Bivariate Time Series: Cross-Correlation and Coherence

Suppose $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ are sequences of observations made across time, and the problem is to describe their sequential relationship. For example, an increase in y_t may tend to occur following some increase or decrease in a linear combination of some of the preceding x_t values. This is the sort of possibility that bivariate time series analysis aims to describe.

Example See Fig. 10 of Kaminski *et al.*, 2001, *Biol. Cybern.*; Fig. 2 of Brovelli *et al.*, 2004, *PNAS*. □

The theoretical framework of such efforts begins, again, with stationarity. A joint process $\{(X_t, Y_t), t \in \mathcal{Z}\}$ is said to be *strictly stationary* if the joint distribution of $\{(X_t, Y_t), \ldots, (X_{t+h}, Y_{t+h})\}$ is the same as that of $\{(X_s, Y_s), \ldots, (X_{s+h}, Y_{s+h})\}$ for all integers s, t, h. The process is *weakly stationary* if each of X_t and Y_t is weakly stationary with means and covariance functions μ_X, $\gamma_X(h)$ and μ_Y, $\gamma_Y(h)$, and, in addition, the cross-covariance function

$$\gamma_{XY}(s,t) = E((X_s - \mu_X)(Y_t - \mu_Y))$$

depends on s and t only through their difference $h = s - t$, in which case we write it in the form

$$\gamma_{XY}(h) = E((X_{t_h} - \mu_X)(Y_t - \mu_Y)).$$

Note that $\gamma_{XY}(h) = \gamma_{YX}(-h)$. The *cross-correlation* function of $\{(X_t, Y_t)\}$ is

$$\rho_{XY}(h) = \frac{\gamma_{XY}(h)}{\sigma_X \sigma_Y}$$

where $\sigma_X = \sqrt{\gamma_X(0)}$ and similarly for Y_t.

An important result is the following. Suppose we have

$$Y_t = \beta X_{t-\ell} + W_t \tag{1}$$

where X_t is stationary and W_t is stationary with mean zero and variance σ_W^2, independently of X_s for all s. Here, ℓ is the lag of Y_t behind X_t.

Then

$$\mu_Y = E(\beta X_{t-\ell} + W_t) = \beta \mu_X$$

and

$$\begin{aligned}
\gamma_{XY}(h) &= E((Y_{t+h} - \mu_Y)(X_t - \mu_X)) \\
&= E((\beta X_{t+h-\ell} + W_t - \mu_Y)(X_t - \mu_X)) \\
&= \beta E((X_{t+h-\ell} - \mu_X)(X_t - \mu_X)) + 0
\end{aligned}$$

which gives

$$\gamma_{XY}(h) = \beta \gamma_X(h - \ell). \tag{2}$$

Thus, the cross-covariance function will look like the autocovariance function of X_t, but shifted by lag ℓ. Similarly, the autocorrelation function will be

$$\begin{aligned}
\rho_{XY}(h) &= \frac{\beta \gamma_X(h-\ell)}{\sqrt{\beta^2 \sigma_X^2 + \sigma_W^2}\sigma_X} \\
&= \left(\frac{\beta \sigma_X^2}{\beta \sigma_X^2 + \sigma_W^2} \right)^{\frac{1}{2}} \rho_X(h).
\end{aligned}$$

This provides one of the basic intuitions behind much practical use of cross-correlations: one examines them to look for lead/lag relationships.

The cross-covariance and cross-correlation functions are estimated by their sample counterparts:

$$\hat{\gamma}_{XY}(h) = \frac{1}{n} \sum_{t=1}^{n-h} (x_{t+h} - \bar{x})(y_t - \bar{y})$$

with $\hat{\gamma}_{XY}(-h) = \hat{\gamma}_{YX}(h)$, and

$$\hat{\rho}(h) = \frac{\hat{\gamma}_{XY}(h)}{\hat{\sigma}_X \hat{\sigma}_Y}.$$

A second use of cross-correlation comes from decomposing the cross-covariance function into its spectral components. The starting point for the intuition behind this class of methods comes from Figure 1. There, a pair of phase-shifted cosine functions have a periodic cross-correlation function. Similarly, if $\{X_t\}$ and $\{Y_t\}$ act somewhat like phase-shifted quasi-periodic signals, one might expect to see quasi-periodic behavior in their cross-covariance function. However, the situation is a bit more subtle than this.

