Assignment 2

36-462, Spring 2009

Due 30 January 2009

When a problem asks you to do a simulation, include your code in your answer.

1. LYAPUNOV EXPONENTS IN THE LOGISTIC MAP For a one-dimensional map Φ, the Lyapunov exponent is the limiting time-average of the log derivative:

$$\lambda = \lim_{T \to \infty} \sum_{t=1}^{T} \log |\Phi'(x_t)|$$

   (a) Use this fact to write a program to calculate the Lyapunov exponent of the logistic map as a function of r.

   (b) Plot λ as a function of r.

   (c) Explain how this plot is related to the bifurcation diagram. *Hint:* make a new plot of λ, with r confined to the chaotic region.

2. THE RÖSSLER ATTRACTOR Install the `tseriesChaos` package from CRAN. (Be sure to install any required packages as well.) Run the command

```
rossler.ts <- sim.cont(rossler.syst, start=0, end=650,
                       dt=0.1, start.x=c(0,0,0),
                       parms=c(0.15, 0.2, 10))
```

   (Or type `example(sim.cont)` to see this again.) This generates a time series from the dynamical system known as the Rössler equations. You could look them up, but then you wouldn't learn much from this exercise.

   (a) Reconstruct the attractor using the method of false nearest neighbors. Explain how you determined the time lag and the embedding dimension. (It is no more than 9.) Include a plot of the reconstructed attractor, if physically possible.

   (b) The embedding dimension k can also be thought of as a setting to be chosen by cross-validation. Which embedding dimension, in the range of 1 to 9, gives the smallest cross-validated prediction error for 1-nearest-neighbor prediction? If this answer is not the same as in the previous part, why do you think they differ?

Hints/warnings: Look carefully at how the Lorentz system example was done in the notes. Also, be sure to allow your computer a *lot* of time to think about this. (There will be partial, but not full, credit if you can write a correct piece of code for this but can't get it to run in time.)

3. MIXING ON THE HÉNON ATTRACTOR For simplicity, consider only the distribution of the x coordinate, so that you can make one-dimensional histograms.

   (a) Find, by simulation, the long-run distribution of points visited from a single trajectory of the Hénon map when $a = 1.29$, $b = 0.3$. Does the answer depend on the initial condition?

   (b) Experiment with different ensembles of initial conditions; how much does the distribution of points at $t = 100$ vary?

   (c) Show by simulation that the time-average of x_t converges to the same value, independent of the initial condition.

   (d) Does $\sqrt{T}\frac{1}{T}\sum_{t=1}^{T} X_t$ go towards a Gaussian distribution for large T?