Sparse Least Squares Estimation

by
Cun-Hui Zhang
Rutgers University
Department of Statistics and Biostatistics, Busch Campus
110 Frelinghuysen Road, Piscataway, NJ 08854, USA
`czhang@stat.rutgers.edu`

Abstract

Estimation of a sparse high-dimensional signal vector is well understood if the signal is directly observed with white noise. If the signal vector belongs to a small ℓ_r ball of radius R, threshold estimators at a certain level λ_{mm} approximately attain the minimax risk. We show that in linear regression models with n data points and $p > n$ unknowns, this result can be extended with penalized least squares estimators, provided that the number of significant components or R^r/λ_{mm}^r of the signal vector is no greater than a certain d_*. The order of this d_* could be as high as $n/\log(p/n)$.