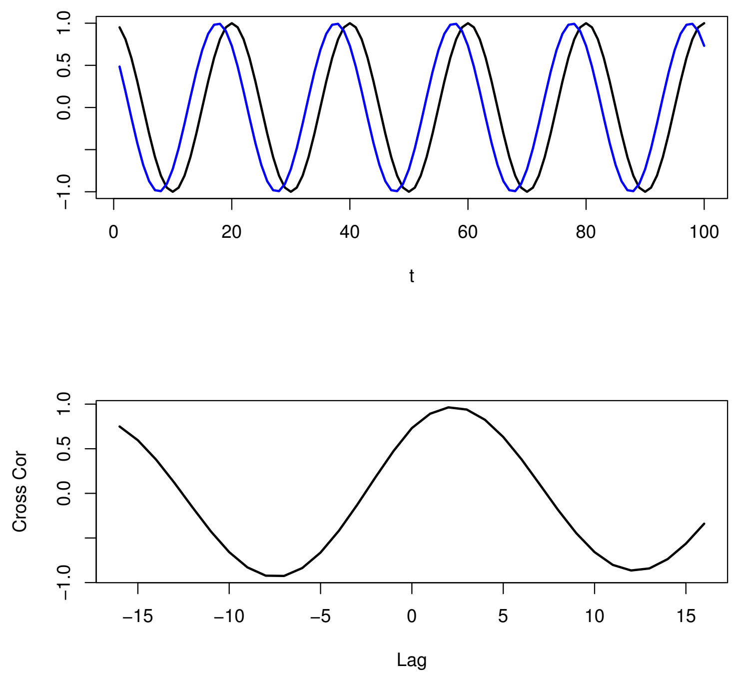

Figure 1: *A pair of phase-shifted cosine functions and their cross-correlation function.*

Recall that for a stationary process $\{X_t; t \in \mathcal{Z}\}$, if

$$\sum_{h=-\infty}^{\infty} |\gamma_X(h)| < \infty \tag{3}$$

then there is a spectral density function $f_X(\omega)$ for which

$$\gamma_X(h) = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{2\pi i\omega h} f_X(\omega) d\omega \tag{4}$$

and

$$f_X(\omega) = \sum_{h=-\infty}^{\infty} \gamma_X(h) e^{-2\pi i\omega h}. \tag{5}$$

We noted that the DFT of the sample autocovariance function was equal to the periodogram (which was also equal to the squared magnitude of the DFT of the data). Smoothed versions of the DFT of the sample autocovariance function thus became estimates of the spectral density.

The bivariate and, more generally, multivariate versions of these basic results are analogous: if

$$\sum_{h=-\infty}^{\infty} |\gamma_{XY}(h)| < \infty$$

then there is a *cross-spectral density function* $f_{XY}(\omega)$ for which

$$\gamma_{XY}(h) = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{2\pi i\omega h} f_{XY}(\omega) d\omega \tag{6}$$

and

$$f_{XY}(\omega) = \sum_{h=-\infty}^{\infty} \gamma_{XY}(h) e^{-2\pi i\omega h}.$$

The cross-spectral density is, in general, complex valued. Because $\gamma_{YX}(h) = \gamma_{XY}(-h)$ we have

$$f_{YX}(\omega) = \overline{f_{XY}(\omega)}. \tag{7}$$

An estimate $\hat{f}_{XY}(\omega)$ of $f_{XY}(\omega)$ may be obtained by smoothing the DFT of sample covariance function $\hat{\gamma}_{XY}(h)$. (The cross-periodogram is the unsmoothed DFT of $\hat{\gamma}_{XY}(h)$.)

A commonly-applied summary of the relationship between $\{X_t\}$ and $\{Y_t\}$ is the *squared coherence*, given by

$$\rho_{XY}^2 = \frac{|f_{XY}(\omega)|^2}{f_X(\omega) f_Y(\omega)}.$$

To gain some interpretation, let us return to (1), generalize it by writing

$$Y_t = \sum_{h=-\infty}^{\infty} \beta_h X_{t-h} + W_t, \tag{8}$$

where W_t is a stationary white noise process independent of $\{X_t\}$, and consider the theoretical least-squares regession problem of finding the coefficients $\{\beta_h\}$ that minimize

$$MSE = E\left(Y_t - \sum_{h=-\infty}^{\infty} \beta_h X_{t-h}\right)^2. \tag{9}$$

In the case of ordinary least squares, where $Y = X\beta + \epsilon$, geometrical arguments (discussed earlier) show that the sum of squares is minimized when β makes the residual orthogonal to the fit, i.e., $\beta = \hat{\beta}$ satisfies $(Y - X\beta)^T X\beta = 0$, and this set of equations (a version of the normal equations) determines the value of $\hat{\beta}$. For later use, let us also note that the residual sum of squares may be written in terms of $R^2 = ||X\hat{\beta}||^2/||Y||^2$ as

$$SS_{error} = ||Y||^2 - ||X\hat{\beta}||^2 = ||Y||^2(1 - R^2). \tag{10}$$

The MSE in (9) is analogous to SS_{error} in (10). Technically, dealing with stochastic processes requires an extension of the theory presented earlier: the set of stationary stochastic processes $Y = \{Y_t, t \in \mathcal{Z}\}$ (assumed, for simplicity, to have mean zero) with inner product

