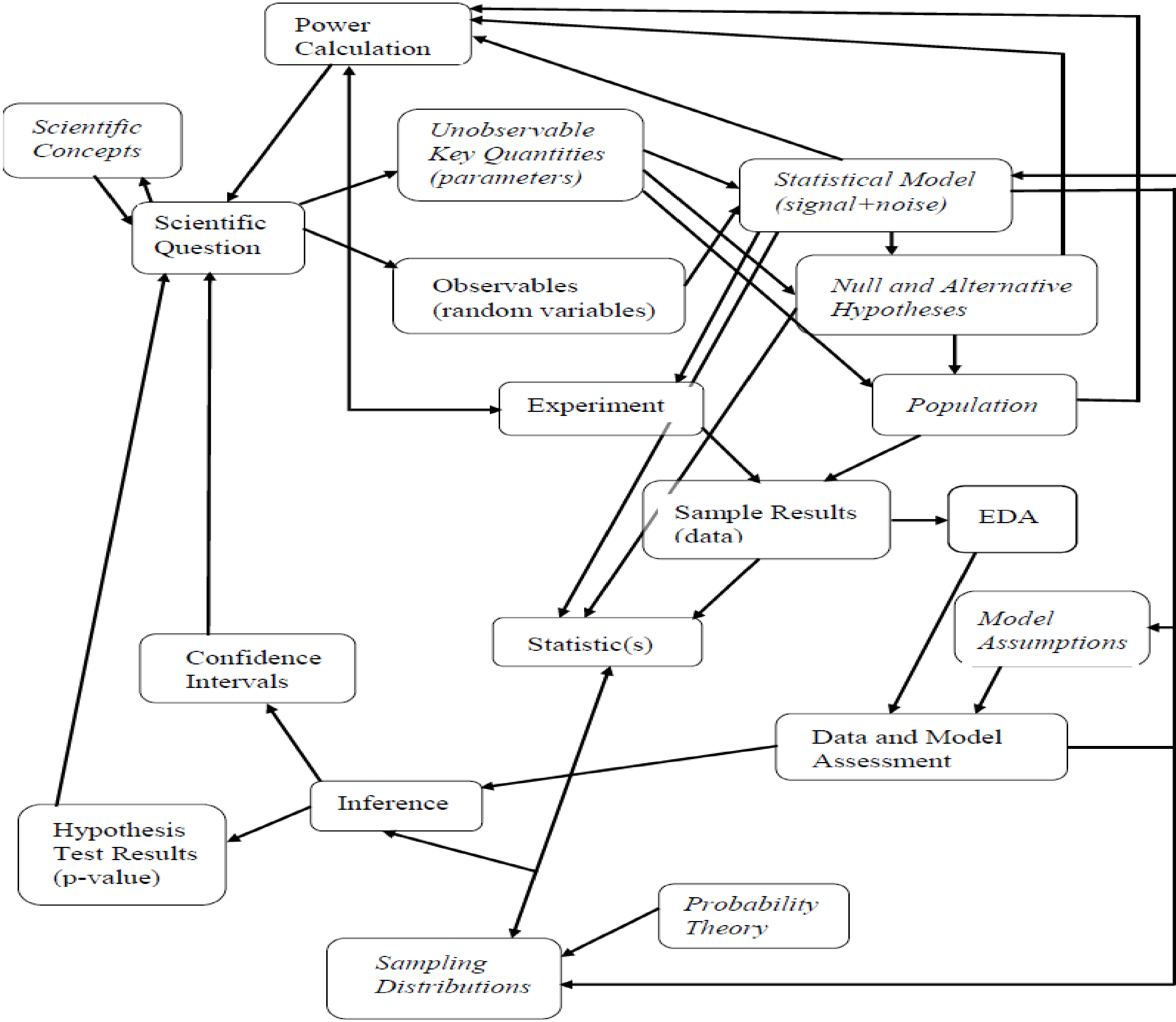
36-309/749
Experimental Design for Behavioral and Social Sciences

Oct. 6, 2015
Lecture 6: 2-way ANOVA


Power Calculation
Scientific Concepts
Unobservable Key Quantities (parameters)
Statistical Model (signal+noise)
Scientific Question
Observables (random variables)
Null and Alternative Hypotheses
Experiment
Population
Sample Results (data)
EDA
Model Assumptions
Statistic(s)
Confidence Intervals
Data and Model Assessment
Inference
Hypothesis Test Results (p-value)
Probability Theory
Sampling Distributions

