Chaos, Complexity, and Inference (36-462)

Lecture 26: Some Inference for Network Models

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Inference for Network Models

General reading: Hunter *et al.* (2008)

Matched Random Networks

"So, you think you've found an interesting network structure, do you? Well isn't that *special*!"

Some kinds of network structure follow automatically from others

e.g., assortative $\Rightarrow$ reciprocal, cluster

Is what you are seeing an artifact or does it mean something?

Question of *what does a random network look like*?

But not just any random network, one that is *close* to yours

Basic Algorithm

Observe interesting feature X in your data graph g
Construct a distribution μ over random graphs G that matches g, but doesn't build in X
Draw many samples $G_1, G_2, \ldots G_m$ from μ
See how many of them have feature X

Simple Matched Random Networks

Erdős-Rényi networks are random...
Matching: same number of nodes and same density of edges
$=$ expected degree
Very random... in fact, too random
In almost any situation, you know that your network doesn't look like *that*
It would be nice to match some more features than just the size and the density!

Exponential Random Graphs as Matched Random Networks

Pick your functionals on the network = sufficient statistics = T_i, $i \in 1:d$, observe values $t_i = T_i(g)$
Then (as discussed) $\widehat{\theta}_{MLE}$ solves

$$\mathbf{E}_{\widehat{\theta}_{MLE}}[T] = t$$

BUT $\mu = \Pr_{\widehat{\theta}_{MLE}}(G)$ also solves

$$\max H_{\mu}[G] \text{ s.t. } \mathbf{E}_{\mu}[T] = t$$

with H = Shannon entropy (Mandelbrot, 1962).

Maximizing likelihood in the exponential family maximizes entropy over all distributions

Why Should You Care about Maximum Shannon Entropy?

Some people see this as a self-justifying ideal

this is hard to take seriously

Gives distribution closest to independence under the constraint

(Amari, 2001)

but observed $\approx$ expected isn't a universal rule of inference!

unless observation is a big average!

Background problem: picking the right statistics to match

Fixing the Degree Distribution

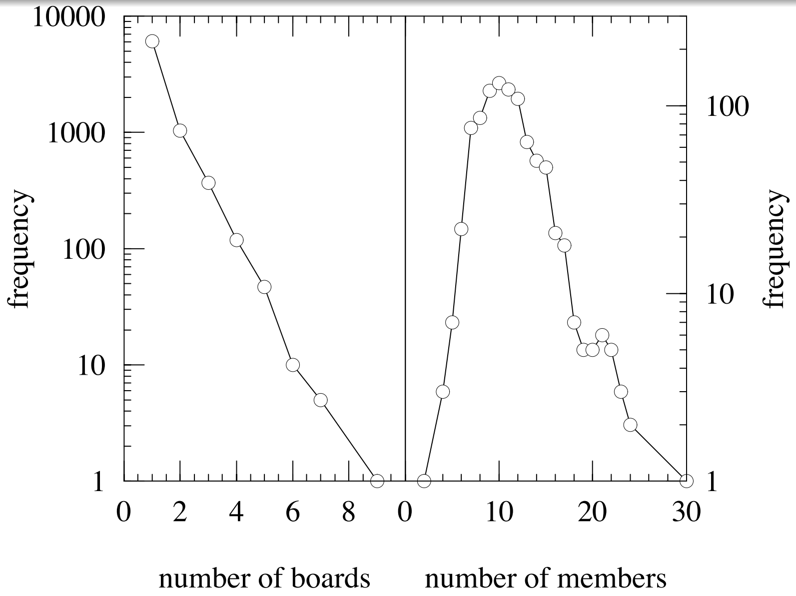
Newman *et al.* (2001)
Fix N
Generate N random numbers K_i from the empirical degree distribution — “stubs”
Choose pairs of free stubs uniformly at random; join them
Equally likely to produce any graph with that degree distribution
Must have even sum-of-degrees but this is not a big issue (if not even, discard and re-simulate)
Modifications required for directed graphs and bipartite graphs to handle summing-up constraints
if sums don’t match, pick one pair, discard their sizes, re-draw; repeat as needed