$$< X, Y >= E(X_t Y_t)$$

forms an *infinite-dimensional* vector space (a *Hilbert space*). However, the geometrical arguments go through by essentially the same arguments. In particular, the minimum-MSE solution satisfies the orthogonality condition

$$E\left(\left(Y_t - \sum_{h=-\infty}^{\infty} \beta_h X_{t-h}\right) X_{t-s}\right) = 0 \tag{11}$$

for all $s \in \mathcal{Z}$. Bringing the expectation inside the summation gives the analogue to the normal equations:

$$\sum_{h=-\infty}^{\infty} \beta_h \gamma_X(s-h) = \gamma_{YX}(s) \tag{12}$$

for all $s \in \mathcal{Z}$. We now decompose this into frequency components. We define

$$B(\omega) = \sum_{h=-\infty}^{\infty} \beta_h e^{-2\pi i \omega h}.$$

We next use (4) on the left-hand side and (6) on the right-hand side of (12). For the left-hand side we have

$$\begin{aligned}
\sum_{h=-\infty}^{\infty} \beta_h \gamma_X(s-h) &= \sum_{h=-\infty}^{\infty} \beta_h \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{2\pi i \omega (s-h)} f_X(\omega) d\omega \\
&= \int_{-\frac{1}{2}}^{\frac{1}{2}} \sum_{h=-\infty}^{\infty} \beta_h e^{-2\pi i \omega h} e^{2\pi i \omega s} f_X(\omega) d\omega \\
&= \int_{-\frac{1}{2}}^{\frac{1}{2}} B(\omega) e^{2\pi i \omega s} f_X(\omega) d\omega
\end{aligned}$$

while on the right-hand side we have

$$\gamma_{YX}(t) = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{2\pi i\omega s} f_{YX}(\omega) d\omega$$

and so, by the uniqueness of the Fourier transform, we get the frequency-based version of the normal equations

$$B(\omega) f_X(\omega) = f_{YX}(\omega). \tag{13}$$

Let us now find a frequency-based analogue to the expression (10). Using (11) we may write the MSE as

$$\begin{aligned} MSE &= E\left(\left(Y_t - \sum \beta_h X_{t-h}\right) Y_t\right) \\ &= \gamma_Y(0) - \sum \beta_h \gamma_{XY}(-h) \end{aligned}$$

and transforming this expression we get

$$\begin{aligned} MSE &= \int_{-\frac{1}{2}}^{\frac{1}{2}} f_Y(\omega) d\omega - \sum \beta_h \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-2\pi i\omega h} f_{XY}(\omega) d\omega \\ &= \int_{-\frac{1}{2}}^{\frac{1}{2}} f_Y(\omega) - B(\omega) f_{XY}(\omega) d\omega. \end{aligned}$$

Substituting for $B(\omega)$ using (13) we have

$$MSE = \int_{-\frac{1}{2}}^{\frac{1}{2}} f_Y(\omega) - \frac{f_{YX}(\omega) f_{XY}(\omega)}{f_X(\omega)} d\omega$$

then using (7) and rewriting gives

$$MSE = \int_{-\frac{1}{2}}^{\frac{1}{2}} f_Y(\omega)(1 - \frac{|f_{YX}(\omega)|^2}{f_Y(\omega) f_X(\omega)}) d\omega$$

which, finally, produces

$$MSE = \int_{-\frac{1}{2}}^{\frac{1}{2}} f_Y(\omega)(1 - \rho_{YX}(\omega)^2) d\omega. \tag{14}$$

Thus, as an analogue to (10), $f_Y(\omega)(1 - \rho_{YX}(\omega)^2)$ is the ω-component of MSE in the minimum-MSE fit of (8).

Two more quick observations help with the interpretation of the coherence. First, in (14) $MSE \geq 0$ and $f_Y(\omega) \geq 0$ imply that $0 \leq \rho_{YX}(\omega) \leq 1$ for all ω. The second begins with the noise-free case

$$Y_t = \sum_{h=-\infty}^{\infty} \beta_h X_{t-h}.$$

For simplicity let us assume X_t and Y_t have mean zero and, as before,

$$\sum_{t=-\infty}^{\infty} |\beta_t| < \infty.$$

Defining $B(\omega)$ as above, some manipulation shows that (13) again holds. On the other hand, in discussing linear filters we noted

$$f_Y(\omega) = |B(\omega)|^2 f_X(\omega).$$

Combining this with (13) we have

$$\begin{aligned} \rho_{YX}(\omega) &= \frac{|f_{YX}(\omega)|^2}{f_X(\omega) f_Y(\omega)} \\ &= \frac{B(\omega)^2 f_X(\omega)^2}{f_X(\omega) f_Y(\omega)} \\ &= \frac{f_Y(\omega) f_X(\omega)}{f_X(\omega) f_Y(\omega)} = 1 \end{aligned}$$

for all ω. Taking this together with (14) gives the interpretation that the coherence is a frequency-component measure of linear association between two theoretical time series.

