Assignment 1

36-462, Spring 2009

Due 23 January 2009

When a problem asks you to do a simulation, attach your code as a separate file (so I can run it if need be).

1. BIFURCATIONS IN THE LOGISTIC MAP

   (a) Find the value of r at which the fixed point bifurcates into a cycle with period 2; call this r_2.

   (b) Verify that the fixed point is stable when $r < r_2$ and unstable when $r > r_2$.

   (c) Verify, using the stability criterion in Flake, that the 2-cycle is stable for values of r just above r_2.

   (d) Find the r at which the 2-cycle becomes unstable, and show that a 4-cycle appears there.

   Hints: (i) look at the material in lecture 2 on how the fixed point at zero becomes unstable; (ii) look up how to solve a quartic (not quadratic) equation.

2. ROTATIONS This is a one-dimensional map for which the state θ_t is an angle on the circle, measured in degrees, i.e. $0 \leq \theta_t < 360$. The circle is rotated by a constant angle α each time-step.

$$\theta_{t+1} = \theta_t + \alpha \bmod 360$$

   (a) Show (by algebra, not simulation) that if α is rational then every point is a periodic point, but if α is irrational than there are no periodic points.

   (b) Show, by simulation, that this map is ergodic when α is irrational, with the invariant distribution being the uniform distribution.[1]

   (c) Show, again by simulation, that this map is not mixing.

[1] This was first proved formally, in rather different terms, by Nicholas Oresme in the 1300s.