Example: Corporate Boards

Public corporations have **directors** who represent shareholders and (supposedly) pick the executives
Board members often sit on many boards
This effectively a coordinating mechanism

> *People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices.*

Also something that doesn't lead to very good decisions, but does shield rich people from market forces (Khurana, 2002)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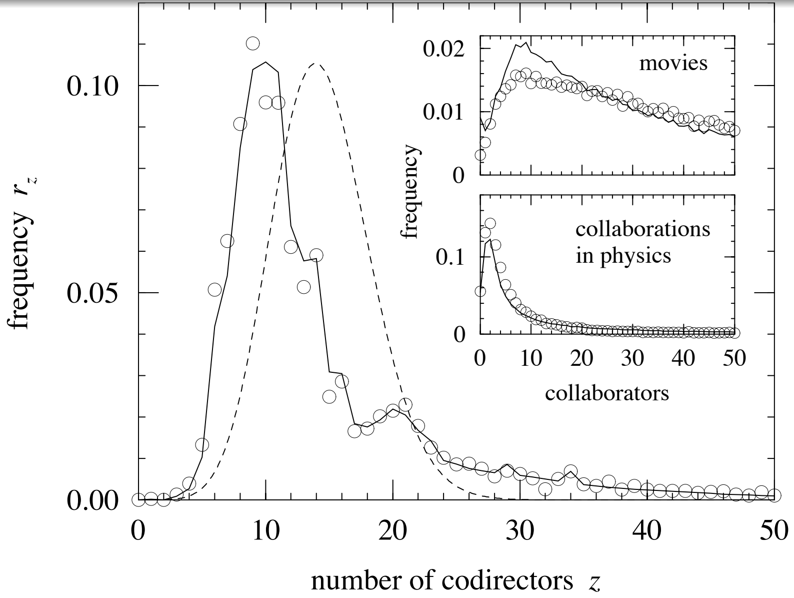
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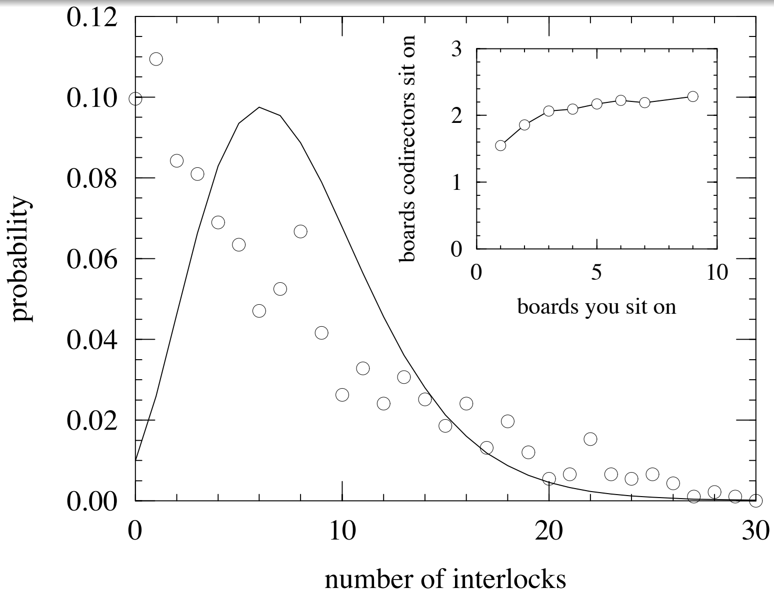
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number of interlocks
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0.02
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30
boards codirectors sit on
boards you sit on
3
2
1
0
5
10

Discriminating Network Growth Modes

Middendorf *et al.* (2005)
Given: different models for how a network grew
Wanted: guess as to which one it was
Simulate many networks from each model
Train a **classifier** to reliably discriminate between them
Need **features** (sub-graph census) and **classifiers** (decision trees)
Validate the classifier by showing it has low error rates
Classifier may or may not look at features important in any one model

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