One additional note. The complex-valued quantity $B_{YX}(\omega)$ is often written in polar form:

$$B_{YX}(\omega) = |B_{YX}| \exp(-i\phi_{YX}(\omega)).$$

Notice that in the special case $Y_t = \beta X_{t-h} + W_t$ we have $f_{YX}(\omega) = \beta \exp(-2\pi i\omega h) f_X(\omega)$ and the phase satisfies the linear function $\phi_{YX}(\omega) = -2\pi i h$.

We now consider data-based evaluation of coherence. The coherence may be estimated by

$$\hat{\rho}^2_{XY} = \frac{|\hat{f}_{XY}(\omega)|^2}{\hat{f}_X(\omega) \hat{f}_Y(\omega)}. \tag{15}$$

However, the smoothing in this estimation process is crucial. The raw cross-periodogram $I_{XY}(\omega)$ satisfies the relationship

$$|I_{XY}(\omega)|^2 = I_X(\omega)I_Y(\omega)$$

so that plugging the raw periodograms into (15) will always yield the value 1. This make interpretation of coherence more subtle: it is not trivial to come up with series $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ that have substantial peaks in their estimated spectral densities, but no substantial peak, in the corresponding range of frequencies, for their estimated cross-spectral density.

Here is such an example, starting with a pair of cosines. It involves two steps (i) phase modulation and (ii) contamination of the cosines by an AR(1) process.

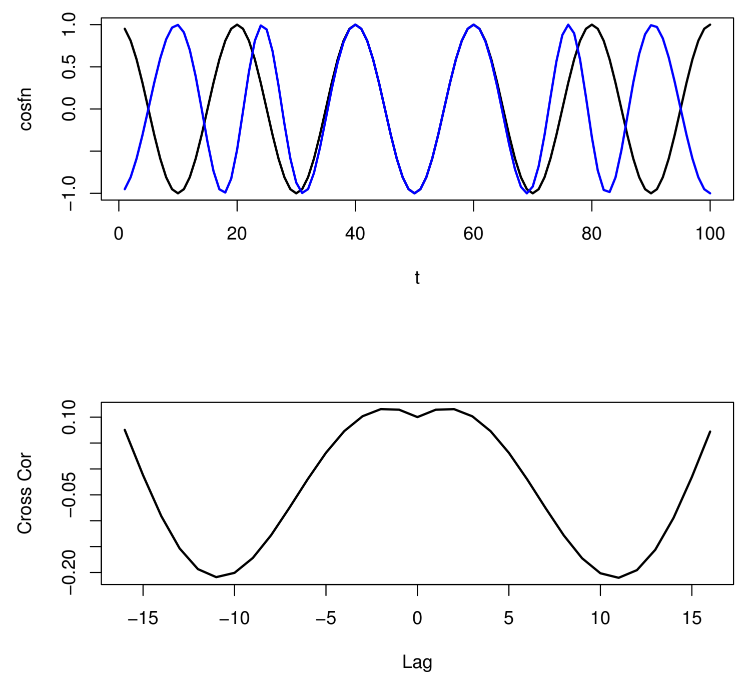

Figure 2: *A pair of cosines, where the second (blue) has its phase modulated so that in the middle of the time interval it is in phase with the other series, but at the ends it drifts to opposite phase. This decreases the maginitude of the cross-correlation.*

