Chaos, Complexity, and Inference (36-462)

Lecture 9: Emergence and Self-Organization, Take 1

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Theme for the next few lectures: how lots of small things, interacting, make interesting large-scale patterns
Some good introductory readings on this:

popular: Pagels (1988); Holland (1998); Ball (1999); Johnson (2001)

socio-economic, less popular: Krugman (1996), also Schelling (1978); Axelrod and Cohen (1999)

biology: Thompson (1942) is old, but beautiful

Levels of Description

Different levels of resolution/precision for same system

Forest vs. individual organisms vs. physiology vs. cells vs. biochemistry vs. molecules

Distinction between relatively **macroscopic** variables/levels and relatively **microscopic**

Macro objects are assemblages of interacting micro objects

Macro variables are coarse-grainings/aggregations of micro ones

sometimes spatial scales (Morrison *et al.*, 1982)

but what really matters is being causally self-contained (DeLanda, 2006)

Macro/Micro Splits

Sometimes macro just sums or averages micro

- Pressure of a gas = average force of molecules per unit area
- GNP = total purchases in economy
- Metabolic rate = total calories consumed by all cells
- Population fitness = mean surviving offspring per adult

Sometimes more complicated relations

- Strength of materials vs. molecular bonds
- Prevailing prices vs. individuals' demand and supply
- Animal behavior vs. neural cell electrical activity

Macro and Micro Models

Different models needed at different levels — the variables are different!

Compatability

- aggregating the micro model should give you the macro
- macro behavior constrains possible micro models

Reductionism: microscopic dynamics explains macroscopic behavior

Reductionism, roughly speaking, is the view that everything in this world is really something else, and that the something else is always in the end unedifying. So lucidly formulated, one can see that this is a luminously true and certain idea. . . . It is important to understand why *it is so indubitably true. It is rooted . . . not in the nature of things, but in* our *ideal of explanation. Genuine explanation, not the grunts which pass for such in "common sense", means subsumption under a structure or schema made up of neutral, impersonal elements. In this sense, explanation is always "dehumanising", and inescapably so.*

— Ernest Gellner (Gellner, 1974, p. 107)

Emergence

What kinds of macro behavior can micro interactions produce?

Emergence: macro patterns relatively indifferent to micro details

This is how we *find* levels

Also how we get to build bridges before mastering quantum mechanics

Central Limit Theorem is the kind of pattern we expect with no or weak micro interactions and an averaging/summing macrovariable

Other behaviors (esp. of averages) need strong interactions!

First, excursion into why interesting behavior is strange

Entropy Again

x = microstate, m = macrostate, $m = M(x)$
$W(m)$ = # microstates y with $M(y) = m$
or volume of such microstates, if continuous
Boltzmann entropy of macroscopic state m:

$$S(m) = k_B \log W(m)$$

use of natural log, Boltzmann's constant k_B are just choice of units
Compare to topological entropy rate
"How many ways are there to realize this macroscopic state?"
Extend definition to Boltzmann entropy of microstate x:

$$S(x) = k_B \log W(M(x))$$

"How many microstates look, macroscopically, just like this one?"
Both depend on choice of macroscopic observables

Photo by Tom Schneider

Suppose system is an assemblage so $x = x_1, x_2, \ldots x_N$, giving some distribution on $\mathcal{X}$

$$S(m) = (k_B \log 2) N H[X_i | M(x) = m] + o(N)$$

connection to our entropy

also to algorithmic information, recall $K(x_1^N) \leq N H[X_i | M(x) = m] + o(N)$

This entropy is connected to heat (Q) and temperature (T):

$$\Delta S \approx \frac{\Delta Q}{T}$$

or really

$$dS = \frac{1}{T} dQ$$

The Second Law

This S tends to a maximum in a closed, thermally isolated system
equilibrium = macrostate of maximum entropy
This is the **second law of thermodynamics**

first law: energy is conserved

The second law of thermodynamics holds, I think, the supreme position among the laws of Nature. If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations — then so much the worse for Maxwell's equations. If it is found to be contradicted by observation, well, these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.

