Chaos, Complexity, and Inference (36-462)

Lecture 10: Cellular Automata

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Some things you can read:

Poundstone (1984) is what got me interested in the subject;

Toffoli and Margolus (1987) is the best introduction to CA modeling

code examples are for obsolete mid-1980s hardware

principles apply

Boccara (2004) has nice chapter on CAs

Chopard and Droz (1998); Ilachinski (2001): easier to read the more physics you know

Important paper collections: Burks (1970); Gutowitz (1991); Griffeath and Moore (2003)

Cellular Automata

Completely discrete, spatially-extended dynamical systems
Time: discrete
Space: divided into discrete **cells**
Each cell is in one of a finite number of **states**, a.k.a. colors
Global **configuration**: the states of all cells at one time
Every cell has a **neighborhood**

includes cell itself

von Neumann neighborhood: cardinal directions (NSEW)

Moore neighborhood: diagonals too

possibly of range > 1

The dynamics

Local rule: Given the states of the neighborhood at time t, what is the state of the cell at $t+1$?

simultaneous vs. random-order updating

Local rule implies a *global* rule for the configuration

Rules can be stochastic!

Globally, always have a Markov chain

Locally, have a **dynamic Markov random field**

Diffusion-Limited Aggregation

Particles appear at outer boundary, move in random walks

some trickery needed to get a random walk in a CA

Particles which hit the crystal freeze and become more crystal

Models aggregation, freezing

Qualitative dynamics:

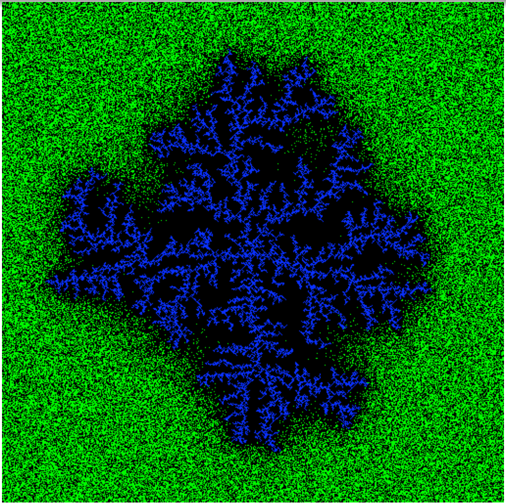
- smooth boundaries on crystal are unstable

  protrusions capture more particles

  ∴ protrusions grow, become spikes
- spikes create their own flat boundaries
- spikes-on-spikes = branching trees, **dendritic growth**

  Gets swamped if inflow of particles is fast enough

from Wikipedia



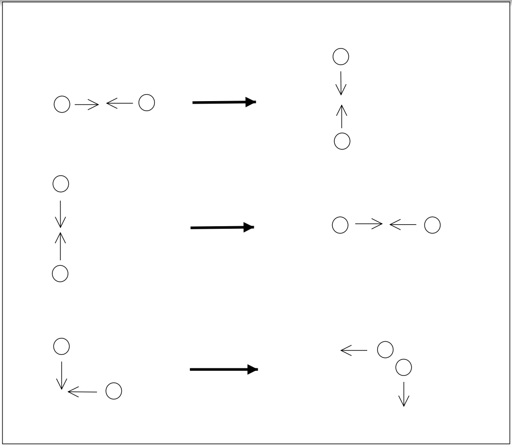
me, me, me!

Andy Goldsworthy

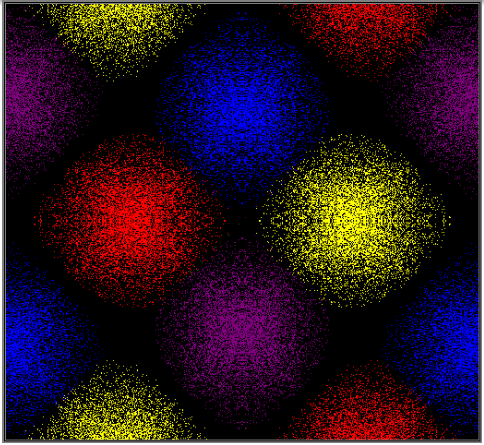
Lattice Gases

(Doolen, 1989, 1991; Rothman and Zaleski, 1997)

- Models of fluid flow
- Fluid consists of discrete particles moving with discrete velocities
- State of each cell is number of particles with different velocities there (possibly none)
  - "atoms and void"
- Collision rules implement basic physics
  - conservation of mass
  - conservation of energy
  - conservation of momentum
- Hydrodynamics emerges from local collisions, averaging over a large scale



Collision rules for “HPP” lattice gas (Hardy *et al.*, 1976), gets some hydrodynamics right but isn’t isotropic (rotationally symmetric) — need a non-rectangular lattice (Frisch *et al.*, 1986)



Diffusion in the HPP rule. Particles were initially clustered in tight blocks with random velocities. Particle color shows cluster of origin. The lattice wraps around at the boundaries. Source: `http://psoup.math.wisc.edu/archive/recipe61.html`

Majority Vote

"Majority vote": change to match the majority of the neighborhood (including self)

Magnetism, strains of organisms, conventions, ...