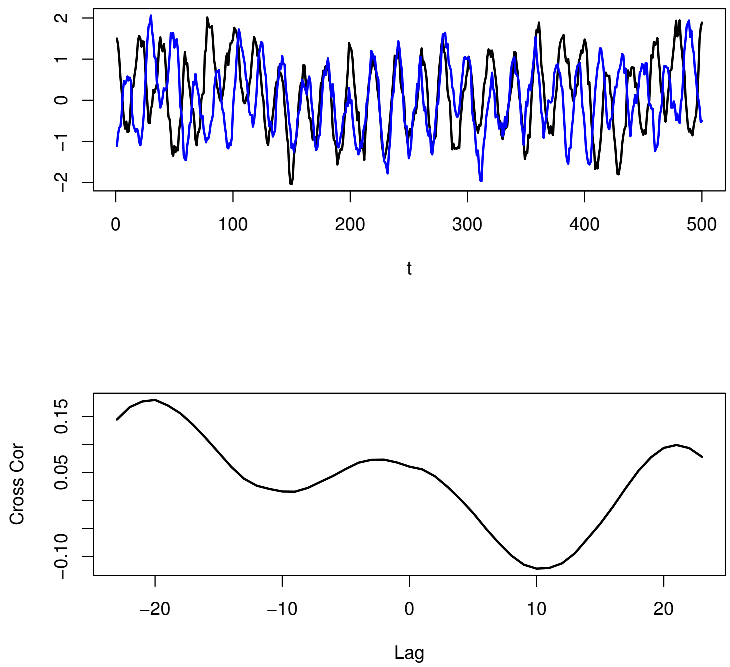

Figure 3: *Cross-correlation for two generated series.*

Granger Causality

A way of examining the directional dependence among two stationary processes $\{X_t, t \in \mathcal{Z}\}$ and $\{Y_t, t \in \mathcal{Z}\}$ begins by assuming they may be represented in the form of time-dependent regression. A paper by Geweke (1982,*J. Amer. Statist. Assoc.*) spelled this out nicely, based on earlier work by Granger.

The idea is very simple. In ordinary regression we assess the influence of a variable (or set of variables) X_2 on Y in the presence of another variable (or set of variables) X_1 by examining the reduction in variance when we compare the regression of Y on (X_1, X_2) with the regression of Y on X_1 alone. If the variance is reduced sufficiently much, then we conclude that X_2 helps explain (predict) Y. Here, we replace Y with Y_t, replace X_1 with $\{Y_s, s < t\}$ and X_2 with $\{X_s, s < t\}$. In other words, we examine the additional contribution to predicting Y_t made by the past observations of X_s after accounting for the autocorrelation in $\{Y_t\}$. The "causality" part comes when the past of X_s helps predict Y_t but the past of Y_s does *not* help predict X_t.

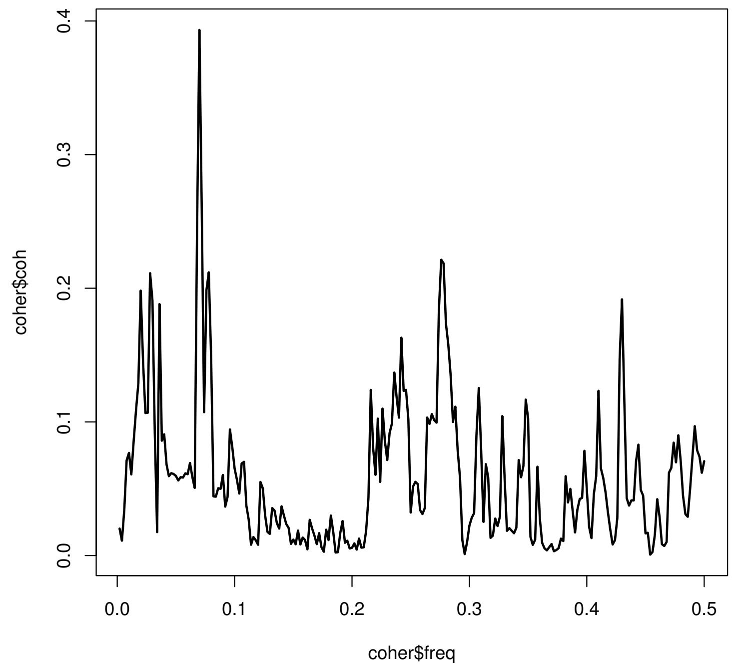

Figure 4: *Estimated spectral densities and coherence for series in Fig 3.*

In Geweke's notation, suppose

$$\begin{aligned} X_t &= \sum_{s=1}^{\infty} E_{1s} X_{t-s} + U_{1t} \\ Y_t &= \sum_{s=1}^{\infty} G_{1s} Y_{t-s} + V_{1t} \end{aligned}$$

with $V(U_{1t}) = \Sigma_1$ and $V(V_{1t}) = T_1$, and then suppose further that

$$\begin{aligned} X_t &= \sum_{s=1}^{\infty} E_{2s} X_{t-s} + \sum_{s=1}^{\infty} F_{2s} Y_{t-s} + U_{2t} \\ Y_t &= \sum_{s=1}^{\infty} G_{2s} Y_{t-s} + \sum_{s=1}^{\infty} H_{2s} X_{t-s} + V_{2t} \end{aligned}$$

where now $V(U_{1t}) = \Sigma_2$ and $V(V_{1t}) = T_2$. The residual variation in predicting Y_t from the past of Y_t is given by T_1 and that in predicting Y_t