— Arthur Eddington (1948)

In fact the second law is "merely" probabilistic

violations are exponentially rare and exponentially brief (both in N)

violations can be observed observed experimentally (Wang *et al.*, 2002; Carberry *et al.*, 2004)

Scale: very small

$1\mathrm{cm}^3$ of argon at STP, $N = 2.5 \times 10^{19}$ atoms

equilibrium (maximum) entropy $= 6.6 \times 10^{20}$ bits

chance of 10% fluctuation in S

$$\approx e^{\Delta S/k_B} = 10^{-1.8\times10^{19}}$$

The second law is, itself, an emergent property (Ruelle, 1991; Lebowitz, 1999)

So what does the Second Law Mean?

Most frequent interpretation: disorder must increase

Makes sense for some contexts, like thermodynamics of heat engines

heat flows from warm things to cold things:

$$-\frac{\Delta Q}{T_H} + \frac{\Delta Q}{T_C} > 0$$

chemical reactions run to equilibrium

milk spreads in coffee

generally: smooth out, spread out, run down

A common conclusion:

> *One of the most basic laws in the universe is the Second Law of Thermodynamics. This states that as time goes by, entropy in an environment will increase. Evolution argues differently against a law that is accepted EVERYWHERE BY EVERYONE. Evolution says that we started out simple, and over time became more complex. That just isn't possible: UNLESS there is a giant outside source of energy supplying the Earth with huge amounts of energy. If there were such a source, scientists would certainly know about it.*
>
> *see* `http://www.fstdt.com/fundies/top100.aspx?archive=1`

sometimes they put it more subtly...

How many ways is this wrong?

1. The sun! (Also, the stars.)
2. Equilibrium in open systems minimizes free energy $F = E - TS$, not $-S$
3. Non-equilibrium, approach need not be monotonic
4. Open, non-equilibrium system need not go to equilibrium at all
5. Misunderstands entropy

Boltzmann entropy is *not* “how disordered is this state?”
Boltzmann entropy *is* “how many ways could this state happen?”
Maximizing Boltzmann entropy *with gravity* can create gradients
It can even create solar systems
— Recognition of the difference between entropy and disorganization is actually pretty old (Needham, 1943)

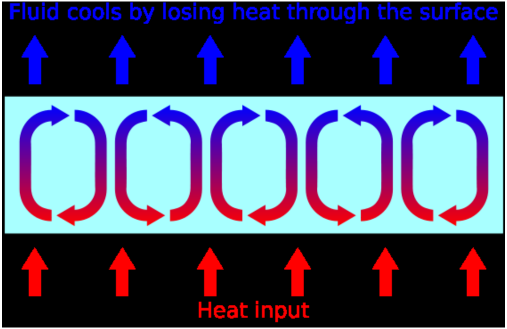
Viral Self-Assembly

Virus consists of DNA or RNA, protein shell ("capsids") in multiple parts
Dissociate parts, put into solution
Proteins *spontaneously* assemble into capsids, enclosing genes
Entropically driven: linking proteins together but excluding water from inside shell actually increases the number of configurations
Result: functioning virus, ready to take over a cell
(Fox, 1988; Zandi *et al.*, 2006)
entropy $\Uparrow$, organization $\Uparrow$

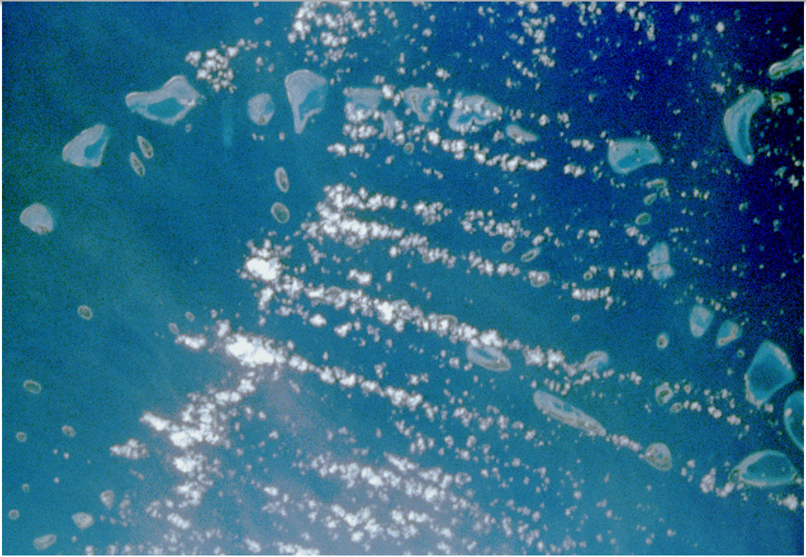
Self-organization: becoming more organized, without an external program
this is change over time
“emergence” is about differences between levels at *one* time
self-organization $\neq$ emergence, though there are links

