Quiz 4(Wednesday June 12th)

AndrewID: Name:

Total: 125 points. Full score: 100 points.

1. (25 pts) Let $X_1 \sim \text{Bernoulli}(p)$ and $X_2 \sim \text{Bernoulli}(1 - X_1)$. Compute $E[X_2]$.

2. (25 pts) Prove that

$$p_X(x) = \begin{cases} \frac{1}{100} \text{ if } x \in \{0, 1/100, 2/100, \ldots, 99/100\} \\ 0 \text{ o/w} \end{cases}$$

   is a proper probability mass function.

3. (25 pts) Find the pmf of Y where $Y = X^2$ with X being a r.v. with pmf

$$p_X(x) = \begin{cases} \frac{1}{5} \text{ if } x \in \{-2, -1, 0, 1, 2\} \\ 0 \text{ o/w} \end{cases}$$

4. (25 pts) Let $X_1, X_2 \overset{iid}{\sim} \text{Poisson}(\lambda)$. Compute

$$E[X_1 X_2 | X_2 = 5].$$

5. (25 pts) Let $X_1, X_2 \overset{iid}{\sim} \text{Poisson}(\lambda)$. Let the realizations of these r.v.'s be $X_1 = x_1, X_2 = x_2$. Find the MLE for λ given $\mathbf{x} = (x_1, x_2)$.