2-Way (Between Subjects) ANOVA

- ***Quantitative outcome*** and ***two categorical explanatory variables*** (“factors”)
- Each subject used once & no collusion, etc.
- Both factors may be of primary interest, or one is of primary interest and the other represents “blocks”. In the latter case the block p-value is usually ignored.
- Each factor may have 2 or more levels.
- “Full factorial design”: every combination of the levels of the two factors is represented by some subjects.
- Shorthand: “2 × 3” or “3 by 5” factorial experiment (shows number of levels)

2-way ANOVA, cont.

- SPSS EDA: clustered boxplots or multiple line plot with 95% CI error bars

- Means models: **additive** (parallel) ***vs.*** with **interaction**
- Error model:

- SPSS formal analysis: 2-way ANOVA with General Linear Model / Univariate, which defaults to automatically creating and using the two-way interaction.

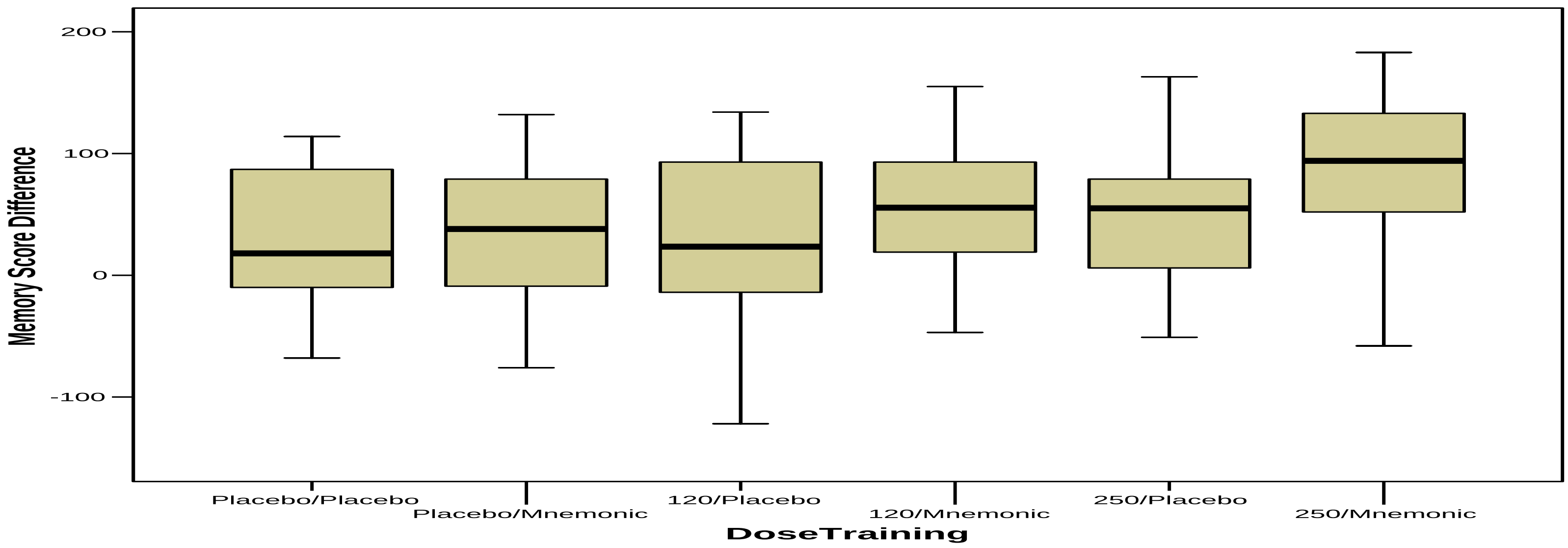
Example 1: Ginkgo for Memory (3x2 ANOVA)

- Test the effects of the herbal medicine Ginkgo biloba (Placebo, 120mg, and 250mg) on memory (treated as categorical because…)
- Also test the effects of "mnemonic training" (no or yes)
- 18 healthy subjects for each factor combination (N = 18 x 6 = 108)
- Memory is tested before the study and after two-months.
- The response variable is the difference (after - before) in the memory test scores [Not violating independent errors.]
- Some data:

