Figure 1. Roulette table

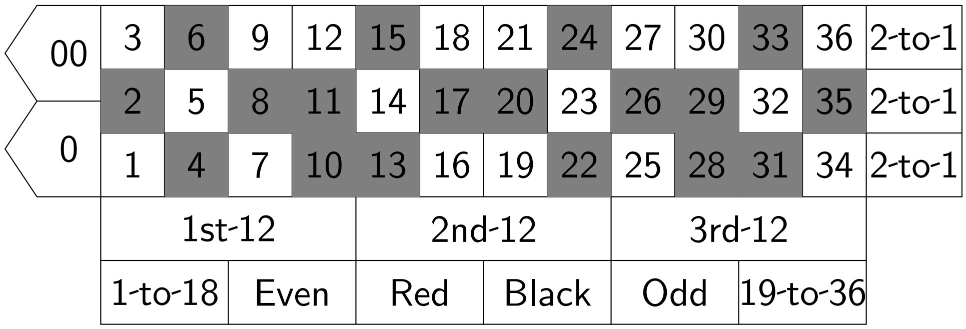


Table 1. Roulette Payoffs

Name of Bet	Winning Numbers	Payoff Odds
Even Money	A set of 18 numbers: red, black, even, odd, 1–18, 19–36	1 to 1
Dozen	One of 1–12, 13–24, or 25–36	2 to 1
Column	12 numbers in one row	2 to 1
Line	6 consecutive numbers (2 columns)	5 to 1
House Special	0, 00, 1, 2, 3	6 to 1
Quarter	4 numbers that share a corner	8 to 1
Street	3 consecutive numbers (1 column)	11 to 1
Split	An adjacent pair of numbers	17 to 1
Straight	A single number	35 to 1

The payoff odds for each play give the ratio of the amount won to the amount bet, should that bet succeed. For instance, the Straight Play is less likely to win than an Even Money Play, but a winning $1 bet pays $35 in the former case and $1 in the latter.

Table 2. One Spin

Name of Play (bet)	Proportion of Wins	Average Value of a $10 bet
Even Money (red)	0	-10
Dozen (3rd 12)	0	-10
Column (1,4,7,. . .)	0	-10
Line (19–24)	0	-10
House Special	1	60
Quarter (2,3,5,6)	0	-10
Street (34–36)	0	-10
Split (8,11)	0	-10
Straight (00)	1	350

Table 3. 100 Spins

Name of Play (bet)	Proportion of Wins	Average Value of a $10 bet
Even Money (red)	0.45	-1.00
Dozen (3rd 12)	0.34	0.20
Column (1,4,7,...)	0.30	-1.00
Line (19–24)	0.15	-1.00
House Special	0.18	2.60
Quarter (2,3,5,6)	0.06	-4.60
Street (34–36)	0.09	0.80
Split (8,11)	0.04	-2.80
Straight (00)	0.07	15.20

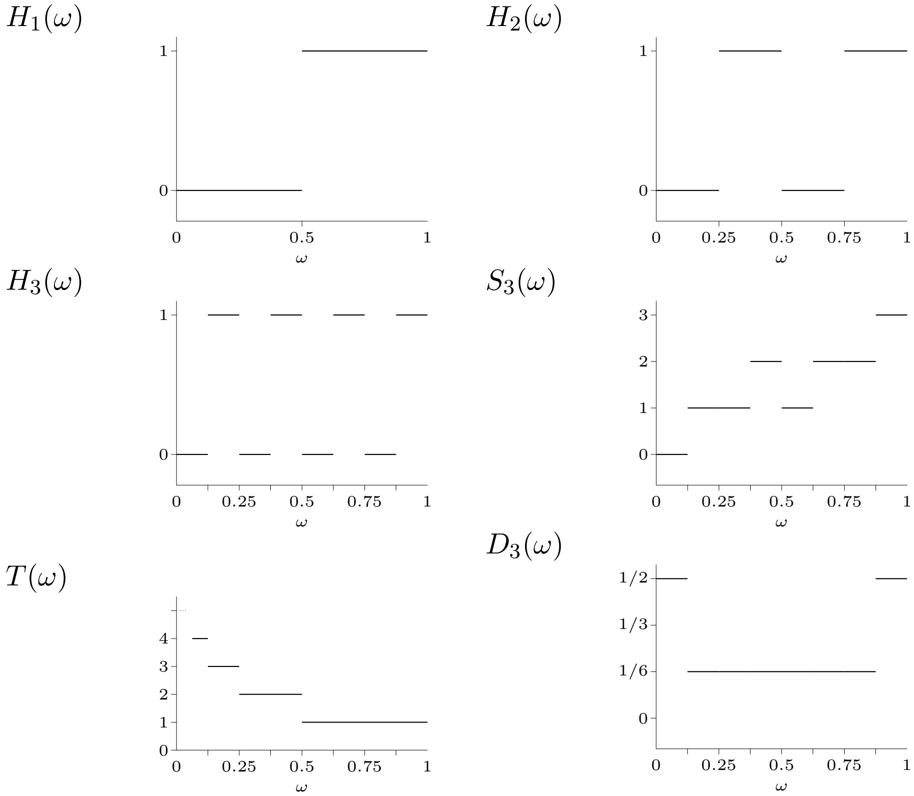
Table 4. 100,000 Spins

Name of Play (bet)	Proportion of Wins	Average Value of a $10 bet
Even Money (red)	0.47255	-0.55
Dozen (3rd 12)	0.31396	-0.58
Column (1,4,7,...)	0.31603	-0.52
Line (19–24)	0.15884	-0.47
House Special	0.13257	-0.72
Quarter (2,3,5,6)	0.10534	-0.52
Street (34–36)	0.07877	-0.55
Split (8,11)	0.05205	-0.63
Straight (00)	0.02613	-0.59

Table 5. In the long run...

