Statistical Computing (36-350)

Lecture 11: Functions as Objects 2: Creating Functions as Values

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Agenda

- Functions as return values
- Example: Linear predictor
- Example: the gradient operator
- Example: `surface`

OPTIONAL RECOMMENDED READING: Chapter 3 of Chambers
MERELY USEFUL READING: Chapter 3 of the textbook

Functions are objects, just like everything else in R
Last time, saw how to pass functions in to other functions as arguments
This time, will look at how to create functions as return values

Mathematically

Something which takes a function in and gives a function back is an **operator**

Differentiation: the operator d/dx takes f and gives a new function

Gradient: the operator ∇ takes f and gives a new function

similarly $\nabla \cdot$, $\nabla \times$, ...

Indefinite integration: $\int_{-\infty}^{x} f(u) du$ takes f and gives a new function

Computationally

Functions are objects
The easiest way to create such an object is by calling `function`
The resulting function object has a body it executes, arguments, and an environment it looks names up in

An admittedly trivial example

```
make.noneuclidean <- function(ratio.to.diameter=pi) {
  circumference <- function(d) { return(ratio.to.diameter*d) }
  return(circumference)
}
```

Define `make.noneuclidean` but don't run it yet

```
> circumference(10)
Error: could not find function "circumference"
> kings.i <- make.noneuclidean(3)
> kings.i(10)
[1] 30
> formals(kings.i)
$d
> body(kings.i)
{
    return(ratio.to.diameter * d)
}
> environment(kings.i)
<environment: 0xe43d64>
> circumference(10)
Error: could not find function "circumference"
```

A Less Trivial Example

Create a linear predictor, based on sample values of two variables

```
make.linear.predictor <- function(x,y) {
  linear.fit <- lm(y~x)
  predictor <- function(x) {
   return(predict(object=linear.fit,newdata=data.frame(x=x)))
  }
  return(predictor)
}
```

The predictor function persists and works, even when the data we used to create it is gone

Example:

```
> independent.variable <- runif(10)
> dependent.variable <- 7 + 3*independent.variable
> my.predictor <- make.linear.predictor(x=independent.variable,
+   y=dependent.variable)
> rm(independent.variable,dependent.variable)
> independent.variable
Error: object 'independent.variable' not found
> dependent.variable
Error: object 'dependent.variable' not found
> my.predictor(5)
 1
22
> my.predictor(runif(10))
       1        2        3        4        5        6        7        8
7.610858 7.704701 7.018500 8.211081 8.820881 9.935522 9.034840 8.219453
       9       10
7.912262 8.125505
```

A more mathematical example

Last time wrote, `gradient` function to find ∇f at a point x

```
nabla <- function(f,...) {
  g <- function(x,...) { gradient(f=f,x=x,...) }
  return(g)
}
```

Re-using the `mse.for.optimization` function from last time

```
> mse.gradient <- nabla(mse.for.optimization)
> mse.gradient(c(6611,0.15),deriv.steps=c(1,1e-6))
[1] 1.646082e+05 1.428795e+10
> gradient(mse.for.optimization,c(6611,0.15),c(1,1e-6))
[1] 1.646082e+05 1.428795e+10
> gradient(mse.for.optimization,c(6611,0.15),c(1,1e-6),Y=2*gmp$pcgmp)
[1] -2.908638e+05 -2.486987e+10
> mse.gradient(c(6611,0.15),deriv.steps=c(1,1e-6),Y=2*gmp$pcgmp)
[1] -2.908638e+05 -2.486987e+10
```

Actually, as I said, the simple first-differences method is not so hot, so use the `grad` function from `numDeriv`

```
del <- function(f,...) {
  require(numDeriv)
  g <- function(x,...) { grad(func=f,x=x, ...)}
  return(g)
}
```

How would you check this?

Example: `surface`

Last time: `curve`, which takes an expression and, as a side-effect, plots a 1-D curve by sweeping over `x`

Suppose we want something like that but sweeping over two variables

Built-in plotting function `contour`:

```
contour(x,y,z, [[other stuff]])
```

`x` and `y` are vectors of coordinates, `z` is a matrix of the corresponding shape

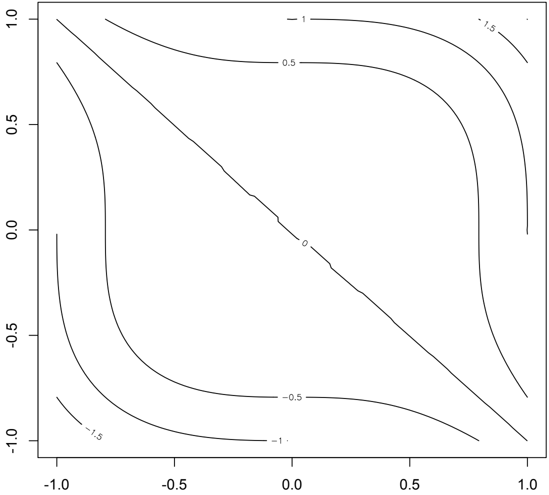
(see `help(contour)` for graphical options)

Strategy: `surface` should make `x` and `y` sequences, evaluate the expression at each combination to get `z`, and then call `contour`