Outcome	Dose	Training
9	1	1
-30	1	1
…		
55	3	2
…		

Example 1, EDA

	DoseTraining		Statistic	Std. Error
Memory Score Difference	Placebo/Placebo	Mean	31.00	13.098
		Std. Deviation	55.571	
	Placebo/Mnemonic	Mean	34.00	15.525
		Std. Deviation	65.865	
	120/Placebo	Mean	29.06	15.908
		Std. Deviation	67.492	
	120/Mnemonic	Mean	55.83	12.508
		Std. Deviation	53.065	
	250/Placebo	Mean	48.56	14.099
		Std. Deviation	59.818	
	250/Mnemonic	Mean	90.44	14.749
		Std. Deviation	62.574	



Example, cont.

One-way ANOVA for the 6 groups (educational use only!)

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	48725.296	5	9745.059	2.623	.028
Within Groups	378948.333	102	3715.180		
Total	427673.630	107			

Example: Two-way ANOVA with interaction

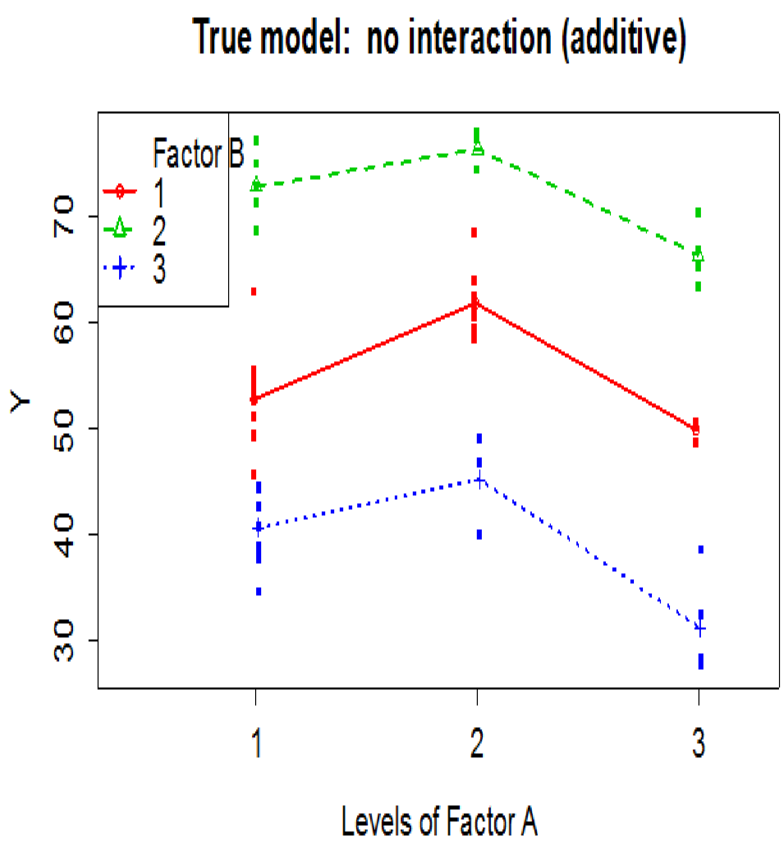
	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	48725.296	5	9745.059	2.623	.028
Within Groups	378948.333	102	3715.180		
Total	427673.630	107			

