36-325/725
Homework 10
due Thursday November 21

(1) Let $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$. We want to test that the median is μ_0, that is, $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$.

(a) Find the Wald test.

(b) Explain how to do a nonparametric Wald test using the plug-in estimate and the bootstrap.

(c) Find the likelihood ratio test.

(d) Here is an exact (small-sample) nonparametric test. Let

$$S_i = \begin{cases} 1 & \text{if } X_i - \mu_0 > 0 \\ -1 & \text{if } X_i - \mu_0 \leq 0. \end{cases}$$

Let $Y \sim \text{Binomial}(n, 1/2)$. Let

$$c = \min\left\{ j : \ \mathbb{P}\left(\left| Y - \frac{n}{2} \right| > j \right) \leq \alpha \right\}.$$

Suppose we reject H_0 when $|T| > c$ where

$$T = \sum_{i=1}^{n} I(S_i = 1) - \frac{n}{2}.$$

Show that this test has type I error that is less than or equal to α. This is called the sign test.

(e) Let us compare these tests. Let $n = 25$, $\mu_0 = 0$ and $\alpha = 0.05$. First, simulate the test under H_0. Do the following steps:

(i) Simulate $X_1, \ldots, X_n \sim N(0, 1)$.

(ii) Compute each test statistic and make the reject/retain decision.

(iii) Repeat steps (i) amd (ii) 1,000 times and see how often you reject. Compare this to the nominal size α.

(f) To check the power of the test, repeat part(e) but simulate from a $N(2, 1)$. Summarize your findings.

(g) Let's check how robust the tests are to the assumption of Normality. Repeat (e) but simulate from a Cauchy distribution. Summarize your findings.

(2) Let $X \sim \text{Multinomial}(n, p)$ and consider testing

$$H_0 : (p_1, \ldots, p_k) = (p_{10}, \ldots, p_{k0}) \quad \text{versus} \quad H_1 : (p_1, \ldots, p_k) \neq (p_{10}, \ldots, p_{k0}).$$

(a) Show that the likelihood ratio test is to reject when $T > \chi^2_{k-1,\alpha}$ where

$$T = 2 \sum_{j=1}^{k} X_j \log \frac{X_j}{np_{0j}}.$$

(b) Analyze Mendel's data (example 11.16) using a χ^2 test and using a likelihood ratio test. Report the test statistic and p-value in each case.

(3) Repeat chapter 11 exercise 8. Treat this as a problem of comparing two multinomials. Find the likelihood ratio test and then use it to analyze the data.

(4) Repeat chapter 11 exercise 9b using FDR.

(5) Chapter 12, exercise 1.

(6) Chapter 12, exercise 2. (Part (e) should read, find a 95 per cent interval for μ.)