First attempt at `surface`

Only works with vector-to-number functions:

```
surface.0 <- function(f,from.x=0,to.x=1,from.y=0,to.y=1,n.x=101,
  n.y=101,...) {
  x.seq <- seq(from=from.x,to=to.x,length.out=n.x)
  y.seq <- seq(from=from.y,to=to.y,length.out=n.y)
  plot.grid <- expand.grid(x=x.seq,y=y.seq)
  z.values <- apply(plot.grid,1,f)
  z.matrix <- matrix(z.values,nrow=n.x)
  contour(x=x.seq,y=y.seq,z=z.matrix,...)
  invisible(list(x=x.seq,y=y.seq,z=z.matrix))
}
```




```
surface.0(function(p){return(sum(p^3))},from.x=-1,from.y=-1)
```

`curve` doesn't require us to write a function every time — what's it's trick?

Expressions are just another class of R object, so they can be created and manipulated

One manipulation is evaluation

```
eval(expr,envir)
```

evaluates the expression `expr` in the environment `envir`, which can be a data frame or even just a list

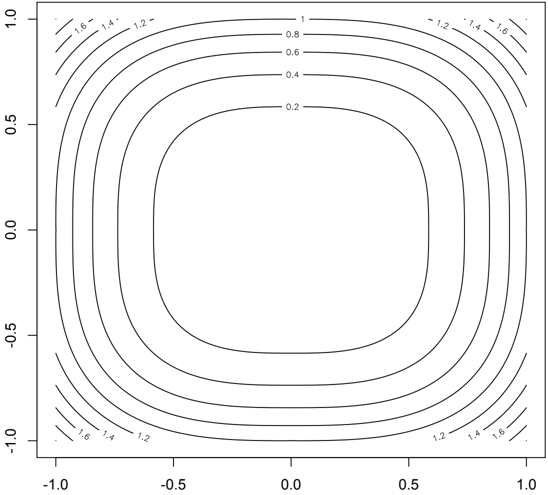
When we type something like `x^2+y^2` as an argument to `curve`, R tries to evaluate it prematurely

`substitute` returns the *unevaluted* expression

`curve` uses first `substitute(expr)` and then `eval(expr,envir)`, having made the right `envir`

Second attempt at `surface`

```
surface.1 <- function(expr,from.x=0,to.x=1,from.y=0,to.y=1,n.x=101,
  n.y=101,...) {
  x.seq <- seq(from=from.x,to=to.x,length.out=n.x)
  y.seq <- seq(from=from.y,to=to.y,length.out=n.y)
  plot.grid <- expand.grid(x=x.seq,y=y.seq)
  unevaluated.expression <- substitute(expr)
  z.values <- eval(unevaluated.expression,envir=plot.grid)
  z.matrix <- matrix(z.values,nrow=n.x)
  contour(x=x.seq,y=y.seq,z=z.matrix,...)
  invisible(list(x=x.seq,y=y.seq,z=z.matrix))
}
```



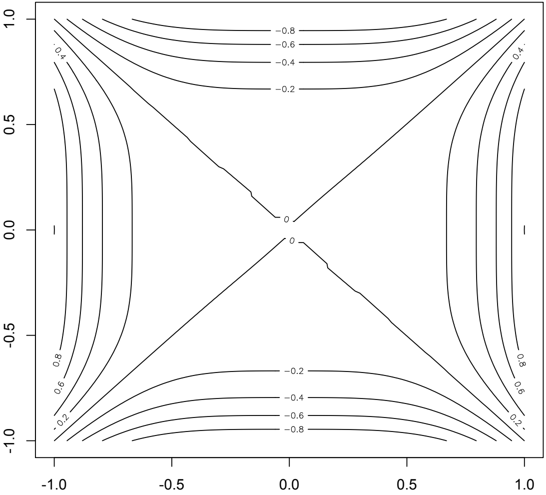

```
surface.1(abs(x^3)+abs(y^3),from.x=-1,from.y=-1)
```

Evaluating a function at every combination of two arguments is a really common task

There is a function to do it for us: `outer` (seen in lecture 3)

```
surface.2 <- function(expr,from.x=0,to.x=1,from.y=0,to.y=1,n.x=101,
  n.y=101,...) {
  x.seq <- seq(from=from.x,to=to.x,length.out=n.x)
  y.seq <- seq(from=from.y,to=to.y,length.out=n.y)
  unevaluated.expression <- substitute(expr)
  z <- function(x,y) {
    return(eval(unevaluated.expression,envir=list(x=x,y=y)))
  }
  z.values <- outer(X=x.seq,Y=y.seq,FUN=z)
  z.matrix <- matrix(z.values,nrow=n.x)
  contour(x=x.seq,y=y.seq,z=z.matrix,...)
  invisible(list(x=x.seq,y=y.seq,z=z.matrix))
}
```

could also include the function as part of the returned list




```
surface.2(x^4-y^4,from.x=-1,from.y=-1)
```

Summary

- Functions can be return values, just like any other object
- Variables other than the arguments to the function are fixed by the environment of creation
- Manipulation of expressions gives us ways of flexibly creating functions

Next week: The split/apply/combine trick