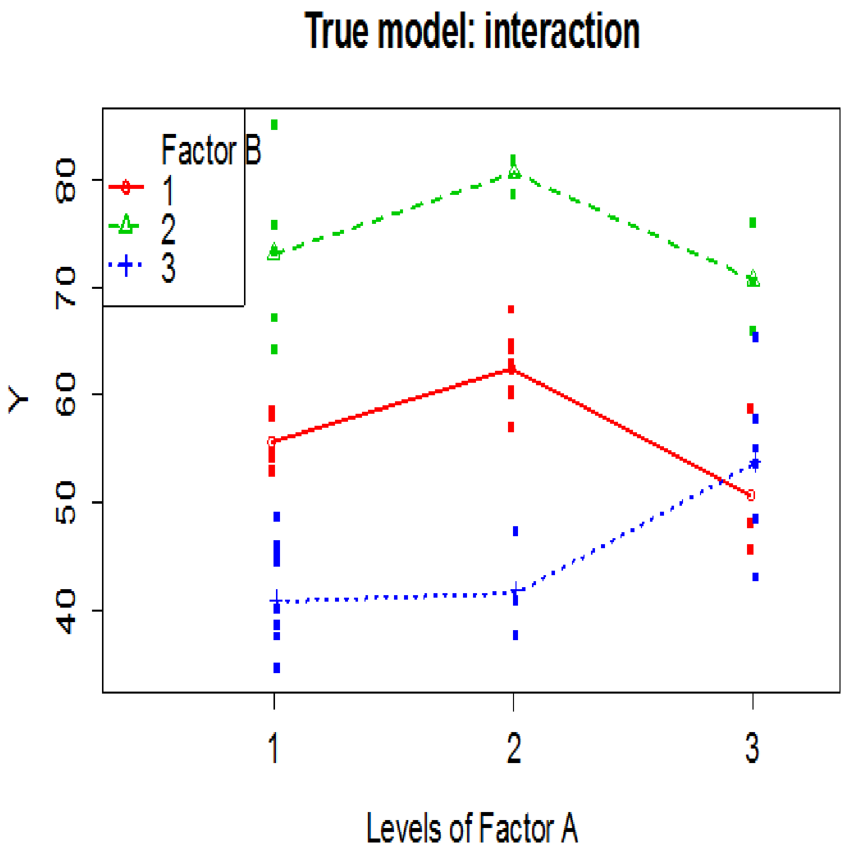
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	48725.296(a)	5	9745.059	2.623	.028
~~Intercept~~	~~250370.370~~	~~1~~	~~250370.370~~	~~67.391~~	~~.000~~
Dose	26398.741	2	13199.370	3.553	.032
Training	15408.333	1	15408.333	4.147	.044
dose * training	6918.222	2	3459.111	.931	.397
Error	378948.333	102	3715.180		
~~Total~~	~~678044.000~~	~~108~~			
Corrected Total	427673.630	107			

The meaning of interaction and interaction (profile) plots

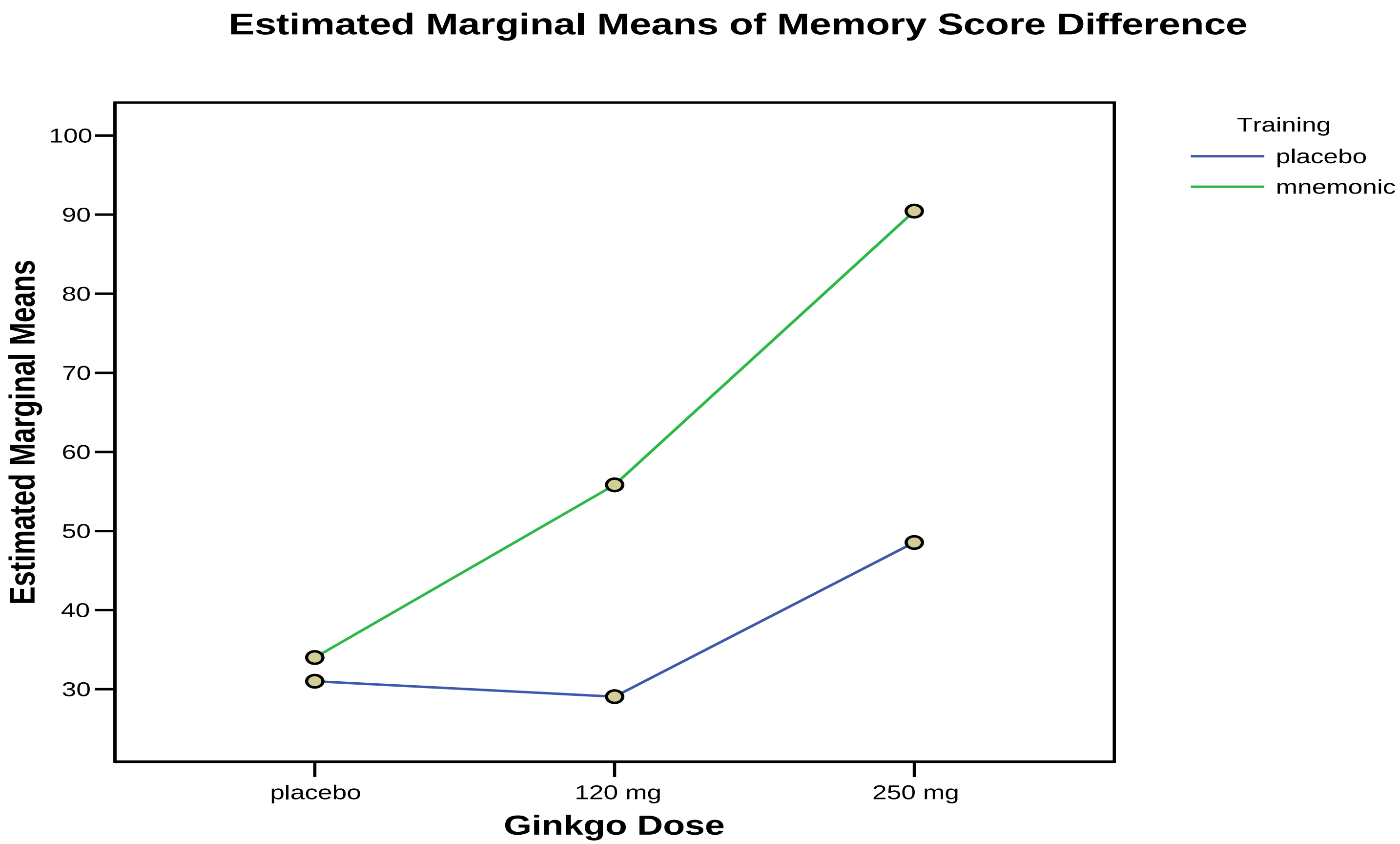
- An interaction is always described as being ***between two (or more)*** ***<u>explanatory</u>*** ***variables,*** and it indicates that ***the effects of changes in level of any one IV on the <u>outcome</u> depends on the level of the other IV***. Also, the interaction is ***not*** between "levels" of one or both of the factors.
- Interaction (profile) plots
  - Be sure to take into account the sampling error.
  - SPSS "profile" plots show the ***current model***, not the truth, i.e. those made with an additive ***model*** are ***always parallel.***
  - "Crossing" is ***<u>not</u>*** synonymous with interaction!!!

