Homework 8

36-705

Due: Thursday October 22 by 3pm.

1. Let $X_1, ..., X_n \sim N(\mu, \sigma^2)$.

- (a) Find the Fisher information matrix.
- (b) Find the limiting distribution of the mle.
- (c) Find the mle of $\psi = \mu/\sigma$ and find its limiting distribution.
- 2. Let $X \sim Bin(n, p)$. We want to test

$$H_0: p = p_0$$
 verus $H_1: p \neq p_0.$

- (a) Find the Wald test statistic and the critical value. (i.e. when do we reject H_0 ?)
- (b) Find the likelihood ratio test and the critical value.
- 3. Let $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$. We want to test

$$H_0: \mu = \mu_0$$
 verus $H_1: \mu \neq \mu_0.$

- (a) Find the Wald test statistic and the critical value. (i.e. when do we reject H_0 ?)
- (b) Find the likelihood ratio test and the critical value.
- 4. Another popular test is the score test. Let $X_1, \ldots, X_n \sim p(x; \theta)$. Suppose that $\theta \in \mathbb{R}$. We want to test

$$H_0: \theta = \theta_0$$
 verus $H_1: \theta \neq \theta_0$.

Let

$$S_n(\theta) = \sum_i \frac{\partial \log p(X_i; \theta)}{\partial \theta}$$

be the score function. We will use $S_n(\theta_0)$ as a test statistic. Find the limiting distribution of $S_n(\theta_0)$ under H_0 . When do we reject H_0 ? Note that, to do this test, we never need to estimate θ .