Start from near-50-50 initial conditions

Quickly: solid-color regions form

More slowly: they unbend; "motion by mean curvature"

opposite of irregularity-amplifying DLA

Variants:

- higher or lower thresholds
- probability of flipping depending on proportion of neighbors
- copy a random neighbor ("voter model")

Difference between finite-size behavior and infinite-size limit

Game of Life

2D, binary, Moore neighborhood
$<$ 2 neighbors on: turn off
$=$ 2 neighbors on: stay the same
$=$ 3 neighbors on: turn on
$>$ 3 neighbors on: turn off
some common life patterns: gliders, blinkers, beehives, ...
build a computer: use glider streams to represent bits, then build logic gates
build a universal computer by attaching memory

Can build machines which construct new configurations according to plans

These can reproduce themselves and copy their plans: mechanical (if mathematical) self-reproduction

This is actually the historical origin of cellular automata (von Neumann, 1966; Burks, 1970)

Poundstone (1984) is a wonderful account

Learning CA rules

Known neighborhood, deterministic rule: wait and see.
Known neighborhood, stochastic rule: counting (as with Markov chain)
Unknown neighborhood: search over neighborhoods to maximize mutual information between neighbors' states and new state (Richards *et al.*, 1990)

Huge number of neighborhoods, use fancy optimization techniques

Could also try: minimize conditional MI between new state and rest of configuration given neighborhood

Boccara, Nino (2004). *Modeling Complex Systems*. Berlin: Springer-Verlag.

Burks, Arthur W. (ed.) (1970). *Essays on Cellular Automata*, Urbana. University of Illinois Press.

Chopard, Bastien and Michel Droz (1998). *Cellular Automata Modeling of Physical Systems*. Cambridge, England: Cambridge University Press.

Doolen, Gary D. (ed.) (1989). *Lattice Gas Methods for Partial Differential Equations: A Volume of Lattice Gas Reprints and Articles*, Reading, Massachusetts. Adidson-Wesley.

— (1991). *Lattice Gas Methods: Theory, Applications, and Hardware*, Cambridge, Massachusetts. MIT Press. Also published as *Physica D* **47 (1–2)**, 1991.

Frisch, Uriel, Boris Hasslacher and Yves Pomeau (1986). "Lattice-gas Automata for the Navier-Stokes Equation." *Physical Review Letters*, **56**: 1505–1508.

Griffeath, David and Cristopher Moore (eds.) (2003). *New Constructions in Cellular Automata*, Oxford. Oxford University Press.

Gutowitz, Howard (ed.) (1991). *Cellular Automata: Theory and Experiment*, Cambridge, Massachusetts. MIT Press. Also published as *Physica D* **45** (1990), nos. 1–3.

Hardy, J., Y. Pomeau and O. de Pazzis (1976). "Molecular Dynamics of a Classical Lattice Gas: Transport Properties and Time Correlation Functions." *Physical Review A*, **13**: 1949–1960.

Ilachinski, Andrew (2001). *Cellular Automata: A Discrete Universe*. Singapore: World Scientific.

Poundstone, William (1984). *The Recursive Universe: Cosmic Complexity and the Limits of Scientific Knowledge*. New York: William Morrow.

Richards, Fred C., Thomas P. Meyer and Norman H. Packard (1990). “Extracting Cellular Automaton Rules Directly from Experimental Data.” *Physica D*, **45**: 189–202. Reprinted in Gutowitz (1991).

Rothman, Daniel H. and Stéphane Zaleski (1997). *Lattice-Gas Cellular Automata: Simple Models of Complex Hydrodynamics*. Cambridge, England: Cambridge University Press.

Toffoli, Tommaso and Norman Margolus (1987). *Cellular Automata Machines: A New Environment for Modeling*. Cambridge, Massachusetts: MIT Press.

von Neumann, John (1966). *Theory of Self-Reproducing Automata*. Urbana: University of Illinois Press. Edited and completed by Arthur W. Burks.