Examples of Interaction Plots





Example, cont. : SPSS profile plot



Example: rerun ANOVA without interaction

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	41807.074(a)	3	13935.691	3.756	.013
~~Intercept~~	~~250370.370~~	~~1~~	~~250370.370~~	~~67.481~~	~~.000~~
dose	26398.741	2	13199.370	3.558	.032
training	15408.333	1	15408.333	4.153	.044
Error	385866.556	104	3710.255		
~~Total~~	~~678044.000~~	~~108~~			
Corrected Total	427673.630	107			

1. Ginkgo Dose

Dependent Variable: Memory Score Difference

Ginkgo Dose	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
placebo	32.500	10.397	11.882	53.118
120 mg	40.972	10.397	20.354	61.590
250 mg	69.500	10.397	48.882	90.118

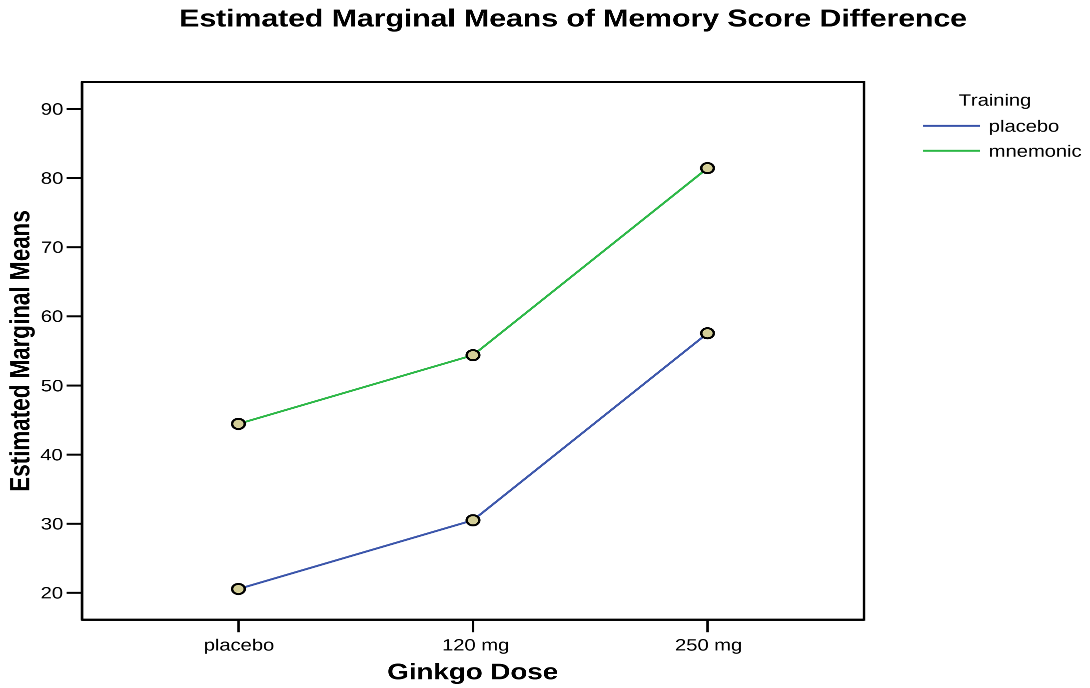
2. Training

Dependent Variable: Memory Score Difference

Training	Mean	Std. Eror	95% Confidence Interval	
			Lower Bound	Upper Bound
placebo	35.222	8.489	18.388	52.057
mnemonic	60.093	8.489	43.258	76.927

Example, cont.

- Additive model profile plot (fitted values):

