Chaos, Complexity, and Inference (36-462)

Lecture 11

Cosma Shalizi

19 February 2008

Today: excitable media

1. Example of common real-world mechanism for pattern formation
2. Can be modeled by cellular automata
3. Can understand the CA in some analytical detail

Two truly excellent books to read if this is at all interesting: Winfree (1980, 1987)

Excitable Media

Some facts about heart tissue:

Muscle cells work by contraction
Contraction needs electrical/chemical stimulation
Contraction works by pumping some chemical out of the cell and others in
Contraction is followed by “refractory period” (reversing the chemical pump)
Refractory cells cannot be excited into contraction
After refractory period cells *can* be excited but do not *spontaneously* contract
Contracting cells produce electrical pulses which stimulate neighbors
The heart is a closed manifold
Pacemaker node
Network of special fibers for conducting stimulation (“anastomosis”)

Wiener and Roseblueth (1946)

regular contractions, spiral waves, scroll waves

Abstracting the mechanism

Three basic sorts of behavior:

- **Quiescent** Standing around waiting; can be excited
- **Excited** Doing something right now
- **Refractory** Recovering from having been excited, cannot be excited but does return to quiescence eventually

Assumption: excitation *spreads*, somehow, *locally*
What this leaves out: nature of excitation, mechanism of spread, geometry of heart, change of geometry due to contraction, nature of quiescence, pacemaker, conducting fibers
"Excitable medium" is an **abstraction** and a **mechanism**

Abstraction

Selects out certain features, hides others as details

math, software engineering

Promises — if the abstraction is valid — that it somehow implements those features, that behavior, can count on it to obey those rules

Requires that you not rely on any of the details of *how* they are implemented, or even get to know what they are

("restricted interface", "handles" (Krieger, 1992))

Emergent macroscopic variables are abstractions

Finding good abstractions is difficult

In the social world, institutions work hard to *make* abstractions valid (Stinchcombe, 2001)

This is essentially *mathematical* thinking (Kitcher, 1983)

> *The first things I found out were that all mathematical reasoning is diagrammatic and that all necessary reasoning is mathematical reasoning, no matter how simple it may be. By diagrammatic reasoning, I mean reasoning which constructs a diagram according to a precept expressed in general terms, performs experiments upon this diagram, notes their results, assures itself that similar experiments performed upon any diagram constructed according to the same precept would have the same results, and expresses this in general terms.*
>
> *— C. S. Peirce (quoted in Haaparanta (2001))*

Mechanism

recurring, recognizable causal pattern, sometimes but not universally valid (Hedström and Swedberg, 1998; Hedström, 2005)

like a "pattern" in software engineering

See Salmon (1984); Giere (1988); Kitcher (1993); DeLanda (2006) for interesting (and useful!) philosophy on mechanisms, explanations, abstractions

Other Exciting Things

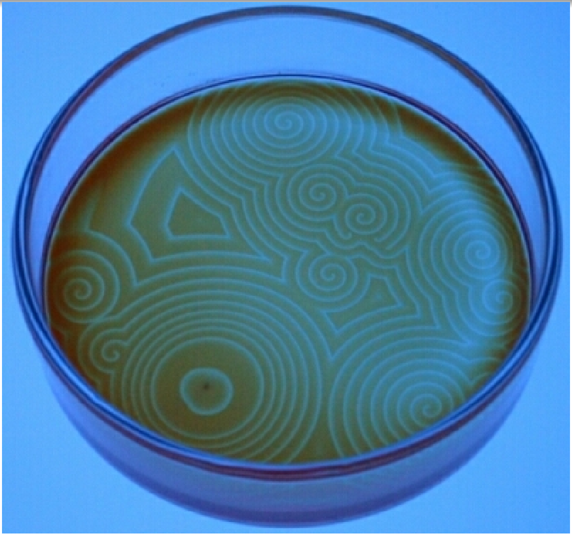
Because mechanism is abstract can think about applying it to other assemblages
need to check that it works!
neural tissue, epilepsy — neurons are also excitable cells; spreading waves of excitation are Bad Things (seizures)
chemical oscillators
slime molds
Martian ice canyons