Classic example: Bénard Cells

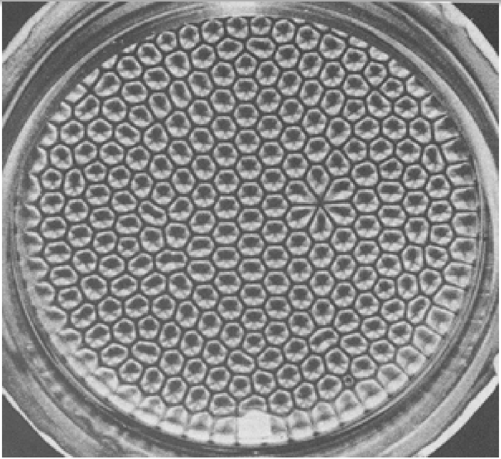
Take fluid; hot below, cold above
heat flows from hot to cold: temperature gradient
but warm fluids expand, become lighter, rise
convection cells form at a critical temperature gap (bifurcation)



Spatial patterns: "rolls" or "streets"; discrete (usually hexagonal) cells; labyrinths

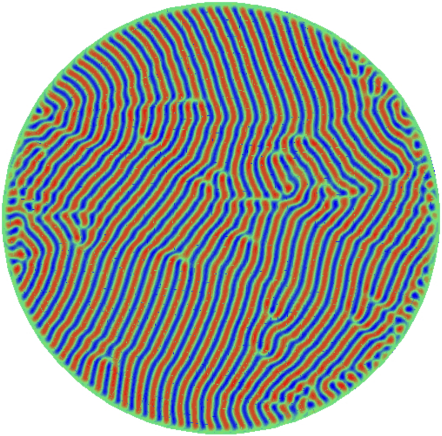


NASA photo from orbit



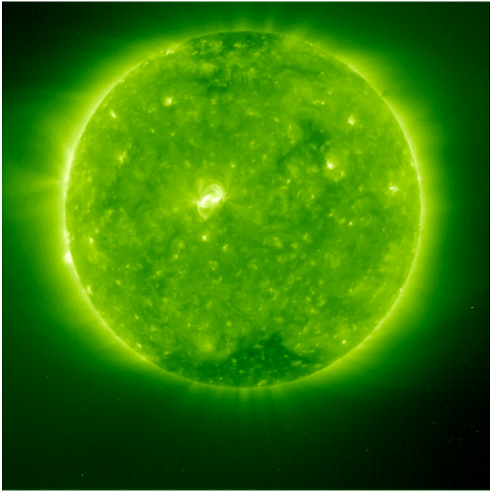
Ultimately from Van Dyke (1982)

Fluid rises in centers of hexagons, falls at edges; shiny light parts are aluminum particles floating at top of convection cells

