On normal approximations to U-statistics

by

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Abstract

Let $X_1, \ldots, X_n$ be i.i.d. random observations. Let $\mathbb{S} = \mathbb{L} + \mathbb{T}$ be a U-statistics of order $k \geq 2$, where $\mathbb{L}$ is a linear statistic having asymptotic normal distribution, and $\mathbb{T}$ is a stochastically smaller statistic. We show that the rate of convergence to normality for $\mathbb{S}$ can be simply expressed as the rate of convergence to normality for the linear part $\mathbb{L}$ plus a correction term, $(\operatorname{var} \mathbb{T}) \ln^2(\operatorname{var} \mathbb{T})$, under the condition $\mathbb{E}\, \mathbb{T}^2 < \infty$. An optimal bound without this log factor is obtained under a lower moment assumption $\mathbb{E}\, |\mathbb{T}|^\alpha < \infty$ for $\alpha < 2$. Some other related results are also obtained in the paper. Our results extend, refine and yield a number of related known results in the literature.

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