PRACTICE TEST #1
36-325/725
FALL 2001

(1) Let S be a sample space and let $C \subset S$ be such that $Pr(C) > 0$. Let $Q(A) = Pr(A \mid C)$. Show that Q satisfies the axioms of probability.

(2) Let X have probability density function

$$f_X(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Let $Y = 0$ if $X < 1/2$ and $Y = 2X$ otherwise. Find the distribution function of Y.

(3) Let X have probability density function

$$f_X(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find a random variable $Y = r(X)$ such at Y has density

$$f_Y(y) = \begin{cases} e^{-y} & y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

(4) Suppose we toss a fair, six-sided die repeatedly until we see a 5 or a 6.

(4a) What is the sample space S?

(4b) Suppose that the die is not fair and, in fact, $Pr(\{1\}) = Pr(\{3\}) = Pr(\{5\}) = 2/9$ and $Pr(\{2\}) = Pr(\{4\}) = Pr(\{6\}) = 1/9$. What is the probability that a 5 appears before a 6.

(5) Let X have probability density function

$$f_X(x) = \begin{cases} \frac{1}{2} & 0 < x < 1 \\ \frac{1}{2} & 3 < x < 4 \\ 0 & \text{otherwise.} \end{cases}$$

Find the cumulative distribution function of X.

(6) Let X and Y be independent random variables, each having a Unif(0,1) distribution, i.e. each has the same density f given by

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Let $Z = \max\{X, Y\}$. Find the density of Z.

Hint: First find the joint density $f(x, y)$. Then find cdf of Z i.e. find $F_Z(z) = P(Z \leq z)$. Then find f_Z.

(7) Let X and Y have joint density

$$f_{X,Y}(x, y) = \begin{cases} c(x + y) & 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(7a) Find c.

(7b) Find $f_{Y|X}(y|x)$.

(7c) Find $P(Y > 1/2|X = 1)$.

(7d) Find $P(Y > 1/2|X < 1/2)$.

(8) Suppose that the density for (X, Y) is defined over a rectangle and that $f_{X,Y}(x, y) = g(x)h(y)$ for some functions g and h. Show that X and Y are independent.

(9) Let $X \sim \text{Unif}(0, 1)$ and let $Y = X^2$. Find the density of Y.