

Please do as much of the following as you can in **20 minutes**. There are four questions.

1

You want to generate a random number with CDF F , which you can compute.

Let U be a Uniform $\langle 0, 1 \rangle$ (continuous) random variable.

Show that $F^{-1}(U)$ has CDF F .

2

A group of $n + 1$ players (including you) play the following game. Each player contributes \$1 to the prize money and then flips a coin once. Assume that all the coin flips are independent and have the same probability $0 < p < 1$ of coming up heads. Every player whose coin comes up heads gets an equal share of the prize; those whose coin comes up tails get nothing. If no heads appear, the money is donated to charity.

Find your expected winnings in this game. (Ignore the positive feelings you experience when the money goes to charity.)

3

A sleep deprived, thirteenth-century monk is hurrying to copy a manuscript with n pages. Each page takes many hours to copy, and if he makes any errors, he must start over from scratch.

Assume that in his sleep deprived state, the monk makes a Poisson $\langle \lambda \rangle$ total number of copying errors, for $\lambda > 0$, without noticing them. Assume that each error is equally likely to be made on any page and that the positions of the errors are independent. The abbot notices the errors and insists that the monk get some sleep before fixing the pages.

Find the expected number of pages the monk must recopy when he awakes.

4

A random point (X, Y) is chosen uniformly from the unit disk in \mathbb{R}^2 .

Find the marginal PDF of X , f_X , and the conditional PDF of Y given X , $f_{Y|X}(v | u)$ for $-1 \leq u \leq 1$.