(1a) $f(x) = 1/3$ for $0 < x < 3$.
(1b) The CDF is

$$F(x) = \begin{cases} 0 & x < 0 \\ x/3 & 0 \le x \le 3 \\ 1 & x > 3. \end{cases}$$

(1c) The CDF is

$$F(x) = \begin{cases} 0 & x < 1 \\ 1/3 & 1 \le x < 2 \\ 2/3 & 2 \le x < 3 \\ 1 & x \ge 3. \end{cases}$$

(1d) For $0 \le z \le 9$,

$$F_Z(z) = \mathbb{P}(Z \le z) = \mathbb{P}(X^2 \le z) = \mathbb{P}(X \le \sqrt{z}) = \frac{\sqrt{z}}{3}.$$

Therefore,

$$f_Z(z) = \frac{1}{6\sqrt{z}}, \quad 0 < z < 9.$$

(2) First nore that $f(x, y) = 1/2$ for (x, y) in the square and 0 otherwise. For $-1 < z < 1$ let A_z be the square with vertices $(-1, 0), (0, -1), ((1 + z)/2, (z - 1)/2), ((z - 1)/2, (z + 1)/2)$. Then

$$F_Z(z) = \mathbb{P}(Z \le z) = \text{area}(A_z) = \frac{z+1}{2}$$

and hence $f_Z(z) = 1/2$ for $-1 \le z \le 1$ that is, $Z \sim \text{Uniform}(-1, 1)$.

(3)

$$\begin{aligned} \mathbb{P}\Big(X > \sqrt{Y}\Big) &= \mathbb{P}\Big(X^2 > Y\Big) \\ &= \mathbb{P}\Big(Y < X^2\Big) \\ &= \int_0^1 \left(\int_0^{x^2} f(x, y)\, dy \right) dx \\ &= \int_0^1 \left(x^3 + \frac{x^4}{2} \right) dx \\ &= \frac{7}{20}. \end{aligned}$$

(4)

$$\begin{aligned}\mathbb{E}\Big(\mathsf{Cov}(X,Y|Z)\Big) &= \mathbb{E}\Big(\mathbb{E}(XY|Z) - \mathbb{E}(X|Z)\mathbb{E}(Y|Z)\Big)\\ &= \mathbb{E}\Big(\mathbb{E}(XY|Z)\Big) - \mathbb{E}\Big(\mathbb{E}(X|Z)\mathbb{E}(Y|Z)\Big)\\ &= \mathbb{E}(XY) - \mathbb{E}\Big(\mathbb{E}(X|Z)\mathbb{E}(Y|Z)\Big).\end{aligned}$$

Let $S = \mathbb{E}(X|Z)$ and $T = \mathbb{E}(Y|Z)$. Then,

$$\begin{aligned}\mathsf{Cov}\Big(\mathbb{E}(X|Z),\mathbb{E}(Y|Z)\Big) &= \mathsf{Cov}\Big(S,T\Big)\\ &= \mathbb{E}(ST) - \mathbb{E}(S)\mathbb{E}(T)\\ &= \mathbb{E}\Big(\mathbb{E}(X|Z)\mathbb{E}(Y|Z)\Big) - \mathbb{E}\Big(\mathbb{E}(X|Z)\Big)\mathbb{E}\Big(\mathbb{E}(Y|Z)\Big)\\ &= \mathbb{E}\Big(\mathbb{E}(X|Z)\mathbb{E}(Y|Z)\Big) - \mathbb{E}(X)\mathbb{E}(Y).\end{aligned}$$

Hence,

$$\mathbb{E}\Big(\mathsf{Cov}(X,Y|Z)\Big)+\mathsf{Cov}\Big(\mathbb{E}(X|Z),\mathbb{E}(Y|Z)\Big) = \mathbb{E}(XY)-\mathbb{E}\Big(\mathbb{E}(X|Z)\mathbb{E}(Y|Z)\Big)++\mathbb{E}\Big(\mathbb{E}(X|Z)\mathbb{E}(Y|Z)\Big)$$