

Comment: How Should Indirect Evidence Be Used?

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Abstract. Indirect evidence is crucial for successful statistical practice. Sometimes, however, it is better used informally. Future efforts should be directed toward understanding better the connection between statistical methods and scientific problems.

Key words and phrases: Bayesian, decision theory, prior information, statistical pragmatism, statistical science.

When Brad Efron speaks about statistical theory and methods we should pay attention. In his talk, as he prefers to call it, he returns to a theme that has surfaced in previous ruminations: his unease with the foundations of statistics and his feeling that there is something missing. In this version he highlights indirect evidence as the aspect of statistical reasoning in need of the theory he yearns for.

The framework of statistical decision theory was created over 50 years ago for small, well-defined problems. Efron seeks an extension to accommodate large datasets where individual observations bear an uncertain relationship to one another. He seems to think such an extension is possible and important for the future of the discipline. Perhaps he is right but, I'm sorry to say, I don't get it. In trying to understand the role of indirect evidence I would examine not theoretical foundations but, instead, the relationship of statistical methodology to scientific inference in the context of specific applications.

Efron begins by citing clinical trials as furnishing “direct evidence” about a question of interest. It is easy to see what he means, but the stereotypical problem in a clinical trial is somewhat special because all the relevant background knowledge has been focused on producing a simple treatment comparison, a comparison that statistical inference will evaluate in a final declarative step. Clinical trials are aimed at treatment policy,

so decision theory is highly relevant. In particular, the concepts of type I and type II error have an unusual immediacy because decisions about patients must be made across a large population.

In the scientific applications I am familiar with, statistical inferences are important, even crucial, but they constitute intermediate steps in a chain of inferences, and they are relatively crude. As Jeffreys pointed out long ago, inferences may be based on estimates and standard errors, and they typically need to be accurate only to first order. Similarly, in using the bootstrap we can get by with a fairly small number of observations from the bootstrap distribution because simulation uncertainty quickly becomes smaller than statistical uncertainty. Furthermore, statistical uncertainty is typically smaller than the unquantified aggregate of the many other uncertainties in a scientific investigation. I tell my students in neurobiology that in claiming statistical significance I get nervous unless the p -value is much smaller than 0.01, and if some refinement of an estimate or p -value changes a conclusion, that indeterminacy itself becomes the story. To be convincing, the science needs solid statistical results, but in the end only a qualitative summary is likely to survive. For instance, in Olson et al. (2000), my first publication involving analysis of neural data, more than a dozen different statistical analyses—some of them pretty meticulous, involving both bootstrap and MCMC—were reduced to the main message that among 84 neurons recorded from the supplementary eye field, “Activity reflecting the direction of the [eye movement] developed more rapidly following spatial than following pattern cues.” The statistical details reported in the paper were important to the process, but

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not for the formulation of the basic finding. Such settings seem to me vastly different than that conceptualized by decision theory. In judging the role of statistical analysis within the general scientific enterprise, I prefer Fisher and Jeffreys to Neyman and Savage.

If science is such a loose and messy process, and inferences so rough and approximate, where does all the statistical effort go? In my view, Jeffreys got it right. State-of-the-art analyses may take months, but they usually come down to estimates and standard errors. The biggest news in the early 1990s was the development, understanding, and propagation of MCMC, which has had an enormous influence on statistical practice. The “Bayesian revolution,” however, in my view, is a misnomer. The most important method in Bayesian inference is what Fisher called the method of maximum likelihood. Most of the time what those people running Markov chains are doing is, essentially, computing MLEs. The “revolution” is really a maximum likelihood/Bayesian synthesis based on EM and Gibbs sampling, and their generalizations. It has shown the power of the insights articulated by Fisher and Jeffreys. (With only a bit of a stretch Dirichlet processes and their relatives may be included as extensions of the basic ideas.) What has advanced over the years is the complexity of the problems we are able to attack, not the fundamental framework.

Data analytic methods comprise both data manipulation—including estimates and standard errors—and interpretation. Manipulation involves the mechanics of statistical inference, interpretation its logic. If I am reading him correctly, Efron seems to be concerned primarily with the latter. To exemplify the kind of “difficult new problems” he has in mind Efron uses a hypothetical issue in applying FDR to neuroimaging, half-brain versus whole-brain analysis. When fMRI first hit the scene, almost 20 years ago, a statistician told me of psychologists who were doing many thousands of voxel-wise t -tests simultaneously. The standard method was to line up the test statistics in ascending order of magnitude, or descending order of p -value, and to pick a threshold that gave them suitable results. In our statistician’s naïvety, we shook our heads with indignation. (I was so much older then . . .) Then FDR came along and provided precisely the same method of data manipulation, but furnished a new interpretation. And it is a wonderful interpretation, very helpful. I think we all appreciate it. However, as its chief accomplishment is to bless the procedure psychologists were already using (but feeling uncomfortable about, due to problems in controlling family-wise

error rate), it is hardly surprising that they like it. I am not by any means an expert in neuroimaging, let alone in diffusion tensor imaging, but I am dubious about the scientific importance of half-brain versus whole brain FDR. I would guess the bigger issues involve connectivity across voxels and the hazards of warping brains from different individuals algorithmically so that their voxels are aligned. I should think a more pressing problem would be to devise within-subject expressions of uncertainty about white matter fibers in regions of potential interest, and a method of combining such things across subjects, within groups. (Apparently initial steps in getting local DTI uncertainty have been taken by Zhu et al., 2007, and by Efron’s former student Armin Schwartzman, 2007, whom he cites.)

