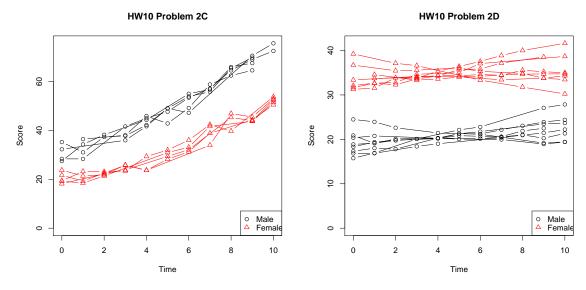
4/1/2010 36-402/608 ADA-II H. Seltman Handout #19: Mixed Models, part 4

1. Review



 $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{b} + \boldsymbol{\epsilon} \quad \mathbf{b} \sim N(\mathbf{0}, \mathbf{G}) \quad \boldsymbol{\epsilon} \sim N(\mathbf{0}, \mathbf{R})$

Means model equation for all times (t), male or female (M=1/0), and for a particular wallaby W:

$$Y_{tW} = [(\beta_0 + \beta_M M) + b_{0W}] + [\beta_T + b_{1W}]t + \epsilon_{tW}$$

or

$$Y_{tW} = [(\beta_0 + \beta_M M) + b_{0W}] + [(\beta_T + \beta_{TM} M) + b_{1W}]t + \epsilon_{tW}$$

- (a) Some generic language: hierarchy is "groups" containing "items"
- (b) Fixed effects and fixed effects parameters (β s) describe the relationship between the outcome and both group-level and item-level explanatory variables, but only for "typical" or "average" groups. $\beta_k = 0$ indicates a useless covariate.
- (c) Random effects describe variation in parameter values from group to group (when that makes sense). A *variance*, say τ_0^2 equal to zero indicates no group-to-group variation in a parameter, i.e., a useless random effect.
- (d) Random effects represent unmeasured group-level covariates and how they affect all items within the group in the same way. Theoretically, if enough group-level covariates are measured and incorporated into the fixed effects model, the random effects will disappear (have zero variance).

- (e) Because the groups in an experiment are thought of as a random selection of groups from a population of groups, estimating the group effect for a particular group is not of interest, and the *variance* of the group effect is estimated instead. In other words, the levels of a group variable are a random selection from a larger set of levels, while fixed effect levels are just the ones of interest, and are the *same* ones that would be used if we were to repeat the experiment.
- (f) You can't have a random slope for an explanatory variable that only varies between groups but not within groups. E.g., $\beta_{\text{school budget}}$ says how much score increases for each additional \$10,000 of school budget. With "school budget" fixed for a given school, there is no information to estimate how the benefit of an additional \$10,000 would vary from school to school.
- (g) You can have a random intercept with no slopes at all, e.g., students within classrooms where the "errors" for students in the same classroom are correlated, but when no quantitative variables are measured.
- (h) You can have many random slopes in the same study, not just for time, time², etc., but also when there are many items in each group with several quantitative per-item measurements.
- 2. Breakout and Discussion