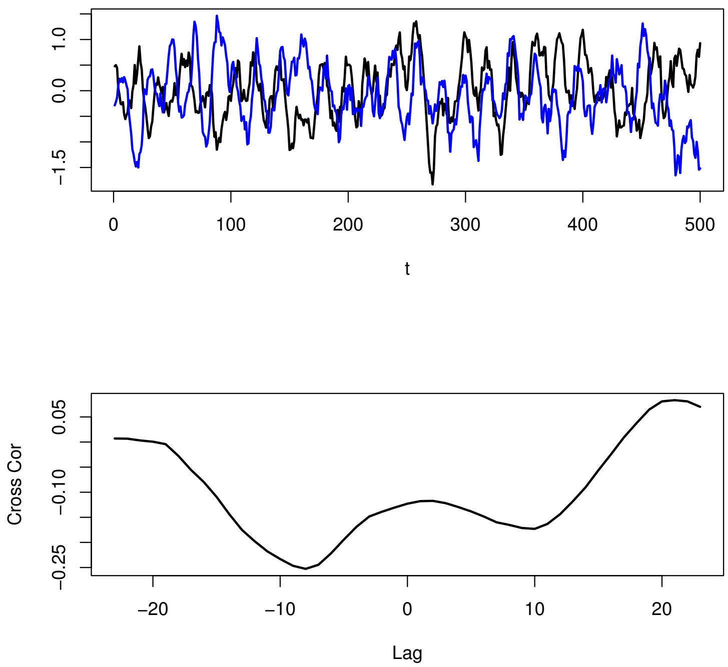

Figure 5: *Cross-correlation for two generated series.*

from the past of Y_t together with the past of X_t is given by T_2. Granger suggested the strength of "causality" (predictability) be measured by

$$GC = 1 - \frac{|T_1|}{|T_2|}.$$

Geweke recommended the modified form

$$F_{X \to Y} = \log\left(\frac{|T_1|}{|T_2|}\right).$$

Geweke also suggested a decomposition of $F_{X \to Y}$ into frequency components. That is, he gave an expression for $f_{X \to Y}(\omega)$ such that

$$F_{X \to Y} = \int_{\frac{1}{2}}^{\frac{1}{2}} f_{X \to Y}(\omega) d\omega.$$

In applications, the basic procedure is to (i) fit a bivariate AR model (using, say, AIC to choose the orders of each of the terms); this modifies the equations above by making them finite-dimensional; then (ii) test

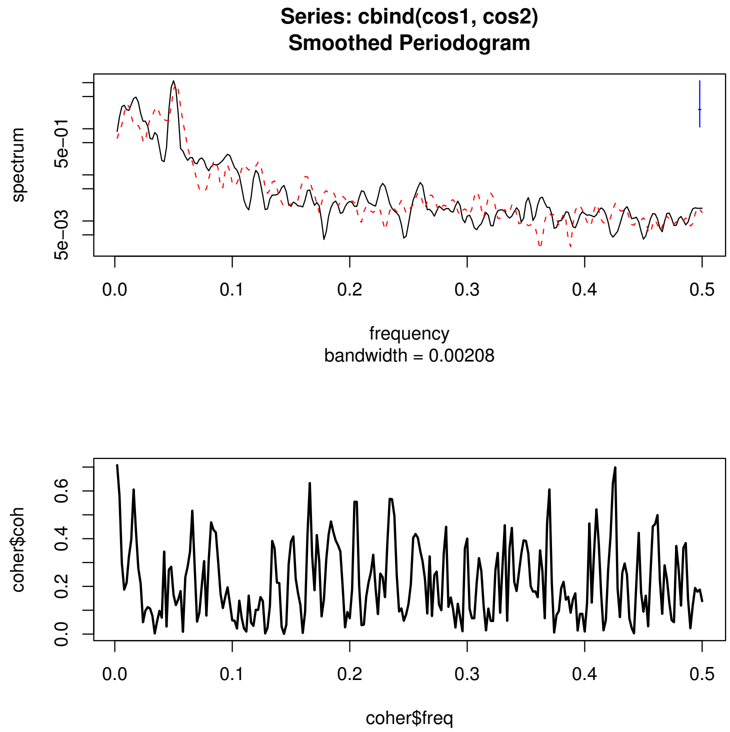

Figure 6: *Estimated spectral densities and coherence for series in Fig 5.*

the hypothesis $H_{2s} = 0$ for all s and also the hypothesis $F_{2s} = 0$ for all s. This tests whether $F_{X \to Y} = 0$ and $F_{Y \to X} = 0$.

As an illustration, I simulated a bivariate time series of length 1000 using the model

$$\begin{aligned} X_t &= .5X_{t-1} + U_t \\ Y_t &= .2Y_{t-1} + .5X_{t-1} + V_t \end{aligned}$$

where $U_t \sim N(0, (.2)^2)$ and $V_t \sim N(0, (.2)^2)$, independently. I then fit a linear regression model of the form

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 X_{t-1} + \epsilon_t$$

and, similarly, fit another model of the same form but with the roles of X and Y reversed.

```
n=1000
x=arima.sim(list(order=c(1,0,0),ar=.5),sd=.5,n=n)
```

