

Useful Facts

If $0 < r < 1$ then

$$\sum_{j=k}^{\infty} r^j = \frac{r^k}{1-r}.$$

For any real number z ,

$$e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!}$$

If $\lim_n a_n = a$ then

$$\lim_n \left(1 + \frac{a_n}{n}\right)^n \rightarrow e^a.$$

The Cauchy-Schwarz inequality:

$$\{E(XY)\}^2 \leq E(X^2)E(Y^2)$$

Jensen's inequality. If g is convex then

$$Eg(X) \geq g(EX)$$

Markov's Inequality: If $X > 0$ and $t > 0$ then

$$P(X > t) \leq \frac{E(X)}{t}$$

Hoeffding's inequality If $X_1, \dots, X_n \sim \text{Bernoulli}(p)$ then

$$P(|\hat{p} - p| > \epsilon) \leq 2e^{-2n\epsilon^2}$$

Binomial Distribution

$$f(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

for $x = 0, \dots, n$. $E(X) = np$, $V(X) = np(1-p)$. Moment generating function:

$$\psi(t) = (pe^t + (1-p))^n.$$

The **Bernoulli Distribution** is the Binomial distribution with $n = 1$.

Hypergeometric Distribution

$$f(x; A, B, n) = \frac{\binom{A}{x} \binom{B}{n-x}}{\binom{A+B}{n}}$$

for $\max\{0, n-B\} \leq x \leq \min\{n, A\}$.

$E(X) = np$ where $p = A/(A+B)$. $V(X) = np(1-p)(N-n)/(N-1)$ where $N = A+B$.

Poisson Distribution

$$f(x; \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}$$

for $x = 0, 1, \dots$

$$E(X) = V(X) = \lambda.$$

$$\psi(t) = e^{\lambda(e^t - 1)}$$

Negative Binomial

$$f(x; r, p) = \binom{r+x-1}{x} p^r (1-p)^x$$

for $x = 0, 1, \dots$

$$E(X) = r(1-p)/p. \quad V(X) = r(1-p)/p^2.$$

$$\psi(t) = \left(\frac{p}{1 - (1-p)e^t} \right)^r$$

for $t < \log(1/(1-p))$.

The **Geometric** is the negative binomial with $r = 1$.

Normal

$$f(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2} \right\}$$

for $-\infty < x < \infty$.

$$E(X) = \mu. \quad V(X) = \sigma^2.$$

$$\psi(t) = e^{\mu t + \frac{1}{2} \sigma^2 t^2}$$

Gamma Distribution

$$f(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

for $x > 0$ where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$.

$$E(X) = \alpha/\beta. \quad V(X) = \alpha/\beta^2.$$

$$\psi(t) = \left(\frac{\beta}{\beta - t} \right)^\alpha$$

for $t < \beta$.

The **Exponential** distribution is a Gamma $(1, \beta)$.

Beta Distribution

$$f(x; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$$

for $x \in (0, 1)$. $E(X) = \alpha/(\alpha + \beta)$.

$$V(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}.$$

$$\psi(t) = 1 + \sum_{i=1}^{\infty} \frac{t^i}{i!} \left(\prod_{r=0}^{i-1} \frac{\alpha + r}{\alpha + \beta + r} \right).$$

Table for Standard Normal

Standard Score z	Pr(Z<z)	Standard Score z	Pr(Z<z)
-3.01	0.00	0.00	0.50
-2.58	0.00	0.03	0.51
-2.33	0.01	0.05	0.52
-2.05	0.02	0.08	0.53
-1.96	0.02	0.10	0.54
-1.88	0.03	0.13	0.55
-1.75	0.04	0.15	0.56
-1.64	0.05	0.18	0.57
-1.55	0.06	0.20	0.58
-1.48	0.07	0.23	0.59
-1.41	0.08	0.25	0.60
-1.34	0.09	0.28	0.61
-1.28	0.10	0.31	0.62
-1.23	0.11	0.33	0.63
-1.17	0.12	0.36	0.64
-1.13	0.13	0.39	0.65
-1.08	0.14	0.41	0.66
-1.04	0.15	0.44	0.67
-0.99	0.16	0.47	0.68
-0.95	0.17	0.50	0.69
-0.92	0.18	0.52	0.70
-0.88	0.19	0.55	0.71
-0.84	0.20	0.58	0.72
-0.81	0.21	0.61	0.73
-0.77	0.22	0.64	0.74
-0.74	0.23	0.67	0.75
-0.71	0.24	0.71	0.76
-0.67	0.25	0.74	0.77
-0.64	0.26	0.77	0.78
-0.61	0.27	0.81	0.79
-0.58	0.28	0.84	0.80
-0.55	0.29	0.88	0.81
-0.52	0.30	0.92	0.82
-0.50	0.31	0.95	0.83
-0.47	0.32	0.99	0.84
-0.44	0.33	1.04	0.85
-0.41	0.34	1.08	0.86
-0.39	0.35	1.13	0.87
-0.36	0.36	1.17	0.88
-0.33	0.37	1.23	0.89
-0.31	0.38	1.28	0.90
-0.28	0.39	1.34	0.91
-0.25	0.40	1.41	0.92
-0.23	0.41	1.48	0.93
-0.20	0.42	1.55	0.94
-0.18	0.43	1.64	0.95
-0.15	0.44	1.75	0.96
-0.13	0.45	1.88	0.97
-0.10	0.46	1.96	0.98
-0.08	0.47	2.05	0.98
-0.05	0.48	2.33	0.99
-0.03	0.49	2.58	1.00
		3.01	1.00
		3.72	1.00

χ^2 Distribution

Let $X \sim \chi_n^2$. The table gives x such that $P(X \leq x) = p$.

n	.90	.95	.975	.99
1	2.706	3.841	5.024	6.635
2	4.605	5.991	7.378	9.210
3	6.251	7.815	9.348	11.345
4	7.779	9.488	11.143	13.277
5	9.236	11.070	12.833	15.086
6	10.645	12.592	14.449	16.812
7	12.017	14.067	16.013	18.475
8	13.362	15.507	17.535	20.090
9	14.684	16.919	19.023	21.666
10	15.987	18.307	20.483	23.209
11	17.275	19.675	21.920	24.725
12	18.549	21.026	23.337	26.217
13	19.812	22.362	24.736	27.688
14	21.064	23.685	26.119	29.141
15	22.307	24.996	27.488	30.578
16	23.542	26.296	28.845	32.000
17	24.769	27.587	30.191	33.409
18	25.989	28.869	31.526	34.805
19	27.204	30.144	32.852	36.191
20	28.412	31.410	34.170	37.566
21	29.615	32.671	35.479	38.932
22	30.813	33.924	36.781	40.289
23	32.007	35.172	38.076	41.638
24	33.196	36.415	39.364	42.980
25	34.382	37.652	40.646	44.314