36-752 – Advanced Probability Overview
Spring 2013

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Lecture: 9:30-10:30 Mon/Wed/Fri, PH A22.

Texts: 1. *Probability Theory & Measure Theory*, 2nd Ed., by R. Ash and C. Doléans-Dale.
2. *Probability, Theory and Examples*, 4th Ed., by Rick Durrett.

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`http://www.stat.cmu.edu/~jinglei/spring13.html`

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Overview: This is a one-semester course designed to cover two semesters of probability and measure theory. Something will have to give way. Mostly, the proofs will give way. But we will try to prove some of the most important theorems when those proofs illustrate important techniques and concepts. All of the challenging new material in this course will involve limiting operations. If you are uncomfortable with limits or lack intuition about them, this course will be very unpleasant.

Topics

1. σ-fields, measures, probabilities, extension and uniqueness of probabilities.

2. Measurable functions, random variables, distributions, and integration.

3. Integration and limits, convergence theorems.

4. Densities, Radon-Nikodym derivatives.

5. Product spaces and product measures, Fubini's Theorem. Independence.

6. Weak law of large numbers. Convergence of random variables.

7. Borel-Cantalli Lemma, almost sure convergence.

8. L^p space and L^p convergence, Kolmogorov's theorems.

9. Strong law of large numbers.

10. Convergence in distribution and the Central Limit Theorem.

11. Characteristic functions.

12. Conditional probability and expectation.

13. Martingales.

Homework: Homework assignments will be posted weekly on Wednesdays and due on the following Wednesday at the beginning of class.

Grade: homework 30%, midterm test 30%, final exam 40%.