Chemical Oscillators: Belousov-Zhabotinsky Type

A ridiculously crude caricature:

$$\begin{aligned} A &\rightarrow B \\ A + B &\rightarrow 2B \\ B + C &\rightarrow 2C \\ C &\rightarrow D \end{aligned}$$

B builds itself up from A autocatalytically
C does the same thing to A
C turns to D
ultimately reaction runs $A \rightarrow D$
autocatalytic build-up: excitation
diffusion spreads excitation
quiescence: waiting for substrate chemicals to diffuse back in



Chemical Oscillators: Turing Type

A different mechanism

$$\begin{aligned} A &\rightarrow A + I \\ I + A &\rightarrow D \end{aligned}$$

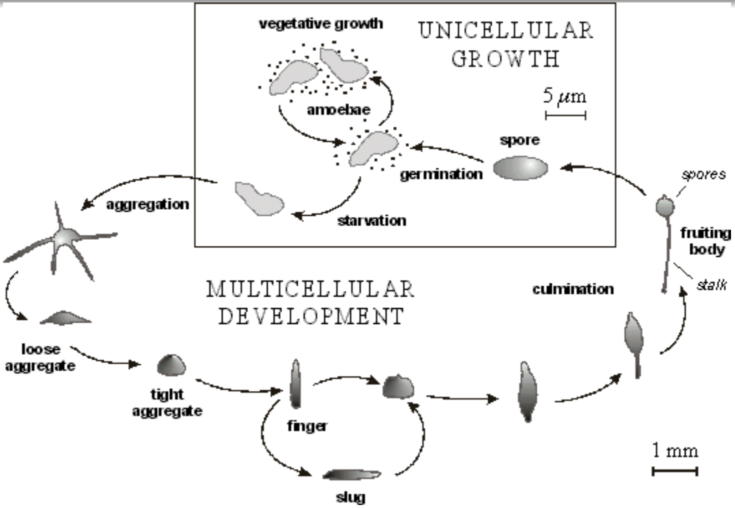
A makes more of itself and I
I makes A inert
Assume I diffuses faster than A
local activation, long-range inhibition
proposed by Turing for plants, later proposed for animals
doesn't seem to work like that
but can be realized chemically
Resulting patterns look *very similar* to excitable medium but it's not

Dictyostelium discoideum

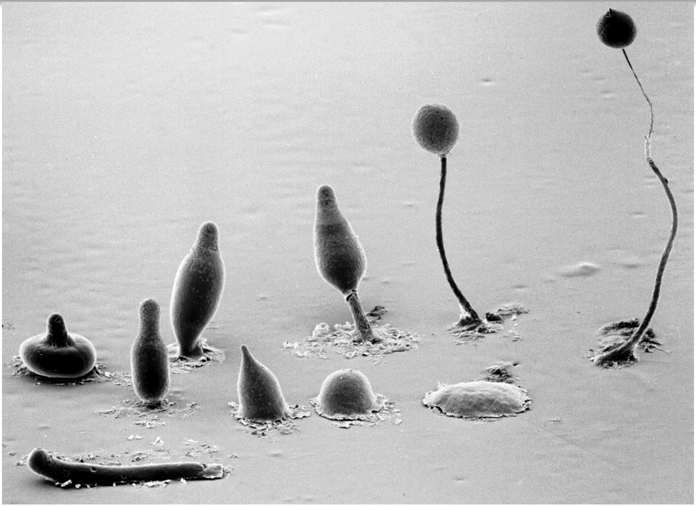
Cellular slime molds (Bonner, 1967, 1974, 1988)
See Johnson (2001) for nice account of the science and its history here
Life cycle:

- **Free-living** amoebae, crawl around eating bacteria, fission
- **Aggregation** response to stress
- **Slug, "grex"** Forms single mass, differentiated, responsive, crawls uphill
- **Fruiting body** Differentiates into stalk/spores, latter disperse

Mathematical modeling of aggregation goes back to Keller and Segel (1970)
Marée and Hogeweg (2001) is first model (CA-based) for full cycle



