Homework 10
36-325/725
due Friday Nov 16

(1) Let $X_1, \ldots, X_n \sim \text{Uniform}(a, b)$ where a and b are unknown parameters and $a < b$.

(1a) Find the mle $\hat{a}$ and $\hat{b}$.

(1b) Let $\tau = \int x dF(x)$. Find the mle of τ.

(1c) Let $\hat{\tau}$ be the mle from (1b). Let $\tilde{\tau}$ be the nonparametric plug-in estimator of $\tau = \int x dF(x)$. Suppose that $a = 1$, $b = 3$ and $n = 10$. Find the MSE of $\hat{\tau}$ by simulation. Find the MSE of $\tilde{\tau}$ analytically. Compare.

(2) Let $X \sim N(\mu, \sigma^2)$. Find the Fisher information matrix $I(\mu, \sigma)$ and find the inverse Fisher informatiom matrix I^{-1}.

(3) Let $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$. Let τ be the .95 percentile, i.e. $P(X < \tau) = .95$. Find the mle of τ. Find an expression for a $1 - \alpha$ confidence interval for τ.

(4) Let $X_1, \ldots, X_n \sim \text{Uniform}(0, \theta)$. Show that the mle is consistent. Hint: Let $Y = \max\{X_1, ..., X_n\}$.. For any c, $P(Y < c) = P(X_1 < c, X_2 < c, ..., X_n < c) = P(X_1 < c)P(X_2 < c)...P(X_n < c)$.

(5) Let $X \sim \text{Poisson}(\lambda)$. Find the Fisher information $I(\lambda)$.

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(6) Let $X_1, ..., X_n \sim N(\theta, 1)$. Define

$$Y_i = \begin{cases} 1 & \text{if } X_i > 0 \\ 0 & \text{if } X_i \leq 0. \end{cases}$$

Let $\psi = Pr(Y_1 = 1)$.

(a) Find the maximum likelihood estimate $\hat{\psi}$ of ψ.

(b) Find an approximate 95 per cent confidence interval for ψ.

(c) Define $\widetilde{\psi} = (1/n)\sum_i Y_i$. Show that $\widetilde{\psi}$ is a consistent estimator of ψ.

(d) Compute the asymptotic relative efficiency of $\widetilde{\psi}$ to $\hat{\psi}$. Hint: Use the delta method to get the standard error of the mle. Then compute the standard error (i.e. the standard deviation) of $\widetilde{\psi}$.

(e) Suppose that the data are not really normal. Show that $\hat{\psi}$ is not consistent. What, if anything, does $\hat{\psi}$ converge to?