## Nonlinear Boundary Crossing Probabilities for Diffusion Processes

by

Liqun Wang University of Manitoba Department of Statistics, University of Manitoba Winnipeg, Manitoba, Canada R3T 2N2 liqun\_wang@umanitoba.ca

## Abstract

The boundary crossing probabilities (BCP), or the first passage time (FPT) distributions, play an important role in many scientific fields including biology, genetics, economics, engineering, epidemiology, finance, physics, seismology and statistics. However, the computation of BCP for nonlinear boundaries is a long-standing and challenging problem, and explicit analytic solutions do not exist except for a few instances. Traditionally, the mainstream of research focuses on solving certain integral or differential equations for the FPT density. These methods are usually limited to one-sided and smooth boundaries only, and the accuracy of the numerical solutions is difficult to assess.

In this talk, I will present a so-called BCP-approach which gives an explicit integral representation of the probabilities that a Brownian motion crosses any piecewise linear boundary. This formula can be used to approximate the crossing probabilities for general nonlinear boundaries. This technique is further extended to a class of commonly used diffusion processes including Ornstein-Uhlenbeck processes and geometric Brownian motion with time-dependent drift. The numerical computation is done using Monte Carlo integration which is straightforward and easy to implement. Further extensions of this approach to jump-diffusion processes will also be discussed. Some numerical examples will be presented to illustrate this technique.