From Wikipedia, s.v. “Dictyostelid”

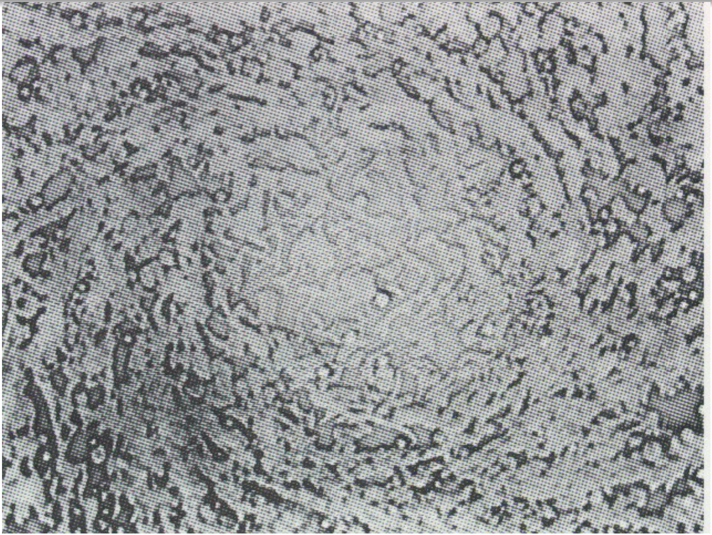


Development stages; copyright, M. J. Grimson & R.L. Blanton, Biological Sciences Electron Microscopy Laboratory, Texas Tech University; via dictybase.org

Aggregation: the amoeba

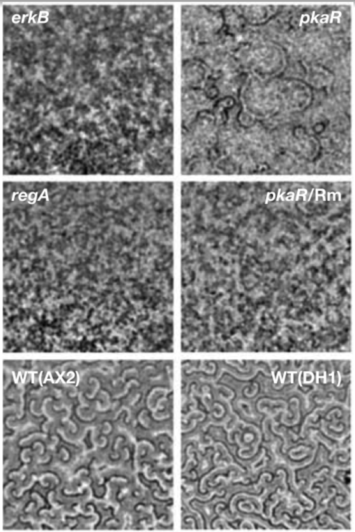
1. senses messenger chemical (cAMP) by receptor protein at cell wall (stimulation)
2. extends pseudopod in direction of chemical, crawls (excitation)
3. emits its own pulse of cAMP (spreading)
4. de-activates receptor (refractory)

messenger identified as cAMP in Konijn *et al.* (1967), just after Bonner (1967)
Someone has to start it; generally response to hunger
Mutations in the genes for these proteins reliably affect pattern formation (Sawai *et al.*, 2005)



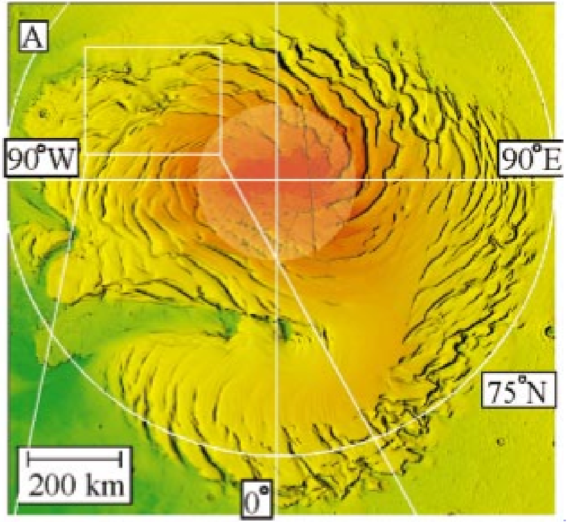
www.uni-magdeburg.de/abp/picturegallery.htm

