Homework 2
36-325/725
due Friday Sept 7

(1) Chapter 1.10 Problem # 11.

Let $A_1, A_2, \ldots$ be an infinite sequence of events such that $A_1 \subset A_2 \subset \cdots$. Show that

$$P\left(\cup_{i=1}^{\infty} A_i\right) = \lim_{n\to\infty} P(A_n).$$

Hint: Define $B_1 = A_1$, $B_2 = A_1^c \cap A_2$, $B_3 = A_1^c \cap A_2^c \cap A_3$, etc. Next show that (i) the $B_n's$ are disjoint, (ii) $\cup_{i=1}^{\infty} A_i = \cup_{i=1}^{\infty} B_i$ and (iii) $\cup_{i=1}^{n} A_i = \cup_{i=1}^{n} B_i$ for every n. Also, it will help to recall the definition of an infinite sum: $\sum_{i=1}^{\infty} x_i = \lim_{n\to\infty} \sum_{i=1}^{n} x_i$.

(2) Chapter 1.11 Problem # 1

Suppose that A and B are independent events. Show that A^c and B^c are independent events.

(3) Chapter 1.13 Problem # 1

Suppose that a fair coin is tossed repeatedly until both a head and tail have appeared at least once.

(a) Describe the sample space S.

(b) What is the probability that three tosses will be required?

(4a) Show that if $P(A) = 0$ or $P(A) = 1$ then A is independent of every other event.

(4b) Show that if A is independent of itself then $P(A)$ is either 0 or 1.

(5) Suppose a coin has probability p of falling heads. If we flip the coin many times, we would expect the proportion of heads to be near p. We will make this formal later. Let's explore the idea now using R (or Splus). Let's pick a value of p and generate n coin flips:

```
p <- .3
n <- 1000
```

```
### generate n coin flips each having prob p
x <- rbinom(n,1,p)
p.empirical <- cumsum(x)/(1:n)
### cumsum computes the cumulative sum
### if you don't see wha this is doing,
### try it for n=5 and look carefully
par(mfrow=c(2,2))   ### put 4 plots per page
plot(1:n,p.empirical,type='l',
     xlab='number of coin flips', ylab='',ylim=c(0,1))
lines(1:n,rep(p,n),lty=3,col=2,lwd=3) ### add the true value of p
```

Experiment with different values of n and p. Hand in a few plots. **Do not** hand in your R code.

(6) Here is a related experiment. Suppose we flip a coin k times. Let X be the number of heads. We call X a binomial random variable. We will discuss this in class in detail. To simulate a value of X:

```
k <- 10
p <- .3
flips <- rbinom(k,1,p)    ### k single flips
print(flips)
X <- sum(flips)
print(X)
```

Alternatively, we can simulate X directly as follows:

```
X <- rbinom(1,k,p)        ### sum of k flips
```

Intuition suggests that X will be close to $k\,p$. To see if this is true, we can repeat this experiment many times and average the X values. Here is one way to do the simulation:

```
nsim <- 1000
output <- rep(0,nsim)
for(i in 1:nsim){
     output[i] <- rbinom(1,k,p)
     }
print(mean(output))
print(k*p)
```

Here is a better way to do the same thing:

```
nsim <- 1000
output <- rbinom(nsim,k,p)
hist(output)          ### draw a histogram of the output
plot(table(output))   ### another way to plot it
print(mean(output))
print(k*p)
```

Try this a few times. How close is X (on average) to $k\,p$? Hand in your histogram.