Stochastic Combinatorial Optimization: From the TSP and MST to Dogapillars

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Perhaps the two most famous problems in Euclidean combinatorial optimization are the traveling salesman problem (TSP) where one considers the shortest tour through n points, and the minimal spanning tree (MST) problem where one considers the tree of minimal length that covers the n points. In this lecture we first review some of the extensive work that has been done on the stochastic versions of these problems where the n points are chosen independently from a given distribution. We then look at problems that interpolate between the TSP and the MST; the simplest example being the "spanning caterpillar." In graph theory, a caterpillar is a graph which has a path that when removed leaves only a collection of disjoint "stars." We then sketch a proof of the fundamental theorem on spanning caterpillars — the analog of the Beardwood, Halton, Hammersley theorem for the TSP. Finally, we consider a new rich class of spanning graphs that are called "dogapillars," or more precisely k-dogapillars. Despite the silly name, these graphs provide a satisfying way to interpolate the full range of graphs between the TSP and the MST. As a consequence, they unify and extend our understanding of the probability theory of the TSP and MST.