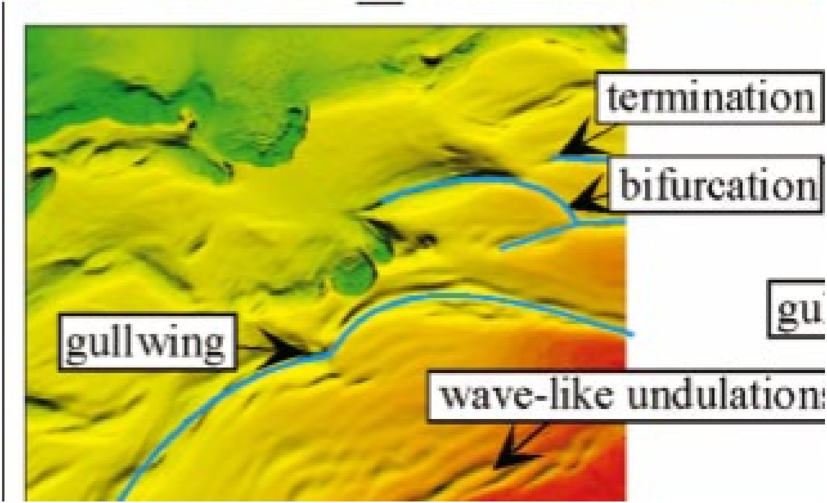
www.uni-magdeburg.de/abp/picturegallery.htm



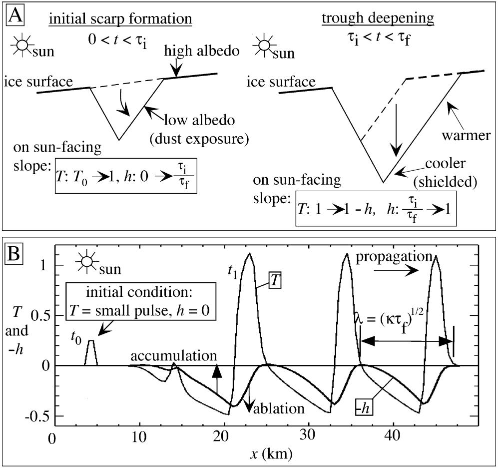
Mutants (top 2 rows), wild type (bottom row) Sawai *et al.* (2005)

Mars: Polar Ice Canyons (Pelletier, 2004)





closeup (Pelletier, 2004)



h = depth, T = temperature (Pelletier, 2004)

ice melts on sunny side, condenses on shady side, hole gets deeper and steeper

City Formation

Where to put your shop/factory/house?

after Krugman (1996); Fujita *et al.* (1999)

but see also Page (1998); Henderson *et al.* (2001)

Near others (positive feedbacks): share supplies/infrastructure/resources, customers, short travel times

Away from others (negative feedbacks): avoid competition, "nobody goes there any more, it's too crowded" (land prices)

Local activation (positive feedback), long-range inhibition (negative feedback)

Chance concentrations will grow, but also inhibit growth of close competitors

principles also apply within cities

see Rietkerk *et al.* (2004) for *ecological* cases

Tysons Corner, VA, *circa* 1962: One corner, gas station
Lots of cross-roads like this in northern VA near DC

Tysons Corner, VA, *circa* 2006
2nd largest single retail center on east coast (after NYC)

http://www.washingtonairports.com/service/Healthy%20Economy.pdf

Greenberg-Hastings CA

Greenberg and Hastings (1978)
$\kappa \geq 3$ states, radius of rule ρ
state 0: resting/quiescent
state 1: excited
state 2–$\kappa - 1$: refractory
$0 \rightarrow 1$ if $\geq \theta$ neighbors are 1
$k \rightarrow k + 1 \bmod \kappa$, automatically, if $1 \leq k \leq \kappa - 1$
$\kappa - 2 =$ length of refractory period

Cyclic CA

Fisch *et al.* (1991a,b)
Get rid of absolute refractory period, make model more symmetric
$\kappa \geq 3$ states, radius of rule ρ
$k \rightarrow k + 1 \mod \kappa$ if $\geq \theta$ neighbors are $k + 1 \mod \kappa$
each color "eats" the one before it in the cycle
think of a closed cycle of autocatalytic reactions

