# Homework 7

1. Download these data:

   `http://stat.cmu.edu/~larry/=stat401/sports.txt`

   These data are from the R package `alr4` and you can find a complete description of the data in that package. The dataset is called `ais` in that package. These are data on 202 athletes. The goal is to predict lean body mass (LBM) from the other variables. For this question, you should ignore the variables `Label` and Sport.

   (a) Use the `pairs` command to plot the data. Comment on any patterns that you see.

   (b) Use a linear regression model to predict `LBM` from Sex, Ht, Wt, RCC, WCC, Hc, Hg, Ferr, BMI, Bfat. Comment on your residual plots.

   (c) Summarize your fitted model.

   (d) Find the eigenvalues of the design matrix.

   (e) Construct a 90 percent confidence rectangle for all the coefficients in the model (except the intercept).

   (f) Let's try a smaller model. Fit a linear regression to predict LBM from Sex, Ht, Wt and RCC. Summarize the fitted model.

   (g) Construct and plot a 95 percent confidence ellipsoid for Ht and Wt.

   (h) Construct an F test to compare the two models that you fit. Summarize and interpret the result of the test.

2. Let $\mathbf{X}$ denote the design matrix for some regression problem. Let $v_j$ denote the $j^{\text{th}}$ column of $\mathbf{X}$. In other words $\mathbf{X} = [v_1 \; v_2 \; \cdots \; v_q]$. Suppose that the coluns are orthogonal. In other words, $v_j^T v_k = 0$ when $j \neq k$.

   (a) Show that $\mathbf{X}^T\mathbf{X}$ is non-singular (invertible) as long as $||v_j|| > 0$ for all $j$.

   (b) Find an explicit expression for $(\mathbf{X}^T\mathbf{X})^{-1}$.

   (c) Now suppose we replace one of the columns of $\mathbf{X}$ with a vector of all zeroes. For example, we set: $v_1 = (0, 0, \ldots, 0)$. Prove or disprove the following statement: the least squares solution is still well-defined (in other words, it exists and is unique).

3. Consider the ridge regression estimator $\widehat{\beta}_\lambda$.

   (a) Show that $\widehat{\beta}_\lambda \to 0$ as $\lambda \to \infty$.

   (b) Consider the following modified ridge estimator:

   $$\widehat{\beta}_\lambda = \lambda(\mathbf{X}^T\mathbf{X} + \lambda I)^{-1}\mathbf{X}^T\mathbf{Y}.$$

   What does this estimator converge to as $\lambda \to \infty$?