36-401/607 Section B (Wasserman), Fall 2017, HOMEWORK 1

Due Friday, September 8 at 3:00

To receive full credit, you must show all steps in your work.

1. Suppose that $X_1, X_2, \ldots, X_m$ are discrete random variables with probability function $p(x_1, \ldots, x_m)$. Let $a_1, \ldots, a_m$ be constants. Show that

$$\mathbb{E}\left[\sum_{j=1}^{m} a_j X_j\right] = \sum_{j=1}^{m} a_j \mathbb{E}[X_j].$$

2. Let X be a discrete random variable such that $X \in \{1, 2, \ldots\}$. Show that $\mathbb{E}[X] = \sum_j P(X \geq j)$.

3. A random variable X is *degenerate* if there exists a number a such that $P(X = a) = 1$.

   (a) Show that, if X is degenerate, then $\mathrm{Var}(X) = 0$.

   (b) Let X be discrete. Suppose that $\mathrm{Var}(X) = 0$. Show that X is degenerate.

4. Let X and Y be random variables. Show that

$$\mathrm{Cov}(a + bX, c + dY) = bd\,\mathrm{Cov}(X, Y).$$

5. Let

$$Y = 5X + \epsilon$$

   where $\epsilon \sim N(0, 1)$ and $X \sim \mathrm{Unif}(-1, 1)$. Assume that X and ϵ are independent.

   (a) Find the mean and variance of Y.

   (b) Find $\mathbb{E}[Y^2]$.

   (c) Find $\mathbb{E}[Y|X = x]$.

   (d) Find $\mathbb{E}[Y^3]$.

   (e) Find $\mathrm{Cov}(\epsilon, \epsilon^2)$. Are ϵ and ϵ^2 independent?