Chaos, Complexity, and Inference (36-462)

Lecture 22

Cosma Shalizi

8 April 2008

Agents and Agent-Based Modeling

What Is an Agent?

Why Agent-Based Models?

Building Up

Why Not?

Miller and Page (2007) is the best book I know on agent-based models

Epstein and Axtell (1996) was extremely (and deservedly) influential

Emphasizing an ecological example in this lecture; Grimm and Railsback (2005) is all about agent models for ecology

Sigmund (1996) may be the best book ever written about mathematical biology, including ecology

What Is an Agent?

Conceptually Stu Kauffman: *"An agent is a thing that does things to things."*

Mathematically One of a set of interacting partially-observable Markov processes (internal state; changes and/or is changed by other things with their own states)

Computationally Agents are objects representing *individuals*

"agent-based model" ≡ "individual-based model"

internal structure

+ methods for acting

+ limited interfaces to the internals (partial observability)

The Ideology

Build up interesting things from *interacting* simple things
Large-scale regularities should *emerge* from interactions
Or: if you can't *generate* it, your model is wrong
Modeling work goes into specifying objects, interfaces, interactions

The Advantages

Mechanistic Represents your hypothesis about the actual mechanism, works out consequences

Granularity/finite-size

Complicated interaction structures

Heterogeneity Different agent parameters, different types of agents, spatial/temporal variation, network structure. . .

Related to econometric distinction between **reduced form** models and **structural form** models

An Example: Dis-assembling an Aggregated Model

Ideas stolen from Flake (1998); Boccara (2004)

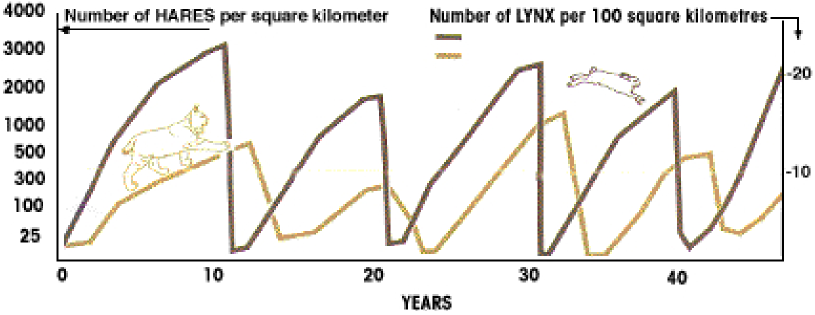
Starting point: **Lotka-Volterra model**

prey density, x_t; predator density, y_t

$$
\begin{aligned}
\frac{dx}{dt} &= x(\alpha - \beta y) \\
\frac{dy}{dt} &= -y(\gamma - \delta x)
\end{aligned}
$$

α: reproductive rate of prey
β: cost to prey of predation
γ: crowding of predators
δ: benefit to predators of predation

Generically generates a limit cycle, with prey leading predators by $\approx 1/4$ cycle



From

`http://www.srd.gov.ab.ca/fishwildlife/wildlifeinalberta/watchablewildlife/rabbitslargerodents/cycles.aspx`

What's Missing from the Lotka-Volterra Model?

Granularity

Noise

Space

Adaptation

Also: crowding term ($-y\gamma$) crucial, but imposed *ad hoc*
Begin to fix this by dis-aggregating the model

First cut individual-based model

X_t, Y_t = number of prey, predator
set per-individual probabilities of births and deaths so that

$$\frac{\mathbf{E}\left[\Delta X_t\right]}{\Delta t} = \frac{dx}{dt}$$

and make Δt small
THOUGHT EXERCISE: What should the probabilities be?
Gives a stochastic population model which $\rightarrow$ ODE model
classical theory of this convergence due to Kurtz (1970, 1971, 1981)

Still missing

Space

Adaptation

but are in a better position to add these

Crude Agent-Based Model

Each agent is either predator or prey
Agents have locations in space
Prey wander randomly
Predators move towards prey
Some probability of eating prey within hunting range
Predators reproduce if they eat
Prey reproduce if they don't get eaten
many choices to make specific here

With most *reasonable* choices, these give crowding *automatically*

because many predators locally $\Rightarrow$ fewer prey to go around

Behavior *like* Lotka-Volterra appears *approximately* for *aggregates*

Need to be careful here because real-world populations don't actually follow Lotka-Volterra!

Further Refinements

Terrain
Evolution on part of prey
Evolution on part of predators
Multiple predator/prey relationships
Harvesting

Limitations

- Have to make a lot of modeling decisions about individuals and interactions
- Many micro models lead to very similar macro consequences
- Macro, aggregated data often much easier to get hold of

Not really limitations

- Where's the theory?
  In the model!
- Lack of closed form/analytic solutions
  So?
- Difficulty of statistics
  Be clever, and/or use indirect inference

Boccara, Nino (2004). *Modeling Complex Systems*. Berlin: Springer-Verlag.

Epstein, Joshua M. and Robert Axtell (1996). *Growing Artificial Societies: Social Science from the Bottom Up*. Cambridge, Massachusetts: MIT Press.

Flake, Gary William (1998). *The Computational Beauty of Nature: Computer Explorations of Fractals, Chaos, Complex Systems and Adaptation*. Cambridge, Massachusetts: MIT Press.

Grimm, Volker and Steven F. Railsback (2005). *Individual-based Modeling and Ecology*. Princeton Studies in Theoretical and Computational Biology. Princeton, New Jersey: Princeton University Press.

Kurtz, Thomas G. (1970). "Solutions of Ordinary Differential Equations as Limits of Pure Jump Markov Processes." *Journal of Applied Probability*, **7**: 49–58. URL http://

`links.jstor.org/sici?sici=0021-9002%28197004%297%3A1%3C49%3ASOODEA%3E2.0.CO%3B2-P`.

— (1971). "Limit Theorems for Sequences of Jump Markov Processes Approximating Ordinary Differential Processes." *Journal of Applied Probability*, **8**: 344–356. URL `http://links.jstor.org/sici?sici=0021-9002%28197106%298%3A2%3C344%3ALTFSOJ%3E2.0.CO%3B2-M`.

— (1981). *Approximation of Population Processes*. Philadelphia: Society for Industrial and Applied Mathematics.

Miller, John H. and Scott E. Page (2007). *Complex Adaptive Systems: An Introduction to Computational Models of Social Life*. Princeton Studies in Complexity. Princeton, New Jersey: Princeton University Press.

Sigmund, Karl (1996). *Games of Life: Explorations in Ecology, Evolution and Behavior*. Penguin.