complex*

*Hydrodynamics*. Cambridge University Press, Cambridge, England, 1997.

[12] J. Hardy, Y. Pomeau, and O. de Pazzis. Molecular dynamics of a classical lattice gas: Transport properties and time correlation functions. *Physical Review A*, 13: 1949–1960, 1976.

[13] Uriel Frisch, Boris Hasslacher, and Yves Pomeau. Lattice-gas automata for the Navier-Stokes equation. *Physical Review Letters*, 56:1505–1508, 1986.

[14] John von Neumann. *Theory of Self-Reproducing Automata*. University of Illinois Press, Urbana, 1966. Edited and completed by Arthur W. Burks.

[15] Fred C. Richards, Thomas P. Meyer, and Norman H. Packard. Extracting cellular automaton rules directly from experimental data. *Physica D*, 45:189–202, 1990. Reprinted in [7].