Homework 10
36-705
Due: Thursday November 5 by 3pm.

1. Let $X_1, \ldots, X_n \sim$ Bernoulli(p) and $Y_1, \ldots, Y_m \sim$ Bernoulli(q). Construct an asymptotic $1 - \alpha$ confidence interval for $\psi = \log(p/q)$.

2. Let $X_1, \ldots, X_n \sim P$ and let $\psi(P) = \int a(x)dP(x)$ for some (known) function $a(x)$. Compute the influence function. Use this to find an asymptotic $1 - \alpha$ confidence interval for ψ.

3. Let $X_1, \ldots, X_n \sim P$ and let $\psi(P) = [\int a(x)dP(x)]^2$ for some (known) function $a(x)$. Compute the influence function. Use this to find an asymptotic $1 - \alpha$ confidence interval for ψ.

4. Let $X_1, \ldots, X_n \sim P$ and assume that P has a strictly positive density p. Let ψ be the median of P. Find the influence function for ψ.

5. Let $X_1, \ldots, X_n \sim P$ and let $\psi(P) = \int a(x)dP(x)$ for some (known) function $a(x)$. Let $X_1^*, \ldots, X_n^*$ denote a bootstrap sample from the empirical distribution. Let $\widehat{\psi}^* = n^{-1} \sum_i a(X_i^*)$.

   (a) Find
   $$\mathbb{E}[\widehat{\psi}^* | X_1, \ldots, X_n]$$
   and
   $$\mathrm{Var}[\widehat{\psi}^* | X_1, \ldots, X_n].$$

   (b) Find $\mathbb{E}[\widehat{\psi}^*]$ and $\mathrm{Var}[\widehat{\psi}^*]$.