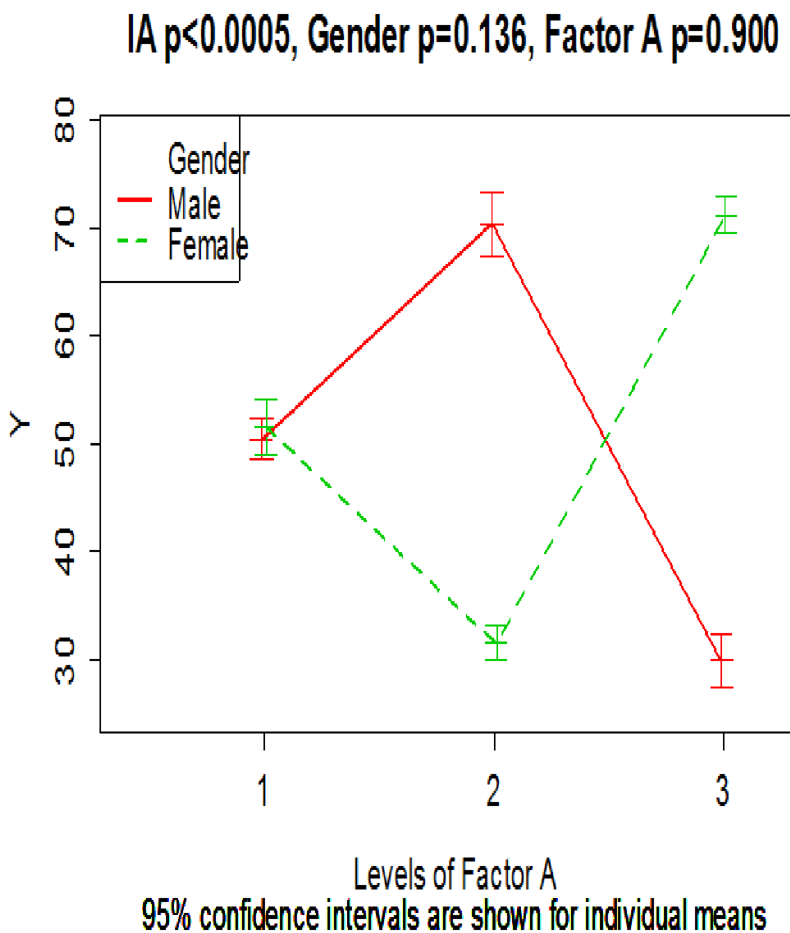


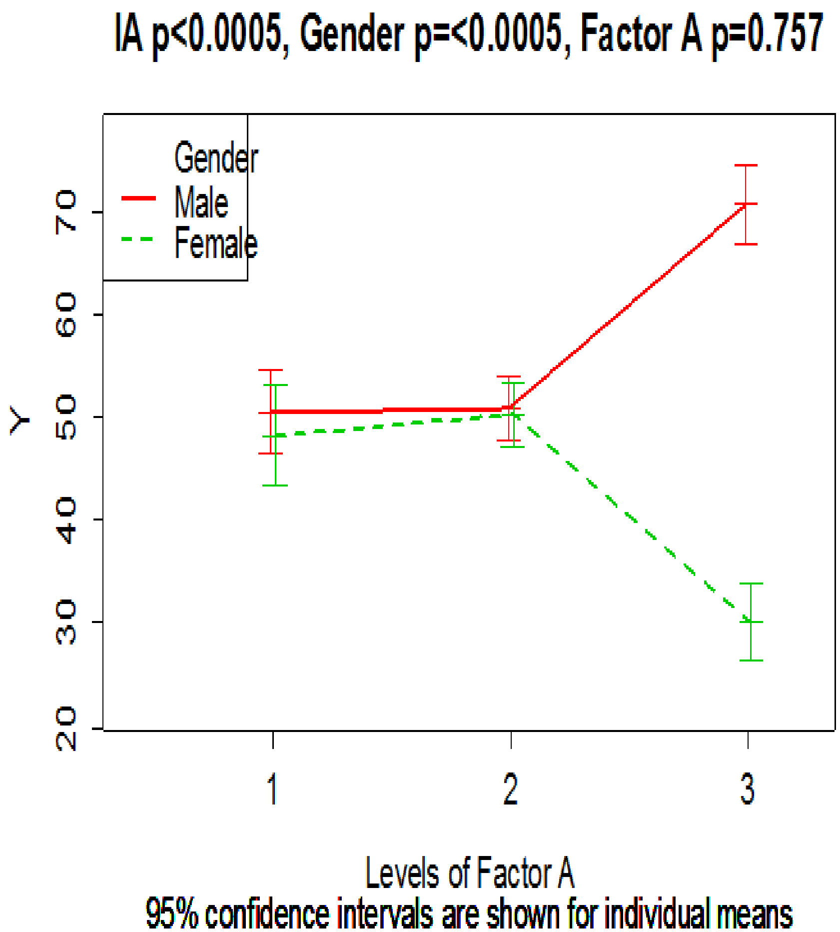
Two-way ANOVA interpretation

- **Which analysis should we use to estimate treatment effects?**
  - If the "no interaction" null hypothesis is rejected, use the "with interaction" model's ANOVA.
  - If the "no interaction" null hypothesis is not rejected ("retained"), use the additive model's ANOVA. (This may be making a false assumption of no interaction. i.e. a type-2 error.)

- If the p-value for ***interaction is significant***, **<u>do not</u>** interpret the "main effects" (dose and training); simply