The Basic CCA Story

- **Debris** regions where nothing will change (immediately)
- **Droplets** region with active dynamics
- **Defects** points where phase is discontinuous, **organizing centers** of spirals
- **Demons** minimum-period spirals

Debris shrinks at the expense of droplets, droplets are organized by defects, demons extend their tentacles and cover everything

Winding Number

Suppose we have a 2D curve around the origin ℓ
winding number $w(\ell)$: how many times it goes around origin counter-clockwise (net)
in calculus:

$$\begin{aligned} w(\ell) &\equiv \int_\ell d\theta \text{ (polar)} = \frac{1}{2\pi}\int_\ell \frac{x}{x^2+y^2}dx - \frac{y}{x^2+y^2}dy \text{ (Cartesian)} \\ &= \frac{1}{2\pi i}\int_\ell \frac{dz}{z} \text{ (complex)} \end{aligned}$$

CCA: we can't draw continuous curves so add up discrete phase changes around loop
Winding number is conserved by the CCA rule (inside droplets)

The Forms of Limiting Behavior

Extinction All cells quiescent. In GH if too few excitations; not possible for CCA

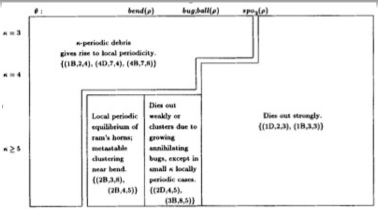
Fixation Configuration at fixed point; for CCA by “bootstrap percolation”

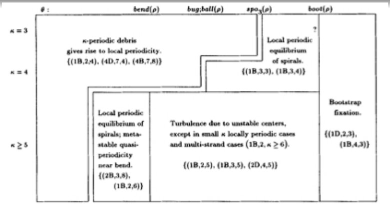
Periodicity: incoherent little spatial order

Periodicity: spirals

Turbulence

[simulation]





From Fisch *et al.* (1991b)

phase diagrams — qualitative behavior as a function of control settings (ρ, θ, κ)

Sharp transitions between different phases *in infinite size limit*, **metastability** at finite sizes

Bonner, John Tyler (1974). *On Development: The Biology of Form*. Cambridge, Massachusetts: Harvard University Press.

— (1988). *The Evolution of Complexity, by Means of Natural Selection*. Princeton, New Jersey: Princeton University Press.

Bonner, Joyn Tyler (1967). *The Cellular Slime Molds*. Princeton, New Jersey: Princeton University Press, 2nd edn.

DeLanda, Manuel (2006). *A New Philosophy of Society: Assemblage Theory and Social Complexity*. London: Continuum.

Fisch, Robert, Janko Gravner and David Griffeath (1991a). "Cyclic Cellular Automata in Two Dimensions." In *Spatial Stochastic Processes: A Festschrift in Honor of Ted Harris on His Seventieth Birthday* (Kenneth Alexander and Joseph Watkins, eds.), pp. 171–188. Boston: Birkhäuser. URL http://psoup.math.wisc.edu/papers/cca.zip.

— (1991b). "Threshold-Range Scaling of Excitable Cellular Automata." *Statistics and Computing*, **1**: 23–39. URL http://psoup.math.wisc.edu/papers/tr.zip.

Fujita, Masahisa, Paul Krugman and Anthony J. Venables (1999). *The Spatial Economy: Cities, Regions, and International Trade*. Cambridge, Massachusetts: MIT Press.

Giere, Ronald N. (1988). *Explaining Science: A Cognitive Approach*. Chicago: University of Chicago Press.

Greenberg, J. and S. Hastings (1978). "Spatial Patterns for Discrete Models of Diffusion in Excitable Media." *SIAM Journal on Applied Mathematics*, **34**: 515–523. URL http://links.jstor.org/sici?sici=0036-1399%28197805%2934%3A3%3C515%3ASPFDMO%3E2.0.CO%3B2-J.

Haaparanta, Leila (2001). "On Peirce's Methodology of Logic and Philosophy." In *Explanatory Connections: Electronic*

Essays Dedicated to Matti Sintonen (Mika Kiikeri and Petri Ylikoski, eds.). Helsinki. URL `http://www.valt.helsinki.fi/kfil/matti/haaparanta.pdf`.