In picking on this example I should acknowledge that everyone who discusses statistical methods per se abstracts away from details of the scientific problem—Fisher and Jeffreys did so, too, and it is unavoidable. I just do not yet understand the logical difficulty Efron is concerned about. While I certainly agree that the use of indirect evidence is a major challenge, especially in dealing with large datasets, it seems to me that with the passage of time our existing logical frameworks are treating us remarkably well. Nor do I see any problem with being Bayesian in one analysis and frequentist in another, or even combining the two in a single swoop. The heyday of decision theory referenced by Efron occurred during a time that emphasized pure theory in many parts of academic life. Now we are in a much more utilitarian period and many of us are content to use whatever seems best suited for the task in front of us. As I have argued elsewhere (Kass, 2010), I believe a straightforward philosophy I have called *statistical pragmatism* can incorporate both Bayesian and frequentist inference.

It is tempting to try to formalize the many aspects of direct and indirect evidence that must get weighed together, and it is possible to do so Bayesianly. Like Efron, however, I am wary. In Kass (1983) I commented on a very nice, but ambitious paper by Du-Mouchel and Harris in which they used a Bayesian hierarchical model to combine evidence about cancer across species:

The Bayesian approach has its difficulties, for while it is surely desirable to express [knowledge] explicitly, in particular through models, it is often difficult to do so accurately. Lurking beside each analysis are the interrelated dangers of oversimplification, overstated precision, and neglect of beliefs other than the analyst’s.

Where I may disagree with Efron is that I do not think it is likely to be fruitful to try some other formalization. The problem in such situations is not inadequacy of logic, but rather the unclear relevance of the related evidence. As I said in Kass (1983), I would not want to apply formal methods in the absence of pretty solid theoretical or empirical knowledge.

In tackling the complexities of real-life science, real-life clinical trials, or real-life policy decisions, statisticians can bring unique insight based on statistical expertise combined with nontrivial experience in the substantive area. They then exercise good sense as they go along. My statistical bioinformatics colleague Kathryn Roeder put this well recently when she told me, “I violate type I error all the time. And do you know why? I actually want *to find* those genes!” As Emery Brown and I emphasized in a recent article (Brown and Kass, 2009), this requires new attitudes about training. It also requires an altered notion of our relationship to our collaborators: as Brown and I said, we should put to rest their characterization (used here by Efron) as “clients” and, instead, agree to share responsibility for all aspects of scientific inference—not just statistical ones. In attempting to understand the anatomical basis of dyslexia, of course it matters which part of the brain we focus on, but the choice can not be made in terms of abstract statistical arguments. It should result from closely-knit statistical, neuroimaging, neuroanatomical, and psychological judgment.

Now, I am pretty confident that Efron will agree about this. I bring it up because we judge statistical methods by the two rather different standards of theoretical performance (evaluated either by mathematics or by simulation studies) and apparent effectiveness in answering an applied question. I find it impossible to think about either one without considering the other, and failure on either front serves to veto further contemplation.

I understand Efron’s “indirect evidence” to include anything that could, in principle, be used to help formulate a prior for a Bayesian analysis. My impulse is to come at indirect evidence from an applied perspective, and I think an uneasiness much like Efron’s motivated me in 1990 to begin organizing the workshop series *Case Studies in Bayesian Statistics*. I had the lofty goal of identifying and describing key steps in using scientific and technological knowledge to build good Bayesian models and priors, so as to help turn the art of Bayesian statistical practice into a science. The idea was to gain understanding of statistical effectiveness by examining methods carefully in an applied context, and I pointed to Mosteller and Wallace (1964)

as the archetype. However, I must admit that while the workshops have been very successful as meetings, they never made much progress on the big agenda. The reason was simply that the audience was too diverse scientifically, so that speakers could not get very far into the details of connecting statistics to science that I originally had in mind. In 2002 Emery Brown and I began a series of meetings *Statistical Analysis of Neural Data* which are broader statistically but, due to their narrower scientific focus, may actually be more successful in providing material for learning about statistical methods.

I have been negative about comprehensive Bayesian analyses, yet I have spent much time and effort trying to understand and promote Bayesian methods. In many circumstances Bayesian methods are great, and very hard to beat. The nonparametric regression method BARS, for example (DiMatteo, Genovese and Kass, 2001), began with existing frequentist and Bayesian results on free-knot splines and used reversible-jump MCMC to great advantage; it was difficult to code properly and takes a long time to run on even modestly sized datasets, but I have not seen another general method produce smaller mean-squared error and more accurate coverage probabilities, and I would be surprised to find an alternative that works much better for the problem we designed BARS to solve, namely Poisson regression with smoothly varying means, which is suitable for fitting neural firing rate intensity functions. BARS illustrates a general truism: we may expect Bayes to work well if there is solid knowledge about the problem that can lead to useful formalization, if one is willing to spend the time it takes to be careful, and if one has the computing resources to get the job done. These are big “ifs.” The challenge of indirect evidence is to figure out when they are satisfied.

ACKNOWLEDGMENT

This work was supported in part by NIH Grant MH064537.

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