Name of Play (bet)	Proportion of Wins	Average Value of a $10 bet
Even Money (red)	$18/38 \approx 0.474$	$-20/38 \approx -0.526$
Dozen (3rd 12)	$12/38 \approx 0.316$	$-20/38 \approx -0.526$
Column (1,4,7,...)	$12/38 \approx 0.316$	$-20/38 \approx -0.526$
Line (19–24)	$6/38 \approx 0.158$	$-20/38 \approx -0.526$
House Special	$5/38 \approx 0.132$	$-30/38 \approx -0.789$
Quarter (2,3,5,6)	$4/38 \approx 0.105$	$-20/38 \approx -0.526$
Street (34–36)	$3/38 \approx 0.079$	$-20/38 \approx -0.526$
Split (8,11)	$2/38 \approx 0.053$	$-20/38 \approx -0.526$
Straight (00)	$1/38 \approx 0.026$	$-20/38 \approx -0.526$

Figure 2. Coin Flipping Variables



Definition 1. A *random experiment* is a repeatable, random process from which we can measure one or more quantities of interest.

Definition 2. Let $\mathcal{F}$ be a collection of subsets of a set Ω. We call $\mathcal{F}$ a *σ-field* (also known as a σ-algebra) if the following are true:

1. $\Omega \in \mathcal{F}$
2. $\mathcal{A} \in \mathcal{F} \implies \mathcal{A}^c \in \mathcal{F}$
3. Given a sequence (necessarily countable) $\mathcal{A}_1, \mathcal{A}_2, \ldots, \in \mathcal{F}$, then $\bigcup_i \mathcal{A}_i \in \mathcal{F}$.

Definition 3. Given a set $\mathcal{X}$ equipped with a σ-field $\mathcal{F}$ of subsets, the tuple $(\mathcal{X}, \mathcal{F})$ is called a *measurable space.*

Definition 4. The *outcome space* (aka sample space) of a random experiment is s meaurable space $(\Omega, \mathcal{F})$ describing the possible outcomes of the experiment. Elements $\omega \in \Omega$ are called *elementary outcomes.* The subsets in $\mathcal{F}$ are called *events.*

Definition 5. Given the outcome space $(\Omega, \mathcal{F})$ of a random experiment and another measurable space $(\mathcal{X}, \mathcal{A})$, a *random variable* is a function mapping $(\Omega, \mathcal{F}) \to (\mathcal{X}, \mathcal{A})$ such that for every $A \in \mathcal{A}$, $X^{-1}(A) \in \mathcal{F}$. (Outside of a probabilistic context, these are known as measurable functions.)

Definition 6. The *expected value* is an operator that acts on random variables over an outcome space $(\Omega, \mathcal{F})$ and satisfies the Basic Expected Value Rules of Table 6. The E operator returns a value in $\mathbb{R}^k$ for $\mathbb{R}^k$-valued random variables. The probability of an event is defined by $\mathsf{P}(A) = \mathsf{E}1_A$ for each $A \in \mathcal{F}$.

Table 6. The Basic Expected Value Rules

Constancy Rule

Given:	The constant random variable 1.
Yields:	$\mathsf{E}1 = 1$.
In words:	A constant is its own expected value.
Analogy:	The average is 1 if all numbers in the list equal 1.

Scaling Rule

Given:	Random variable X and a constant $c \in \mathbb{R}$.
Yields:	$\mathsf{E}(cX) = c\mathsf{E}X$.
In words:	Constants can be taken out of the expected value.
Analogy:	Scaling all numbers in a list by the same constant scales the average of the list by that constant as well.

Additivity Rule

Given:	Random variables $X_1, \ldots, X_n$ for positive integer n.
Yields:	$\mathsf{E}(X_1 + \cdots + X_n) = \mathsf{E}X_1 + \cdots + \mathsf{E}X_n$.
In words:	The expected value of a sum is the sum of the expected values.
Analogy:	The average of an (elementwise) sum of lists is the sum of the averages.

Non-negativity Rule

Given:	A random variable X that is always non-negative, i.e., $X \geq 0$.
Yields:	$\mathsf{E}X \geq 0$.
In words:	The expected value of a non-negative random variable is non-negative.
Analogy:	The average is non-negative if all numbers in the list are non-negative.

Monotone Limits Rule

Given:	A random variable X and random variables $X_1 \leq X_2 \leq \cdots$ such that $\lim_{i\to\infty} X_i = X$ and $\mathsf{E}X_i > -\infty$ for some i.
Yields:	$\lim_{i\to\infty} \mathsf{E}X_i = \mathsf{E}(\lim_{i\to\infty} X_i) = \mathsf{E}X$.
In words:	The order of limits and E can be exchanged if the sequence is increasing.
Analogy:	There is one, but it is too technical to be helpful.

Definition 7. A measurable space $(\Omega, \mathcal{F})$ equipped with an expected value operator E is called a *probability space* and denoted by $(\Omega, \mathcal{F}, \mathsf{E})$. Equivalently, we can define the corresponding probability measure P and denote the space by $(\Omega, \mathcal{F}, \mathsf{P})$. In some conventions, which I like, P is used for both operators transparently.

Definition 8. The elementary outcome selected during the run of a random experiment, called the *selected elementary outcome*, is denoted by ω^*. An event $\mathcal{A}$ is said to have occurred during the experiment if $\omega^* \in \mathcal{A}$.