which has maximum conditional power function for $\theta_1 > \theta_1^0$ and minimum conditional power function for $\theta_1 < \theta_1^0$ subject to the power being $\beta_\psi(\theta_1^0)$ at $\theta_1 = \theta_1^0$. Since ϕ in (4.129) minimizes and maximizes the power in precisely the right places uniformly in u among all tests in a class that contains ψ, it follows that $R(\theta, \phi) \le R(\theta\psi)$ for all θ if a hypothesis-testing loss function is being used. It follows that tests of the form (4.129) form an essentially complete class. Other hypotheses can be handled in a similar way.

4.5.3 Linear Combinations of Natural Parameters

Most of the popular tests in the theory of normal distributions and linear models involve linear combinations of the natural parameters of an exponential family. In a k-parameter exponential family with natural parameter Θ, let $\Psi_1 = \sum_{i=1}^k c_i\Theta_i$ with $c_1 \neq 0$. Let $\Psi_i = \Theta_i$ for $i = 2, \ldots, k$, and set $Y_1 = X_1/c_1$ and $Y_i = X_i - c_iX_1/c_1$ for $i = 2, \ldots, k$. Then $Y = (Y_1, \ldots, Y_k)$ has exponential family distribution with natural parameter Ψ. If we want to test a hypothesis concerning Ψ_1, we can proceed as above.

Example 4.130 (Continuation of Example 4.127; see page 270). The natural parameters are $\Theta_1 = \mathrm{M}/\Sigma^2$ and $\Theta_2 = -1/[2\Sigma^2]$. Testing $\mathrm{M} = \mu_0$ is then equivalent to testing $\Theta_1 + 2\mu_0\Theta_2 = 0$. So, we set $\Psi_1 = \Theta_1 + 2\mu_0\Theta_2$ and the usual two-sided t-test will result exactly as in Example 4.127.

Example 4.131. Suppose that X and Y are conditionally independent given $(\Lambda, \mathrm{M}) = (\lambda, \mu)$ with $X \sim Poi(\lambda)$ and $Y \sim Poi(\mu)$. Suppose that $H : \Lambda = \mathrm{M}$. In natural parameter form, $\Theta_1 = \log\Lambda$ and $\Theta_2 = \log\mathrm{M}$. So $H : \Theta_1 - \Theta_2 = 0$. Set $\Psi_1 = \Theta_1 - \Theta_2$ and $\Psi_2 = \Theta_2$. Let $Z_1 = X$ and $Z_2 = X + Y$. Then (Z_1, Z_2) has exponential family distribution with natural parameter Ψ. We need the conditional distribution of Z_1 given Z_2. For $z_1 = 0, \ldots, z_2$, and $z_2 = 0, 1 \ldots,$ we have

$$
\begin{aligned}
f_{Z_1,Z_2|\Psi}(z_1, z_2|\psi) &= c(\psi)\frac{1}{z_1!(z_2 - z_1)!}\exp(z_1\psi_1 + z_2\psi_2), \\
f_{Z_2|\Psi}(z_2|\psi) &= d(\psi)\frac{1}{z_2!}\exp(\psi_2 z_2)[1 + \exp(\psi_1)]^{z_2}, \\
f_{Z_1|Z_2,\Psi}(z_1|z_2, \psi) &= r(\psi)k(z_1, z_2)\exp(z_1\psi_1)[1 + \exp(\psi_1)]^{-z_2} \\
&= \binom{z_2}{z_1}\left(\frac{\lambda}{\lambda+\mu}\right)^{z_1}\left(\frac{\mu}{\lambda+\mu}\right)^{z_2 - z_1},
\end{aligned}
$$

for $z_1 = 0, \ldots, z_2$. So Z_1 given $Z_2 = z_2$ and $(\Theta_1, \Theta_2) = (\log\lambda, \log\mu)$ has $Bin(z_2, \lambda/(\lambda + \mu))$ distribution. The UMPU level α test for the binomial parameter equal to $1/2$, conditional on Z_2 is the UMPU level α test of $\Lambda = \mathrm{M}$.

4.5.4 Other Two-Sided Cases*

In Theorem 4.124, we saw how to deal with cases in which the hypothesis or alternative is two-sided in a natural parameter. But other two-sided cases

*This section may be skipped without interrupting the flow of ideas.