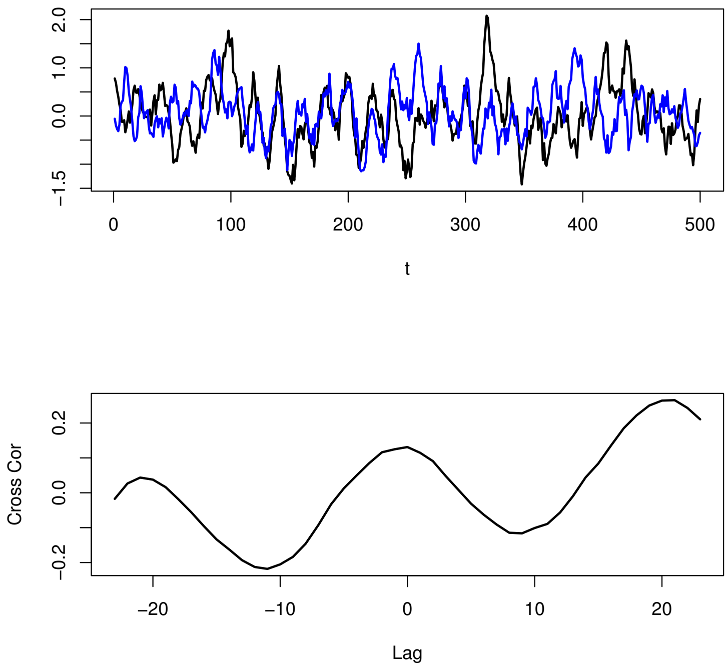

Figure 7: *Cross-correlation for two generated series.*

```
y=null
y[1]=rnorm(1,0,.2)
for(i in 2:n){
y[i]=.5*x[i-1]+.2*y[i-1] + rnorm(1,0,.2)
}
y.xy=lm(y[2:n]~x[1:(n-1)]+y[1:(n-1)])
x.xy=lm(x[2:n]~x[1:(n-1)]+y[1:(n-1)])
summary(y.xy)
summary(x.xy)
#####################
# OUTPUT
#
#Call:
#lm(formula = y[2:n] ~ x[1:(n - 1)] + y[1:(n - 1)])
#
#Residuals:
#      Min         1Q    Median        3Q       Max
#-0.605066 -0.132689  0.001135  0.126644  0.657045
#
```

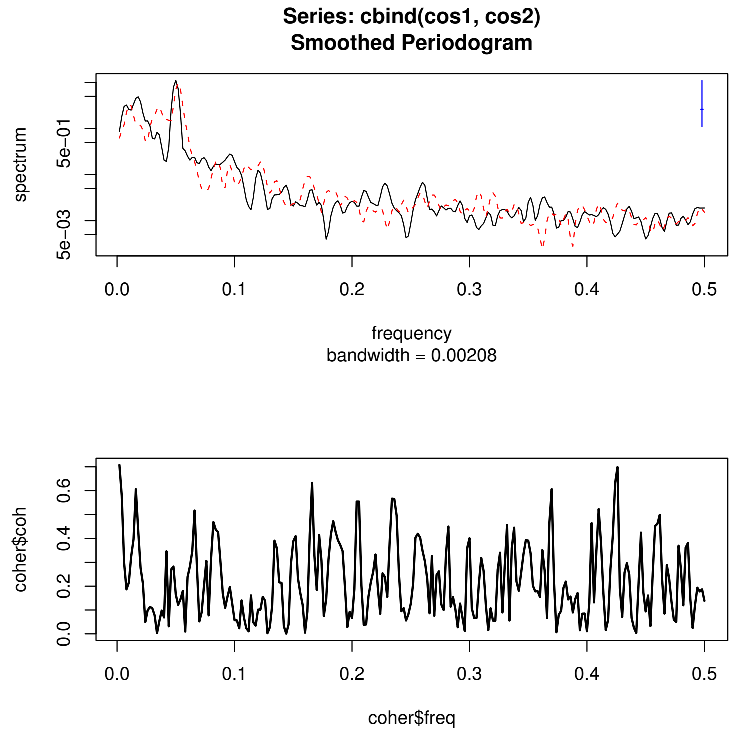

Figure 8: *Estimated spectral densities and coherence for series in Fig 7.*

```
#Coefficients:
#               Estimate Std. Error t value Pr(>|t|)
#(Intercept)   -0.001310   0.006223  -0.211    0.833
#x[1:(n - 1)]   0.495680   0.011589  42.773   <2e-16 ***
#y[1:(n - 1)]   0.191902   0.017811  10.774   <2e-16 ***
#
#Call:
#lm(formula = x[2:n] ~ x[1:(n - 1)] + y[1:(n - 1)])
#
#Residuals:
#      Min        1Q    Median        3Q       Max
#-1.766162 -0.336188 -0.009646  0.319320  1.520826
#
#Coefficients:
#               Estimate Std. Error t value Pr(>|t|)
#(Intercept)    0.008571   0.016003   0.536    0.592
#x[1:(n - 1)]   0.508429   0.029802  17.060   <2e-16 ***
#y[1:(n - 1)]  -0.055286   0.045803  -1.207    0.228
```

As expected, the first fit indicates that X_{t-1} provides additional information beyond Y_{t-1} in predicting Y_t, but Y_{t-1} does *not* provide additional information beyond X_{t-1} in predicting X_t. This is sometimes summarized by saying X_t is *causally* related to Y_t, but we must keep in mind that "causal" is used in a predictive, time-directed sense.

