Homework 6

36-705

Due: Thursday October 8st by 3pm.

1. A geometric distribution is the distribution of the number of coin flips needed to see one head. Suppose that $X_1, \ldots, X_n \sim \text{Geom}(p)$, i.e. the samples have a geometric distribution with parameter p.

   (a) Find a minimal sufficient statistic.

   (b) Show that this is an exponential family.

   (c) Compute the MLE.

2. Let $X_1, \ldots, X_n \sim \text{Gamma}(\alpha, \beta)$. Compute the method of moments estimators for α, β.

3. Let $X_1, \ldots, X_n \sim \text{Unif}[a, b]$.

   (a) Find a minimal sufficient statistic.

   (b) Is this an exponential family?

   (c) Compute the MLE for a and b.

4. Let $X_1, \ldots, X_n \sim \text{Poisson}(\lambda)$.

   (a) Find the method of moments estimator, the maximum likelihood estimator and the Fisher information $I(\lambda)$.

   (b) Use the fact that the mean and variance of the Poisson distribution are both λ to propose two *unbiased* estimators of λ. Show that one of these estimators has a larger variance than the other.

5. Let $X_1, \ldots, X_n$ be a random sample from a distribution with density:

$$p(x; \theta) = \theta x^{-2}, \quad 0 < \theta \le x < \infty.$$

   (a) Find the MLE for θ.

   (b) Find the Method of Moments estimator for θ.