state that both of the explanatory variables affect the outcome in such a way that ***the effects of each factor depends on the level of the other factor***. (Contrasts and the profile plots add more interpretability.)

Two-way ANOVA interpretation





Two way ANOVA interpretation

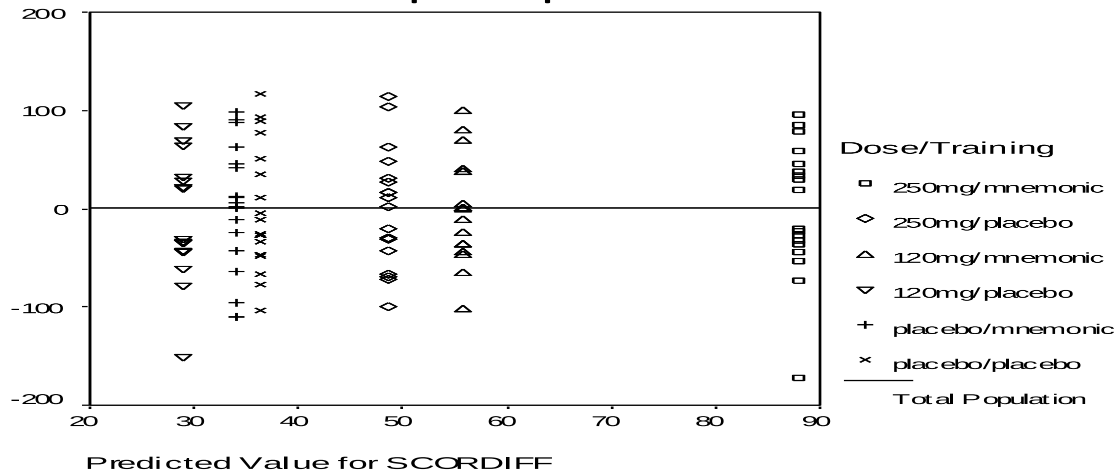
- ***When the interaction is <u>not</u> statistically significant*** interpret each main effect relative to the null hypothesis of equal means across all levels using language like "ignoring" or "averaging over" or "at each level of" the other factor(s).