This illustration sweeps under the rug the model selection part of the problem, item (i) mentioned above. In applications this is non-trivial. It is often handled by assuming the *bivariate* process follows an $AR(p)$. This means that we write, say, $Z_t = (X_t, Y_t)^T$ and, in the case of $AR(1)$,

$$Z_t = \alpha + \Phi Z_{t-1} + W_t$$

where Φ is a 2×2 matrix so that we have a pair of equations

$$\begin{aligned} X_t &= \alpha_1 + \Phi_{11} X_{t-1} + \Phi_{12} Y_{t-1} + W_{t1} \\ Y_t &= \alpha_2 + \Phi_{21} X_{t-1} + \Phi_{22} Y_{t-1} + W_{t2}. \end{aligned}$$

It is straightforward to perform bivariate regression, or ML estimation; model selection is then often based on AIC. This simplification avoids separate consideration of the X and Y terms for each response variable, but of course it comes at the cost of possible lack of fit.

R Code for Figs

```
# FUNCTION DEFINITIONS
plotl=function(x,y,lwd=2,xlab="t",ylab=" ",...){
  plot(x,y,type="l",lwd=lwd,xlab=xlab,ylab=ylab,...)}
phasemod=function(n,p=.2){
  nr=floor(n*p/2)
  ans=rep(0,n)
  F=pbeta(((1:nr)-.5)/nr,5,5)
  ans[1:nr]=pi*F-pi
  ans[(n+1-nr):n]=pi*F
  ans}
# FIG PRODUCTION
t=1:100
cosfn=cos(2*pi*t*.05)
cosfn.ph=cos(2*pi*t*.05+.75)
plotl(t,cosfn)
lines(t,cosfn.ph,type="l",col="blue",lwd=2)
cross=ccf(cosfn,cosfn.ph,plot=FALSE)
```

```
plot(cross$lag,cross$acf,type="l",lwd=2,ylab="Cross Cor",xlab="Lag")
# dev.print...
cosfn.phmod2=cos(2*pi*t*.05+phasemod(100,p=.9))
plot(t,cosfn,type="l",lwd=2)
lines(t,cosfn.phmod2,type="l",col="blue",lwd=2)
cross=ccf(cosfn,cosfn.phmod2,plot=FALSE)
plot(cross$lag,cross$acf,type="l",lwd=2,ylab="Cross Cor",xlab="Lag")
dev.print(device=postscript,file="cos.crosscor.phmod.ps")
```

3 through 8

```
cos.cross=function(a=.2,sig1=.2,sig2=.2){
cosfn=cos(2*pi*t*.05)
cos1=a*cosfn+arima.sim(list(order=c(1,0,0),ar=.9),n=nt,sd=sig2)
cos2=a*cosfn+arima.sim(list(order=c(1,0,0),ar=.9),n=nt,sd=sig2)
list(cos1=cos1,cos2=cos2)
}
cos.plot1=function(cos1,cos2){
par(mfrow=c(2,1))
plot(t,cos1,type="l",lwd=2,ylab="",
      ylim=c(min(c(cos1,cos2)),max(c(cos1,cos2))))
lines(t,cos2,type="l",col="blue",lwd=2)
cross=ccf(cos1,cos2,plot=FALSE)
plot(cross$lag,cross$acf,type="l",lwd=2,ylab="Cross Cor",xlab="Lag")
}
cos.plot2=function(cos1,cos2){
par(mfrow=c(2,1))
coher=spec.pgram(cbind(cos1,cos2),spans=c(3,3))
plot(coher$freq,coher$coh,type="l",lwd=2)
}
# FIG PRODUCTION STARTS HERE
nt=500
t=1:nt
# For first pair of figs
out=cos.cross.ph(1,.2,.2)
cos.plot1(out$cos1,out$cos2)
dev.print(device=postscript,file="cc3.ps")
cos.plot2(out$cos1,out$cos2)
dev.print(device=postscript,file="coh3.ps")
# For latter two pairs of figs
out=cos.cross.ph(.5,.2,.2)
```

```
cos.plot1(out$cos1,out$cos2)
dev.print(device=postscript,file="cc3.ps")
cos.plot2(out$cos1,out$cos2)
dev.print(device=postscript,file="coh3.ps")
```