(1a)

$$S = \{s = (s_1, \ldots, s_n) : \ s_i \in \{0,1\}, \ s_n = 1, \ \sum_{i=1}^{n} s_i = 2, \ n = 2, 3, \ldots, \}$$

or

$$S = \{s = (s_1, \ldots, s_n) : \ s_i \in \{H, T\}, s_n = H, \ s_j = T \text{ for all but one } j < n, \ n = 2, 3, \ldots, \}.$$

(1b) $P(k\text{ tosses}) = P(\{H,T,T,\ldots,T,H\} + P(\{T,H,T,\ldots,T,H\} + P(\{T,T,H,\ldots,T,H\} + \cdots + P(\{T,T,T,\ldots,H,H\} = (k-1)(1/2)^k$ for $k \geq 2$.

(2a)

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{4} & 0 \leq x < 1 \\ \frac{1}{4} & 1 \leq x < 3 \\ \frac{1}{4} + \frac{3}{8}(x-3) & 3 \leq x < 5 \\ 1 & x > 5. \end{cases}$$

(2b) $F_Y(y) = P(Y \leq y) = P(1/X \leq y) = P(X > 1/y)$. The three "zones" for x, $0 \leq x \leq 1$, $1 \leq x \leq 3$, $3 \leq x \leq 5$, correspond to the following zones for y: $1 \leq y \leq \infty$, $1/3 \leq y \leq 1$ and $1/5 \leq y \leq 1/3$. So

$$F_Y(y) = P\left(X > \frac{1}{y}\right) = \int_{1/y}^{5} f_X(x)dx = \begin{cases} 0 & y < 0 \\ \frac{3}{8}\left(5 - \frac{1}{y}\right) & \frac{1}{5} \leq y \leq \frac{1}{3} \quad (3 \leq x \leq 5) \\ \frac{3}{4} & \frac{1}{3} \leq y \leq 1 \quad (1 \leq x \leq 3) \\ \frac{3}{4} + \frac{1}{4}\left(1 - \frac{1}{y}\right) & y \geq 1 \quad (0 \leq x \leq 1). \end{cases}$$

(3) Suppose that X and Y are independent. Then $P(X \in A, Y \in B) = P(X \in A)P(Y \in B)$ for all A and B. Thus, $f_{X,Y}(x,y) = P(X = x, Y = y) = P(X = x)P(Y = y) = f_X(x)f_Y(y)$. Conversely, suppose that $f_{X,Y}(x,y) = f_X(x)f_Y(y)$ for all x, y. Then $P(X \in A, Y \in B) = \sum_{x \in A}\sum_{y \in B} f_{X,Y}(x,y) = \sum_{x \in A}\sum_{y \in B} f_X(x)f_Y(y) = \sum_{x \in A} f_X(x)\sum_{y \in B} f_Y(y) = P(X \in A)P(Y \in B)$.

(4) $P(Y = 1) = P(X \in A) = \int_A f_X(x)dx$ and $P(Y = 0) = P(X \notin A) =$

$\int_{A^c} f_X(x)dx$. So,

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \int_{A^c} f_X(x)dx & 0 \leq y < 1 \\ 1 & y \geq 1. \end{cases}$$

(5) Let A be the square with vertices $(z,z),(z,1),(1,1),(1,z),$. Then, $F_Z(z) = P(Z \leq z) = P(\min\{X,Y\} \leq z) = P((X,Y) \in A^c) = 1 - \text{area}(A) = 1 - (1-z)^2$. Therefore, $f_Z(z) = 2(1-z)$ for $0 \leq z \leq 1$.