Hedström, Peter (2005). *Dissecting the Social: On the Principles of Analytical Sociology*. Cambridge, England: Cambridge University Press.

Hedström, Peter and Richard Swedberg (eds.) (1998). *Social Mechanisms: An Analytical Approach to Social Theory*, Studies in Rationality and Social Change, Cambridge, England. Cambridge University Press.

Henderson, J. Vernon, Zmarak Shalizi and Anthony J. Venables (2001). "Geography and Development." *Journal of Economic Geography*, **1**: 81–105. URL `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=632526`. doi:10.1093/jeg/1.1.81.

Johnson, Steven (2001). *Emergence: The Connected Lives of Ants, Brains, Cities, and Software*. New York: Scribner.

Keller, Evelyn Fox and Lee Segel (1970). "Initiation of slime mold aggregation viewed as an instability." *Journal of Theoretical Biology*, **26**: 399–415.

Kitcher, Philip (1983). *The Nature of Mathematical Knowledge*. Oxford: Oxford University Press.

— (1993). *The Advancement of Science: Science without Legend, Objectivity without Illusions*. Oxford: Oxford University Press.

Konijn, Theo M., J. G. C. van de Meene, John T. Bonner and David S. Barkley (1967). "The Acrasin Activity of adenosine-3′, 5′-cyclic Phosphate." *Proceedings of the National Academy of Sciences (USA)*, **58**: 1152–1154. URL `http://www.pnas.org/cgi/reprint/58/3/1152.pdf`.

Krieger, Martin H. (1992). *Doing Physics: How Physicists Take Hold of the World*. Bloomington: Indiana University Press.

Krugman, Paul R. (1996). *The Self-Organizing Economy*. Oxford: Blackwell.

Marée, Athanasius F. M. and Paulien Hogeweg (2001). "How amoeboids self-organize into a fruiting body: Multicellular coordination in *Dictyostelium discoideum*." *Proceedings of the National Academy of Sciences (USA)*, **98**: 3879–3883. URL http://www.pnas.org/cgi/content/abstract/98/7/3879.

Page, Scott E. (1998). "On the Emergence of Cities." *Journal of Urban Economics*, **45**: 184–208. URL http://www.santafe.edu/research/publications/wpabstract/199808075.

Pelletier, Jon D. (2004). "How Do Spiral Troughs Form on Mars?" *Geology*, **32**: 365–367. doi:10.1130/G20228.2.

Rietkerk, Max, Stefan C. Dekker, Peter C. de Ruiter and Johan van de Koppel (2004). "Self-Organized Patchiness and Catastrophic Shifts in Ecosystems." *Science*, **305**: 1926–1929. doi:10.1126/science.1101867.

Salmon, Wesley C. (1984). *Scientific Explanation and the Causal Structure of the World*. Princeton: Princeton University Press.

Sawai, Satoshi, Peter A. Thomason and Edward C. Cox (2005). "An autoregulatory circuit for long-range self-organization in *Dictyostelium* cell populations." *Nature*, **433**: 323–326. doi:10.1038/nature03228.

Stinchcombe, Arthur L. (2001). *When Formality Works: Authority and Abstraction in Law and Organizations*. Chicago: University of Chicago Press.

Wiener, Norbert (1974). *Collected Works: with commentaries*, vol. 10 of *Mathematicians of Our Time*. Cambridge, Massachusetts: MIT Press.

Wiener, Norbert and Arturo Roseblueth (1946). "The mathematical formulation of the problem of conduction of impulses in a network of connected excitable elements, specifically in cardiac muscle." *Arch. Inst. Cardiologia de Mexico*, **16**: 205–265. Reprinted in (Wiener, 1974, vol. 4).

Winfree, Arthur T. (1980). *The Geometry of Biological Time*. Berlin: Springer-Verlag.

— (1987). *When Time Breaks Down: The Three-Dimensional Dynamics of Electrochemical Waves and Cardiac Arrhythmias*. Princeton: Princeton University Press.