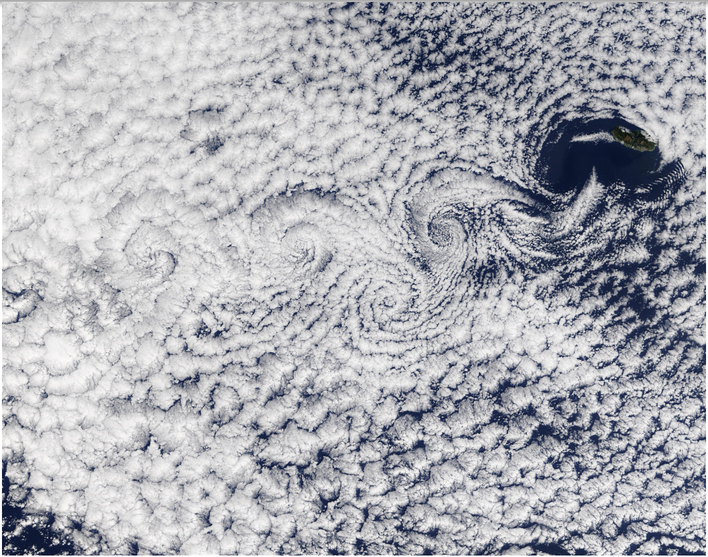


false color, showing temperature; A. Jayaraman

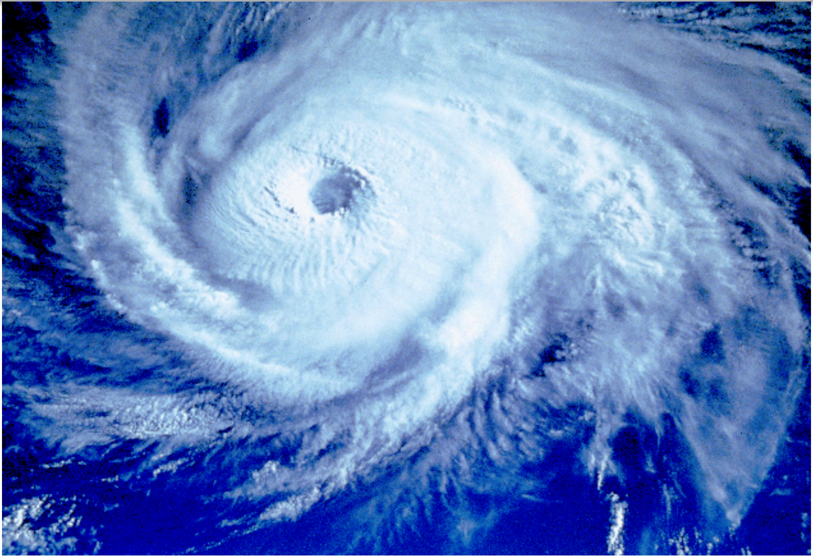
As heating is ramped up, can drive it to chaos



via Steve Lantz



http://www.solarviews.com/cap/earth/vortexstreet.htm



http://www.solarviews.com/cap/earth/typhoon.htm

Common story: for pattern formation: successive instabilities as drivers are increased

Not however the only one; can be stable until perturbed into pattern formation (different attractors)

Other Examples

Chemical oscillations (including mixed drinks)

Development of animals and plants; genetically controlled self-organization (Thompson, 1942; Camazine *et al.*, 2001; Gerhart and Kirschner, 1997)

Social insect colonies (Camazine *et al.*, 2001)

Ecological succession

"Large-scale social cooperation in the East African Plains Ape": markets (Lange and Taylor, 1938; Hayek, 1937, 1945; Schelling, 1978), conventions and customs (Young, 1998), fads and panics (Chamley, 2004)

Note: emergent levels of description in all these cases

Huge, very hard research area, few things *completely* well-understood
Also, real models need lots of background!
Will instead turn to more well-behaved mathematical models which show:

- strong interactions
- micro-macro distinction
- emergent phenomena
- self-organization (sometimes)
- at least qualitative links to reality

[Consider] what would happen in a new world, if God were now to create somewhere in the imaginary spaces matter sufficient to compose one, and were to agitate variously and confusedly the different parts of this matter, so that there resulted a chaos as disordered as the poets ever feigned, and after that did nothing more than lend his ordinary concurrence to nature, and allow her to act in accordance with the laws which He had established .... I showed how the greatest part of the matter of this chaos must, in accordance with these laws, dispose and arrange itself in such a way as to present the appearance of heavens; how in the meantime some of its parts must compose an earth and some planets and comets, and others a sun and fixed stars. ... I came next to speak of the earth in particular, and to show how ... the mountains, seas, fountains, and rivers might naturally be formed in it, and the metals produced in the mines, and the plants grow in the fields and in general, how all the bodies which are commonly denominated mixed or composite might be generated

(continued)

... [S]o that even although He had from the beginning given it no other form than that of chaos, provided only He had established certain laws of nature, and had lent it His concurrence to enable it to act as it is wont to do, it may be believed, without discredit to the miracle of creation, that, in this way alone, things purely material might, in course of time, have become such as we observe them at present; and their nature is much more easily conceived when they are beheld coming in this manner gradually into existence, than when they are only considered as produced at once in a finished and perfect state.
—René Descartes (Descartes, 1637, part 5)

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