  - If a factor's p-value is statistically significant (≤0.05) and the factor has just ***two levels*** (like training) look at which of the two levels has the higher mean and make a statement like "mnemonic training improves memory at each dosage" or, better, "mnemonic training improves memory by 24.1 points on average at each dosage". Adding a 95% CI is even better (1.1 to 48.7 point rise).

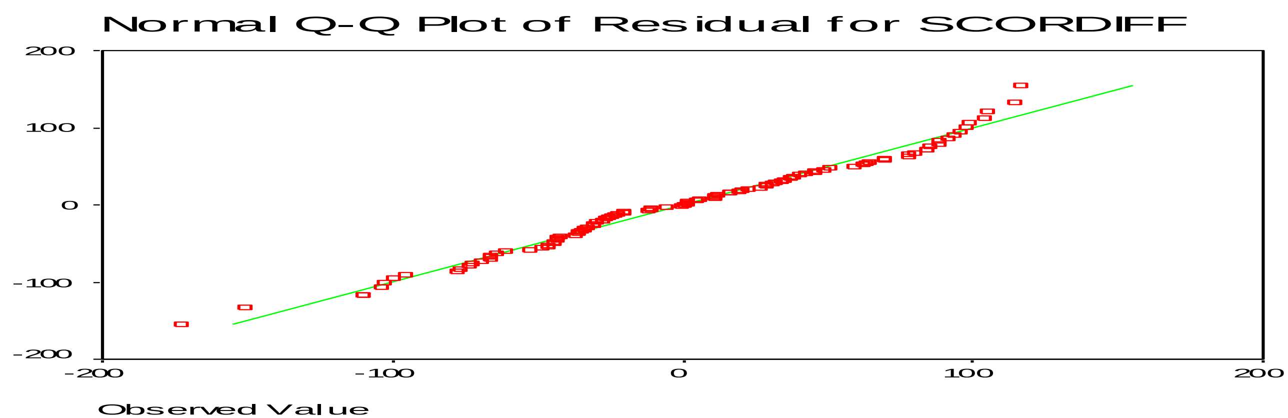
  - If a factor's p-value is statistically significant (≤0.05) and the factor has ***more than two levels*** (like dose), then we reject $H_0{:}\mu_1=\ldots=\mu_k$. Simply state that the (population) mean of the memory score difference for both training types "varies by dose" or "depends on dose" or "differs for at least two doses" for both levels of training. (With contrast testing we can make more detailed conclusions.)

Model checking: Residual Analysis

- Residual vs. fit checks equal spread and need for interaction



- Quantile normal checks for Normality (robust)



Three way ANOVA

Subjects: infants

Setup: new toy (one per child) introduced along with distracting sounds

Outcome: attention (amount of time till distraction)

Explanatory variables: Age of child (8,10,12 months)
Size of toy (small vs. large)
Color of toy (red vs. yellow vs. transparent)

Color:size interaction (but no three-way interaction):

Three way ANOVA, cont.

Three way interaction:

Three way ANOVA, cont.

Results:

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	25000	8	3125	6.25	0.000
Intercept	4000	1	4000	8.00	0.006
Color	7000	2	3500	7.00	0.002
Size	3000	1	3000	6.00	0.017
Color*Size	5000	2	2500	5.00	0.009
Age	6000	2	3000	6.00	0.004
Error	35000	70	500		
Corrected Total	60000	78	769		

Summary

- Multi-way between-subjects ANOVA is used for a ***quantitative DV***, ***independent errors*** and any number of ***categorical IVs***.
- With no interactions (additive model) it is assumed that the ***effect on the DV of any level change in each specific IV*** is a fixed value ***unaffected by the level(s) of the other factor(s)***.
- ***Interactions*** may be present ***between any 2 or more IVs*** (whole variable, not levels!!!) in their effects on the DV. This gives a ***non-parallel plot*** and says that for at least some level changes of a specific IV, the effect on the DV ***depends on (varies with)*** the level(s) of some other IV(s).